29th EPS Conference on Plasma Phys. and Contr. Fusion Montreux, 17-21 June 2002 ECA Vol. **26B**, P-4.069 (2002)

Stabilization of ITG Modes and Destabilization of Trapped Particle Modes with $E \times B$ Flows

L. Villard, A. Bottino, S. Allfrey, and O. Sauter

Centre de Recherches en Physique des Plasmas
Association Euratom - Confédération Suisse
EPFL, 1015 Lausanne, Switzerland

Abstract. The development of radial electric fields (E_r) is an important mechanism leading to the formation of transport barriers through turbulence suppression when the $E \times B$ shearing rate exceeds the turbulence inverse correlation time γ_t [1]. Often a simplified stabilization criterion is used in the case of applied (equilibrium) E_r, replacing γ_t with the linear growth rate without flow γ_0, while the theory was established in the context of turbulence self-generated E_r (zonal flows) which are fluctuating quantities. In this paper, the applicability of this simplified criterion is examined for a variety of cases. Three geometries are considered: axisymmetric, helical and cylindrical. Various radial profiles of the shearing rate are studied. Full radius linear gyrokinetic simulations yield the following results: (a) the toroidal-ITG, slab-ITG and helical-ITG growth rates have a quadratic dependence on the shearing rate; (b) these modes are fully stabilized when the shearing rate is comparable to the linear growth rate, within a factor of about 2; (c) the critical gradient for marginal stability increases quadratically with the value of the shearing rate; (d) applied radial electric fields can be destabilizing when the dominant drive is from trapped ions or when the trapped electron dynamics are taken into account.

1. Global linear gyrokinetic model

We consider low β magnetic configurations with axisymmetry or helical symmetry, with given profiles of $q(\psi)$, $n_0(\psi)$, $T_e(\psi)$, $T_i(\psi)$ and applied electrostatic potentiel $\Phi_0(\psi)$, where ψ is the poloidal or helical flux. Electrostatic perturbations are assumed to follow the usual gyrokinetic ordering $\omega/\Omega \sim k_{||}/k_\perp \sim e\delta\phi/T_e \sim \rho_L/L_n \sim \rho_L/L_T \sim \mathcal{O}(\epsilon_g)$ and $\rho_L/L_B \sim \mathcal{O}(\epsilon_B)$. With $f = f_0 + \delta f$ and $\phi = \Phi_0 + \delta\phi$, the linearized equations are

$$
\begin{aligned}
\frac{d\mathbf{R}}{dt} &= v_{||}\mathbf{e}_{||} + \mathbf{v}_d + \mathbf{v}_E \ , \quad \frac{dv_{||}}{dt} = \frac{1}{2}v_\perp^2 \nabla\cdot\mathbf{e}_{||} \ , \quad \frac{dv_\perp}{dt} = -\frac{1}{2}v_\perp v_{||}\nabla\cdot\mathbf{e}_{||} \ , \\
\mathbf{v}_E &= \frac{\mathbf{e}_{||}\times\nabla\psi}{B}\,\frac{d\Phi_0}{d\psi} \ , \quad \mathbf{v}_d = \frac{v_{||}^2}{\Omega}\mathbf{e}_{||}\times\Big((\nabla\times\mathbf{e}_{||})\times\mathbf{e}_{||}\Big) + \frac{v_\perp^2}{2\Omega}\mathbf{e}_{||}\times\nabla\ln B \ , \\
\frac{d\,\delta f}{dt} &= \frac{<\nabla\delta\phi>}{B} \\
&\cdot \left[\mathbf{e}_{||}\times\left(\nabla\psi\frac{\partial f_0}{\partial\psi} - \frac{1}{2}\nabla\ln B\ v_\perp\frac{\partial f_0}{\partial v_\perp} - \mathbf{e}_{||}\times\nabla\times\mathbf{e}_{||}\ v_{||}\frac{\partial f_0}{\partial v_{||}}\right) + \Omega\frac{\partial f_0}{\partial v_{||}}\mathbf{e}_{||}\right] \ , \quad (1)
\end{aligned}
$$

where the brackets <> indicate Larmor averaging. We consider small Mach numbers of v_E and have neglected terms of order $(v_E/v_{thi})^2$ and $\epsilon_B(v_E/v_{thi})$. The system is closed with the quasi-neutrality equation. Two models for electrons are considered: either all electrons are adiabatic or trapped electrons are drift-kinetic and only passing electrons are adiabatic. The equations are solved with a finite element, PIC, full radius code in magnetic coordinates [2].

The shearing rate of the applied E_r field is [1]

$$\omega_{E\times B} = (1/q)(sd\psi/ds)d^2\Phi_0/d\psi^2 \approx (\rho/q)(\partial/\partial\rho)(qv_E/\rho) \quad (2)$$

in which $s = \sqrt{\psi/\psi_a}$ and the last expression is valid only in circular large aspect ratio configurations: it shows the combined effect of magnetic shear with the *value* of v_E.

2. Stabilization of toroidal-ITG, helical-ITG and slab-ITG modes

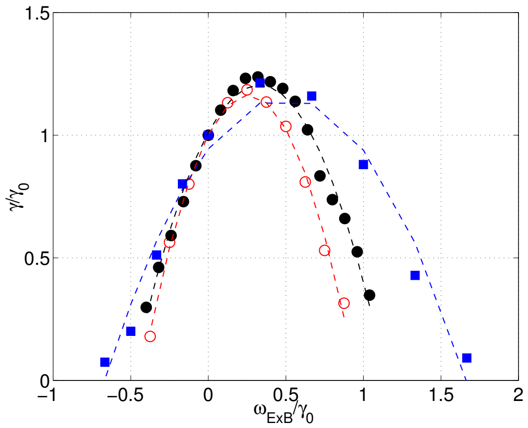

Figure 1: *Growth rates of toroidal-ITG (circles) and helical-ITG (squares) as function of the shearing rate $\omega_{E\times B}$; filled symbols with $v_E \propto s(s - s_0)$, open symbols with v_E/ρ=const. Dashed lines: quadratic fits.*

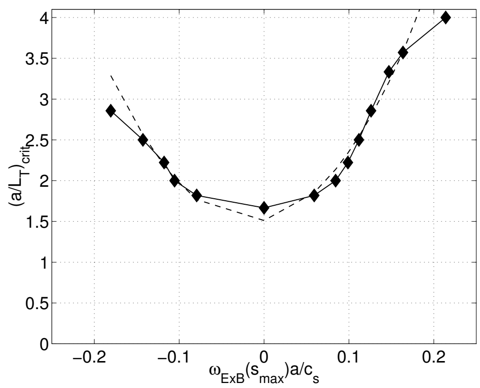

Figure 2: *Critical gradient as function of $\omega_{E\times B}$ for slab-like ITG modes.*

In this section we consider fully adiabatic electrons and gyrokinetic ions. We apply the model to 3 different configurations: (a) a circular cross-section tokamak with $R/a = 5.5$ and $q(s) = 1.25 + 3s^2$, (b) a shearless helically symmetric heliac configuration and (c) a cylinder. First, the most unstable mode in the absence of applied electric field is searched, and then an external electric field is applied. Fig.1 shows how the growth rate depends on the shearing rate of the applied E_r in the tokamak and heliac cases and for different E_r profiles. In all cases the behaviour is quadratic, and the stabilization occurs when $\omega_{E\times B} \approx \gamma_0$, the growth rate in the absence of applied E_r. Note in particular the result of the shearless v_E profile ($v_E \propto \rho$), for which the shearing rate is the product of magnetic shear times the value of v_E/ρ. It is therefore justified to use the expression of Eq.(2) for $\omega_{E\times B}$. A more detailed analysis of the cases shown here is made in Ref.[3]. Slab-ITG modes have been studied in Ref. [4], and the results confirm the generic nature of the quadratic dependence of the growth rate on $\omega_{E\times B}$. We show in Fig.2 the critical gradient $(a/L_T)_{crit}$ as a function of $\omega_{E\times B}$. There is clearly an upshift, and this upshift is proportional to the square of the shearing rate $\omega_{E\times B}$. We conclude that the often used stabilization criterion ($\omega_{E\times B} > \gamma_0$) seems to hold in all these cases. Note that a seemingly similar criterion for the suppression of turbulence has been theoretically derived [1] in which one should compare $\omega_{E\times B}$ not with the linear growth rate γ_0 but with the inverse decorrelation time of turbulence. The stabilization mechanisms are different: linear in our case and nonlinear for turbulence studies. What is common is the expression for the shearing rate, Eq.(2).

3. Destabilization of trapped particle modes

In this section we consider drift-kinetic trapped electrons and adiabatic passing electrons. All ions are gyrokinetic. The purpose of this study is to examine the effect of applied E_r on microinstabilities for which the dominant contributions to the drive come from trapped particles. We start from a tokamak configuration with $R/a = 3$, $L_T/a = 0.3$, $L_n/a = 3$, $q(\rho/a) = 1 + 2.3(\rho/a)^3$. The most unstable mode in the absence of applied E_r is a trapped ion mode (TIM). This has been verified by running a simulation in which $v_{||} = const$ was artificially imposed: a toroidal ITG mode has been found but with a 10 times smaller growth rate.

Applying an external E_r with a shearless profile of $v_E \propto \rho$ we find the growth rates and frequencies of Fig.3. The remarkable result is an overall destabilization. In order to understand the reason for such a contrasted behaviour (as compared to the toroidal-ITG, helical-ITG and slab-ITG cases shown in the previous section), we have performed a power transfer analysis. The power transfer from the particles to the field is

$$-dE_{kin}/dt = \sum_{\alpha=i,e} \int q_\alpha \, \delta f \, (v_{||}\mathbf{e}_{||} + \mathbf{v}_d + \mathbf{v}_E) \cdot < \nabla \delta\phi > d\mathbf{R} d\mathbf{v} \tag{3}$$

and should be equal to the time rate of change of the field energy, dE_{field}/dt, with

$$E_{field} = (1/2) \int \left((n_0 e/T_e) < \delta\phi >^2 + (n_0/B\Omega) | < \nabla_\perp \delta\phi > |^2 \right) d\mathbf{x} \tag{4}$$

Checking this power balance is a way to verify the quality of the numerical simulation. In all results shown here it is satisfied to better than 5%. The power transfer can be broken up into its contributions (ions, electrons, parallel motion, magnetic and $E \times B$ drifts). Fig.4 shows these contributions, divided by $2E_{field}$.

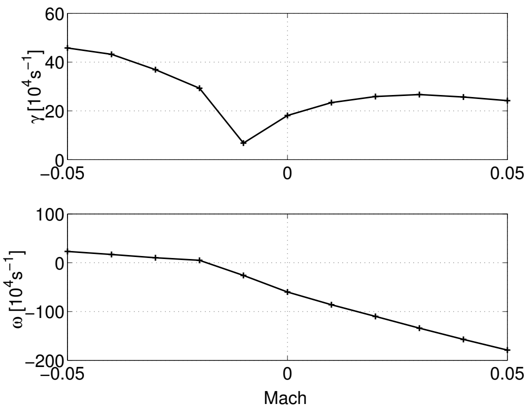

Figure 3: *Growth rate and frequency of TIM / TEM vs Mach number.*

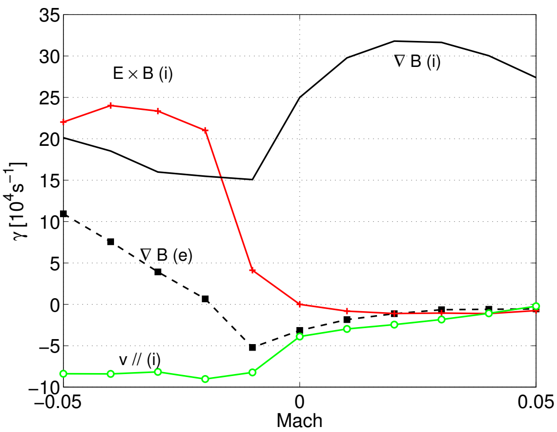

Figure 4: *Contributions to the growth rate from v_E on ions (line with +), v_d on ions (plain line), $v_{||}$ on ions (line with o), and v_d on electrons (dashed line with squares).*

Without applied E_r (Mach=0) the dominant contribution to the instability drive comes from the magnetic drift term on ions. This is to be expected since the mode is a trapped ion mode. The parallel ion dynamics are weakly stabilizing, as is the trapped electron drive. That electrons seem to be stabilizing could be misleading. As a matter of fact, running the same simulation but with *all* electrons adiabatic results is a much lower growth rate (by a factor of about 3). Thus taking into account the trapped electron dynamics is destabilizing rather than stabilizing because the adiabatic electron response is reduced (this mechanism is similar when considering larger T_e/T_i).

When a positive E_r is applied, the main effect is to further increase the contribution to the instability drive coming from the magnetic drift term on ions. The other contributions remain small and are little modified.

When a negative E_r is applied, first the ion instability drive is reduced, but then the trapped electron contribution increases substantially and becomes destabilizing. Another striking feature is the strong destabilization contribution of the $E \times B$ drift on ions, which becomes the dominant drive mechanism. The destabilization for $E_r < 0$ is therefore a combined effect of the trapped electron dynamics and of the $E \times B$ flow on ion dynamics. Note that there is no contribution from $E \times B$ on electrons. Running simulations with all electrons adiabatic the destabilization at $E_r < 0$ disappears. We note that the frequency is positive (Fig.3), meaning that the mode rotates in the electron diamagnetic direction. But the linear behaviour of ω with Mach number is simply due to the Doppler shift due to the poloidal angular $E \times B$ velocity: running without trapped electron dynamics also results in $\omega > 0$ for Mach < 0.02. However, since the mode at $M < -0.01$ exists due to the presence of trapped electrons, it is justified to call it a Trapped Electron Mode.

In order to determine whether the *value* of v_E or its *shearing rate* is causing the destabilization of the TEM, we have run a series of simulations with a linear profile of v_E having zero value at the maximum temperature gradient position s_0, $d\Phi_0/d\psi \propto (s - s_0)$. In this case the TEM destabilization does not occur. We conclude that the *value* of v_E, combined with the indirect effect of trapped electron dynamics, is responsible for the TEM destabilization for $E_r < 0$ in this case.

Conclusion. While the simplified stabilization criterion $|\omega_{E\times B}| > \gamma_0$ appears to be verified for toroidal-ITG, helical-ITG and slab-ITG modes, it can be clearly violated when trapped particle effects dominate the instability drive, in which case the destabilization comes from the *value* of v_E rather than its shearing rate.

Acknowlegements. This work was partly supported by the Swiss National Science Foundation. The computations were performed on the ORIGIN-3000 of the SIC-EPFL.

References

[1] T.S. Hahm and K.H. Burrell, Phys. Plasmas **2**, 1648 (1995). T.S. Hahm, Phys. Plasmas **4**, 4074 (1997).
[2] M. Fivaz, *et al.*, Comput. Phys. Commun. **111**, 27 (1998).
[3] L. Villard, *et al.*, Phys. Plasmas **9**, (to appear in June 2002).
[4] S.J. Allfrey, *et al.*, New Journal of Physics **4**, 29 (2002) (http://www.njp.org)