*29th EPS Conference on Plasma Phys. and Contr. Fusion Montreux, 17-21 June 2002 ECA Vol. **26B**, P-4.063 (2002)*

Generalized transport threshold model of neoclassical tearing modes

S. V. Konovalov[1], A. B. Mikhailovskii[1], M. S. Shirokov [1] and V. S. Tsypin[2]

[1]Institute of Nuclear Fusion, RRC “Kurchatov Institute”, Moscow, Russia

[2]Institute of Physics, University of São Paulo, São Paulo, Brasil

A series of transport threshold models of neoclassical tearing modes (NTMs) in tokamaks was suggested earlier [1-3]. These models are based on the idea that the bootstrap drive of these modes is weakened due to anomalously large perpendicular heat conductivity and diffusion. Meanwhile, according to [4, 5], the ion perpendicular viscosity in tokamaks can also be anomalous. The present report is addressed to analysis of its effect on the bootstrap drive of NTMs.

As a starting point, we use the parallel projection of the electron motion equation of the form

$$0 = e_e n_0 E_\parallel - \nabla_\parallel p_e - \mathbf{b}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{\pi}^e_\parallel + \frac{e_i n_0}{\sigma_\parallel} j_\parallel . \tag{1}$$

Here $\boldsymbol{\pi}^e_\parallel$ is the electron parallel viscosity tensor. It hence follows that the bootstrap current averaged over magnetic island surface, $\langle J_{bs}\rangle$, is given by

$$\langle J_{bs}\rangle = \frac{\sigma_\parallel}{e n_0}\left\langle \left\langle \mathbf{b}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{\pi}^e_\parallel\right\rangle_\theta\right\rangle , \tag{2}$$

where $\langle ...\rangle_\theta$ denotes averaging over the poloidal oscillations of the equilibrium magnetic field.

According to [6], we have approximately

$$\mathbf{b}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{\pi}^e_\parallel = \frac{3}{2}\pi^e_\parallel \boldsymbol{\nabla}\cdot\mathbf{b}, \tag{3}$$

where $\pi^e_\parallel$ is the electron parallel viscosity scalar of the form

$$\pi^e_\parallel = -\frac{4}{3} n_0 M_e \frac{R^2 q^2}{\epsilon^{3/2}} \nu_e \mathbf{V}_e\cdot\boldsymbol{\nabla}\ln B_0 . \tag{4}$$

Then representing $\sigma_\parallel = e^2 n_0/\left(M_e\nu_e\right)$, equation (2) reduces to

$$\langle J_{bs}\rangle = -2en_0 \frac{R^2 q^2}{\epsilon^{3/2}} \left\langle\left\langle \left(\boldsymbol{\nabla}\cdot\mathbf{b}\right)\left(\mathbf{V}_e\cdot\boldsymbol{\nabla}\ln B_0\right)\right\rangle_\theta\right\rangle . \tag{5}$$

In the simplest tokamak equilibrium $\boldsymbol{\nabla}\cdot\mathbf{b} = (\epsilon/Rq)\sin\theta$, $(\boldsymbol{\nabla}\ln B_0)_\theta = -\sin\theta/r$. Then, after averaging over θ, one has from (5)

$$\langle J_{bs}\rangle = en_0\frac{q}{\epsilon^{1/2}}\langle V_{e\theta}\rangle\,. \tag{6}$$

The projection $V_{e\theta}$ is represented in the form

$$V_{e\theta} = \frac{j_y}{en_0} + V_y + \frac{\epsilon}{q}V_{\|}. \tag{7}$$

Here direction $\mathbf{y}$ is defined according to relation $\boldsymbol{\theta} = \mathbf{y} + (B_{0\theta}/B_{0\zeta})\,\boldsymbol{\zeta}$. Leaving the dominant terms in the radial projection of the plasma motion equation, we express the current j_y in terms of the radial derivative of the plasma pressure,

$$j_y = \frac{c}{B_0}\frac{\partial p}{\partial r}. \tag{8}$$

At the same time, the y-projection of the plasma perpendicular velocity, $\mathbf{V}_\perp = \mathbf{V}_E + \mathbf{V}_{pi}$, takes the form

$$V_y = -\frac{cE_r}{B_0} + \frac{c}{en_0B_0}\frac{\partial p_i}{\partial r}. \tag{9}$$

Then (6) is transformed to

$$\langle J_{bs}\rangle = -\frac{\epsilon^{1/2}c}{B_\theta}\left[\left\langle\frac{\partial p_e}{\partial r}\right\rangle + en_0\left(\langle E_r\rangle - \frac{B_\theta}{c}\left\langle V_{\|}\right\rangle\right)\right]. \tag{10}$$

To find $\left\langle V_{\|}\right\rangle$ one should turn to the equation of parallel plasma motion. Averaging this equation over the poloidal oscillations of the equilibrium magnetic field and the magnetic island surfaces, one has

$$\left\langle\left\langle \mathbf{b}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{\pi}^i_{\|}\right\rangle_\theta\right\rangle + \left\langle\left\langle \mathbf{b}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{\pi}^i_{\perp}\right\rangle_\theta\right\rangle = 0. \tag{11}$$

Here $\boldsymbol{\pi}^i_{\|}$ and $\boldsymbol{\pi}^i_{\perp}$ are the ion parallel and perpendicular viscosity tensors, respectively. The contribution of the parallel viscosity into (11) could be represented as it was done for electrons (see e.g., (3), (4)). The term with perpendicular viscosity is approximated by [7] $\langle \mathbf{b}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{\pi}^i_{\perp}\rangle_\theta = -4M_in_0\mu_{\perp i}\partial^2 V_{\|i}/\partial x^2$, where $\mu_{\perp i}$ is the ion perpendicular viscosity coefficient. For magnetic island problem we can use an estimation $\partial^2/\partial x^2 \simeq -1/W^2$, where W is the magnetic island width. Then, qualitatively, (11) could be represented as

$$\epsilon^{1/2}\nu_i\left[\left\langle V_{\|}\right\rangle - \frac{c}{B_\theta}\left(\langle E_r\rangle - \frac{1}{en_0}\left\langle\frac{\partial p_i}{\partial r}\right\rangle\right)\right] + \frac{\mu_{\perp i}\left\langle V_{\|}\right\rangle}{W^2} = 0. \tag{12}$$

Substituting $V_{\|}$ from (12) into (10) we arrive at the following expression for the averaged bootstrap current

$$\langle J_{bs}\rangle = -\frac{\epsilon^{1/2}c}{B_\theta}\left[\left\langle\frac{\partial p_e}{\partial r}\right\rangle + \frac{W^2}{W^2+W_\mu^2}\left\langle\frac{\partial p_i}{\partial r}\right\rangle + \frac{W_\mu^2}{W^2+W_\mu^2}en_0\left\langle E_r\right\rangle\right]. \qquad (13)$$

Here we have introduced W_μ to be the characteristic island width governed by the ion perpendicular viscosity

$$W_\mu \simeq \left(\mu_{\perp i}/\epsilon^{1/2}\nu_i\right)^{1/2}. \qquad (14)$$

If perpendicular viscosity is weak, $\mu_\perp \to 0$, the contribution of radial electric field drops out of the bootstrap current and we arrive at the conventional expression $\langle J_{bs}\rangle \sim \langle p'\rangle$. However in the opposite limit, radial electric field term dominates in (13), reflecting the fact, that in the presence of perpendicular viscosity the ion response to the radial electric field differs from the electron one.

The value Δ_{bs} characterizing the bootstrap drive depends only on the perturbed values of plasma pressure and electric field. Neglecting the perpendicular viscosity and taking the plasma pressure to be flattened inside the island, Eq.(13) leads to the standard expression $\Delta_{bs} \sim \beta_p/W$.

When perpendicular viscosity is neglected but the unflattening is taken into account, one arrives at the transport threshold models described in [1-3]. Their essence is that not too small perpendicular heat conductivity (and/or particle diffusion) modifies the perturbed plasma pressure profile resulting in the weakening of the bootstrap drive for islands of width smaller than critical one, $\Delta_{bs} \sim \beta_p\left(W/(W^2+W_*^2)\right)$. As was shown in [3], the critical island width W_* describing such a weakening is given by

$$\frac{1}{W_*^2} = \frac{1}{W_r^2} + \frac{1}{W_{col}^2} + \frac{1}{W_{conv}^2}. \qquad (15)$$

Here $W_r \sim (\chi_\perp/\omega)^{1/2}$, $W_{col} \sim \left(\chi_\perp/\chi_\|\right)^{1/4}$, $W_{conv} \sim \left(\chi_\perp^{(e)}/V_{Te}\right)^{1/3}$ are the characteristic spatial scales defined by competition of perpendicular transport with island rotation, collisions (parallel diffusion) and parallel convection respectively.

In the case of sufficiently strong perpendicular viscosity one has $\Delta_{bs} \sim (\beta_p/W)\left(\omega/\omega_{*pi}\right)$ where ω_{*pi} is the ion diamagnetic drift frequency and ω is the island rotation frequency which appears due to dependence of the perturbed radial field E_x on the poloidal island rotation velocity ω/k_y [3].

Combining the cases of weak and strong perpendicular viscosity we arrive at the generalized transport threshold model of NTM in the form

$$\frac{\tau_s}{r_s}\frac{dW}{dt} = r_s\Delta' + \beta_p \frac{r_s C_{bs}}{W}\left(\frac{W^2}{W^2+W_*^2} - \frac{C_\mu W_\mu^2}{W^2+W_\mu^2}\frac{\omega}{\omega_{*pi}}\right), \quad (16)$$

where C_μ is a numerical coefficient of the order of unity.

According to (14), the value W_μ increases with increasing the perpendicular viscosity, while, according to (15), the critical island width W_* increases with increasing the perpendicular heat conductivity. Therefore, when all the transport coefficients are sufficiently large, so that $(W_\mu, W_*) > W$, and the island rotation frequency is of the order of characteristic diamagnetic drift frequencies, $\omega \simeq \omega_{*pi}$, equation (16) reduces to

$$\frac{\tau_s}{r_s}\frac{dW}{dt} = r_s\Delta' - \beta_p \frac{r_s C_{bs} C_\mu}{W}\frac{\omega}{\omega_{*pi}}. \quad (17)$$

Thus, in the presence of the perpendicular viscosity the conventional viewpoint that the bootstrap current always drives the NTMs proves to be non-universal. According to (17), bootstrap effect is stabilizing for islands rotating to the ion diamagnetic drift direction, $\omega/\omega_{*pi} > 0$, and destabilizing in the contrary case, $\omega/\omega_{*pi} < 0$.

References

[1] Fitzpatrick R 1995 *Phys. Plasmas* **2** 825

[2] Gorelenkov N N, Budny R V, Chang Z, Gorelenkova M V and Zakharov L E 1996 *Phys. Plasmas* **3** 3379

[3] Konovalov S V, Mikhailovskii A B, Shirokov M S and Tsypin V S 2002 *Plasma Phys. Control. Fusion* **44** 579

[4] Kadomtsev B B, Morozov D K and Pogutse O P 1984 *JETP Lett.* **40** 818

[5] Kadomtsev B B 1992 *Tokamak Plasma: A Complex Physical System*, Institute of Physics, Bristol

[6] Mikhailovskii A B 1998 *Plasma Phys. Control. Fusion* **40** 1907

[7] Mikhailovskii A B and Lakhin V P 1995 *Plasma Phys. Rep.* **21** 271