Nonlinear Electron Cyclotron Current Drive with 2\textsuperscript{nd} Harmonic X-Mode *

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1. Introduction

The linear theory of wave absorption and the quasilinear theory of the evolution of the distribution function are presently the main tools for a quantitative description of ECRH and ECCD in fusion devices. However, the applicability of these theories is violated for some ECRH (ECCD) scenarios in typical experimental conditions. In particular, this is true for one of the basic scenarios where the 2\textsuperscript{nd} harmonic electron cyclotron resonance for the extraordinary mode (X-mode) is used. Formally, the quasilinear description of wave-particle interaction remains valid if the particle flight time through the radiation beam, $t_f$, is small compared to the oscillation period of a particle trapped in the wave, $t_{bE}$. This condition is satisfied only for particles with relatively large parallel velocities as compared to the thermal velocity. On the other hand, in the opposite limit case, $t_f \gg t_{bE}$, whenever typical experimental conditions are considered, the adiabatic theory which is only applicable in the region of phase space with small parallel velocities does not give correct quantitative results for both the particle distribution function and the absorbed power as long as calculations of these quantities are done without taking into account the combined effect of wave-particle interactions and collisional processes.

Since the domain of phase space which is neither covered by the (quasi)linear nor by the adiabatic approximation occupies a significant part of this space, the problem of wave-particle interaction has to be treated numerically.

2. Formulation of the problem

During ECCD (ECRH), the electron distribution function is determined by resonant wave-particle interaction processes which take place in the small power deposition region as well as by the effects of particle drift motion and Coulomb collisions in the main plasma volume. In the present study, the main interest is in the development of a proper model of wave-particle interaction, which in turn can be used for particular cases of magnetic field geometry. The influence of the device geometry in its full extent can be handled by the stochastic mapping technique [1] (SMT). Also in the

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present report, a formulation of equations in terms of the conservation of the particle flux density through Poincaré cuts in phase space, adopted in the SMT approach is used (see Ref. [2]). Thus, a simplified geometry is assumed which should qualitatively represent typical experimental conditions. Within this model, the uniform main magnetic field is directed along the Z-axis and a narrow Gaussian radiation beam propagates across the main magnetic field in the XZ-plane. The system is periodic in the Z-direction with the period $L$. Such a simple model geometry is representative, e.g., for the magnetic axis of a tokamak. With more proximity, it can be used also for the case of tokamak mid-plane heating with the resonance located at the high field side and with safety factor values distant from low order rationals. The effects of trapped particles are discarded in this case.

The particle motion in the wave electric field $\mathbf{E} = E_0 \text{Re} \left[ f F(r)e^{i(kr-\omega t)} \right]$ is described by the Hamiltonian

$$\hat{H} = \Omega w - \frac{1}{2} w^2 + \varepsilon w e^{-r^2} \cos \psi, \quad \Omega = \sqrt{2L_b} \left[ \frac{v_{\parallel}}{v_{\parallel}} \right] \left( k_{\parallel} v_{\parallel} - n_0 \omega_{e0} \left( 1 - \frac{v_{\parallel}^2}{2 \varepsilon^2} \right) - \omega \right), \quad (1)$$

$$\varepsilon = \sqrt{2L_b} \omega_E \frac{v_E}{|v_{\parallel}|}, \quad \omega_E = \frac{v_E}{|J|} \left| \frac{2m_0 \omega_{e0} n_0 \tau}{(n_0 - 1)!} \right| \left( \frac{k_{\perp}}{2m_0 \omega_{e0}} \right)^{n_0 - 1}. \quad (2)$$

Here, $E_0 = \text{const}$, $f \equiv \mathbf{E}/|\mathbf{E}| = \text{const}$ and $F(r) = F(z) = \exp \left( -\alpha(z^2)/2 \right)$ are the wave amplitude, the polarization vector and the parallel form factor for the Gaussian beam shape, respectively. In equations (1) and (2) $\tau = |v_{\parallel}|/|J|$, $w$, $\psi$, $L_b$, $\omega$, $c$ are the dimensionless time, the dimensionless particle perpendicular energy, the wave-particle phase, the beam width, the wave frequency and the speed of light, respectively. The number of the cyclotron harmonic is denoted by $n_0$ (here $n_0 = 2$ is considered), $v_E \equiv eE_0/(m_0 \omega)$, $m_0$ is the particle mass, $\omega_{e0}$ is the gyrofrequency at rest and $e$ is the electron charge.

For the purpose of numerical computations of the distribution function, cuts are introduced at positions $z = \text{const}$ (see Ref. [2]). These cuts are placed at each side of the beam so that the relation $L_b \ll \delta L \ll L$ is realized. Here, $\delta L$ is the distance between the cuts. The kinetic equation is reduced to a set of relations which map the pseudo-scalar particle flux densities through the neighboring cuts, $J = |v_{\parallel}|/|J|$, where $J = v_{\perp}$ and $f$ are the phase space Jacobian and the distribution function, respectively. Combined together using the periodicity of the problem, the relations which map the flux through the wave beam and through the outer region form an integral equation (see Ref. [2]), which is then solved using a Monte Carlo method.

### 3. Computation results

In Fig. 1 a typical particle distribution in the nonlinear case is shown for each of both cuts at the side of the beam. This distribution function is asymmetric and a plateau-like structure is formed around the resonance zone. With such a distribution function, the distribution of the field line integrated absorbed power in velocity space is computed (Fig. 2, left). The distribution of the current density in velocity space (Fig. 2 right), $e(\Gamma(v_{\parallel}) - \Gamma(-v_{\parallel}))$ is computed in the same way and shows that the current is generated by particles above the resonance line (black line in Fig. 2 and 3). The left picture in Fig. 3 shows that most of the current, $j_{\parallel}$, is generated mainly
by suprathermal particles. Moreover, the current density distribution over the pitch angle $\chi$ (Fig. 3) indicates that the highest current is produced by particles with pitch angles bigger than 45. Those particles interact with the wave in the nonlinear regime. All the results mentioned above where obtained for $N_{||} = 0.3$. In Fig. 4 the dependence of the driven current (left) and of the current drive efficiency (right), $\eta$, on the launching angle are compared between the linear, the quasilinear and the nonlinear model. For this computation the following quasilinear diffusion coefficient has been used

$$D_{QL}^{v_{\perp}, v_{\parallel}} = \frac{\pi e^2 E_0^2 |f^-|^2 k_{\perp}^2 v_{\perp}^2 L_{b}^2}{8 \omega^2 m_0^2 v_{\parallel}^2 \tau_b} \exp \left( - \frac{L_{b}^2 (\omega - k_{\parallel} v_{\parallel} - 2 |\omega_c|)^2}{v_{\parallel}^2} \right),$$

where $\tau_b = L_{b} / v_{\parallel}$, $k_{\perp}$ and $f^-$ are the perpendicular component of the wave vector and the left hand polarized component of the polarization vector, respectively.

4. Conclusions

A numerical model for ECCD which consistently takes into account nonlinear wave-particle interaction has been developed. The modelling of ECCD at the 2nd harmonic X-mode resonance shows that results for the current density are different for the linear, the quasilinear and the nonlinear models. The linear and the nonlinear models give close values of the current drive efficiency. This is connected with the fact that the distributions of the absorbed power along the resonance line in this two models are closer to each other as compared with the quasilinear case. However, the dependence of the absorbed power on the launching angle is different for the linear model and the nonlinear model and, consequently, the parallel driven current is also different. Therefore, for ECCD the absorption should be taken into account correctly by using nonlinear computations. It should be mentioned that the noise in Fig. 3 due to poor statistics in the tale of the distribution function where the main part of the current is generated can be resolved using the weight-windows algorithm [3], that will be realized in future computations.

References


Figure 1: Distribution function at the cut first crossed by particles with $v_\parallel > 0$ (left) and at cut first crossed by particles with $v_\parallel < 0$ (right). Here $N_\parallel = 0, 3$.

Figure 2: Distribution in velocity space of the absorbed power (left) and of the parallel current density (right).

Figure 3: Distribution of the parallel current density over $v_\parallel$ (left) and distribution of the same quantity over the pitch angle (right).

Figure 4: Parallel current density (left) and current drive efficiency (right) versus the launching angle.