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Subbanana magnetic islands in tokamaks

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At the present, extensive theoretical and experimental investigations on the magnetic islands in tokamaks are performed (see the review [1] and references therein). Up to now, the main attention has been addressed to studying the islands of width w essentially larger than the drift orbit size, Λ, $w > \Lambda$. In addition, in a series of papers, for instance in [2], the islands of width smaller than the ion Larmor radius ρ_i, $w < \rho_i$ were studied. In this context, the question is reasonable: which physical effects govern the island dynamics in the interval

$$\rho_i < w < \Lambda. \tag{1}$$

Elucidating the role of the effect of interest reduces to analyzing the contribution of finite orbit particles into the generalized Rutherford equation of the form

$$\frac{dw}{dt} \sim \frac{\Delta'}{4} + \Delta^{\Lambda} + \dots . \tag{2}$$

Here the value Δ' is the standard tearing mode stability parameter. In the case of a "well organized discharge", one has $\Delta' < 0$, i.e. the Δ' effect is stabilizing. The terms symbolically designated as "...", characterize the effects of bootstrap current, Δ_{bs}, polarization current, Δ_p and magnetic well, Δ_{mw}, (see, for details, [1]). These effects have been studied in detail in preceding publications in the approximation of sufficiently large scale islands, $w > \rho_i/\epsilon^{1/2}$. One can suggest that the corresponding terms should be modified in the case of interest, i.e. when $w < \rho_i/\epsilon^{1/2}$. Studying such a modification could be the subject of the following publications. As for the scope of the present work, we analyze only the effect characterized by the term Δ^{Λ}. Here superscript Λ emphasizes that we deal with contribution due to particles with large drift orbits Λ.

For calculating Δ^{Λ} we use the standard variables ψ, ξ, characterizing the island magnetic flux and cyclic variable, respectively. In addition, we introduce the dimensionless magnetic flux function $\Omega = -\psi/\widetilde{\psi}$ and the function $j_{\parallel}^{\Lambda} = j_{\parallel}^{\Lambda}(\Omega, \xi)$, characterizing the parallel current density in the "island configuration" due to the presence of large orbit particles. Then, according to [1], the value Δ^{Λ} is defined by

$$\Delta^{\Lambda} = -\frac{2^{3/2} Rq}{cswB_0} \sum_{\sigma_x} \int_1^{\infty} d\Omega \oint \frac{j_{\parallel}^{\Lambda} \cos\xi d\xi}{(\Omega + \cos\xi)^{1/2}}. \tag{3}$$

Here c is the speed of light, $\sigma_x = \mathrm{sign} x = \pm 1$. Physically, the integration region in (3) corresponds to the space lying outside the magnetic island separatrix. The contour integration over ξ in (3) means integrating in the limits from 0 to 2π.

We calculate $j_{\parallel}^{\Lambda}$ guiding ourselves by the following considerations. Let us introduce the total density of the "Λ - group" of ions, n_{Λ}, understood as the sum of the equilibrium and perturbed densities, $n_{0\Lambda}$ and $\widetilde{n}_{\Lambda}$, i.e. $n_{\Lambda} = n_{0\Lambda} + \widetilde{n}_{\Lambda}$. We allow for that the electric charge of this ion group, $e_i n_{\Lambda}$, is compensated by the charge of the corresponding part of the electron plasma component, $e_e n_e^{\Lambda}$, i.e. in the simplest case $e_i = e_e = e$ we have $n_e^{\Lambda} = n_{\Lambda}$. It is the parallel motion of these compensating electrons that causes the parallel current $j_{\parallel}^{\Lambda}$. Assuming the electrons to be cold and allowing for their perpendicular motion in the crossed electric and magnetic fields, we conclude that $j_{\parallel}^{\Lambda}$ can be found by means of the electron continuity equation, which, using above relations, is represented in the form

$$-e\frac{d_0}{dt}(n_{0\Lambda} + \widetilde{n}_{\Lambda}) + \nabla_{\parallel} j_{\parallel}^{\Lambda} = 0. \tag{4}$$

To calculate $\widetilde{n}_{\Lambda}$ we turn to the drift kinetic equation (see, e.g., equation (16.70) of the book [3]). In the limiting case of large drift orbits, $k_{\perp}\Lambda \gg 1$, assuming particle velocity distribution to be Maxwellian with the temperature T_{Λ} we find

$$\widetilde{n}_{\Lambda} = -\frac{e\phi}{T_{\Lambda}} n_{0\Lambda}, \tag{5}$$

The relation (5) means physically that, in small-scale perturbations, the ion group considered is distributed in the space according to the Boltzmann law.

For cold electrons, the electrostatic potential $\phi = (B_0\omega/ck_y)[x - h(\Omega)]$, where $k_y = m/r_s$, $h(\Omega)$ is the so-called electrostatic potential profile function [1, 4]. In terms of the variables (Ω, ξ), the operator d_0/dt, in neglecting the nonstationarities of the island width and of rotation frequency, is defined by $d_0/dt = -\omega\Omega_x h' \partial/\partial\xi$, where $h' \equiv dh/d\Omega$, $\Omega_x = (\partial\Omega/\partial x)_\xi$. We allow also for that $n_{0\Lambda}(r) = n_{0\Lambda}(r_s) + x\,(\partial n_{0\Lambda}/\partial r)_{r=r_s}$. In addition, we have $x = \sigma_x \left(2L_s\widetilde{\psi}/B_0\right)^{1/2}(\Omega + \cos\xi)^{1/2}$, $\nabla_\parallel = k_\parallel \partial/\partial\xi$, where $k_\parallel = -k_y x/L_s$. Then (4) reduces to

$$\frac{\partial j_\parallel^\Lambda}{\partial\xi} = c_\Lambda g(\Omega)\,\frac{\sin\xi}{(\Omega + \cos\xi)^{1/2}}, \tag{6}$$

where

$$c_\Lambda = \frac{\pi}{4\sqrt{2}}\frac{\omega^2}{k_y^2}\frac{e_\Lambda^2 n_{0\Lambda} B_0 L_s}{cT_\Lambda}\left(1 - \frac{\omega_{\star\Lambda}}{\omega}\right), \tag{7}$$

$\omega_{\star\Lambda} = (k_y c T_\Lambda/eB_0)\,(\partial \ln n_{0\Lambda}/\partial r)_{r=r_s}$, is the diamagnetic drift frequency of the ion group considered, and the function $h(\Omega)$ is defined by (see [4]) $g(\Omega) = (8\sigma_x/\pi w)\,h'(\Omega)$. Integrating (6) over ξ, we obtain

$$j_\parallel^\Lambda = -2c_\Lambda g(\Omega)\left[(\Omega + \cos\xi)^{1/2} - \left\langle(\Omega + \cos\xi)^{1/2}\right\rangle\right], \tag{8}$$

where $\langle\ldots\rangle$ means averaging over ξ with the weight $(\Omega + \cos\xi)^{-1/2}$.

Substituting (8) into (3), we find

$$\Delta^\Lambda = \frac{4\pi e_\Lambda^2 n_{0\Lambda}}{T_\Lambda c^2}\frac{\omega^2}{k_y^2}\frac{q^2R^2}{s^2w}\left(1 - \frac{\omega_{\star\Lambda}}{\omega}\right)I, \tag{9}$$

Integral I has been calculated numerically yielding $I = 1.50$.

According to (9), the value Δ^Λ depends on the shear and magnetic island halfwidth as $(s^2w)^{-1}$, which is the same as the dependence of the value Δ_{mw} characterizing the magnetic well effect (see, for details, [1]). Consequently, the large orbit effect, called below as "Λ - effect", can be treated as a variety of the magnetic-well effect.

To estimate the order of magnitude of Δ^Λ let us consider the case when the ions of the"Λ - group" are the trapped and weakly circulating ions of the core plasma, i.e. $n_{0\Lambda} \simeq \epsilon^{1/2} n_0$, $T_\Lambda = T_i$, where n_0 and T_i are the core plasma density and ion temperature, respectively. In addition, we assume the island rotation frequency ω to be of the order of the diamagnetic drift frequency, so that $\omega/k_y \simeq v_{Ti}\rho_i/L_n$, where v_{Ti} is the ion thermal velocity, L_n is the characteristic length of the plasma density inhomogeneity, which is assumed, for simplicity, to be of the order of r_s. Then one finds from (9) the estimation

$$\Delta^{\Lambda} \simeq \frac{\epsilon^{1/2}\beta_p}{s^2 w}, \tag{10}$$

where β_p is the "poloidal beta", $\beta_p = 8\pi n_0 T_i / B_\theta^2$, B_θ is the poloidal magnetic field . On the other hand, according to [1], $\Delta_{mw} \simeq \epsilon^2 \beta_p / (s^2 w)$. Consequently, $\Delta^{\Lambda}/\Delta_{mw} \simeq \epsilon^{-3/2}$, i.e. the Λ - effect is essentially stronger than the standard magnetic-well effect. Note also that for $s \simeq 1$ the estimation (10) for Δ^{Λ} is the same as that for Δ_{bs}, i.e. for bootstrap current contribution into the equation for island width evolution (2). It hence follows that, in studying the islands of sufficiently small width, along with the bootstrap current effect, one should obligatory take into account the Λ - effect as well.

According to (9), for $\omega/\omega_{\star\Lambda} > 1$, i.e. in the case of "superdrift" magnetic islands rotating in the direction of ion diamagnetic drift $\Delta^{\Lambda} > 0$, i.e. the Λ - effect is destabilizing. On the other hand, for the "subdrift" magnetic islands rotating in the ion diamagnetic drift direction, $0 < \omega/\omega_{\star\Lambda} < 1$, one has $\Delta^{\Lambda} < 0$, i.e in this case the Λ - effect is stabilizing. These results are in correspondence with the predictions of the paper [2] concerning the islands with $w < \rho_i$. In addition, according to (9), the Λ - effect is destabilizing, $\Delta^{\Lambda} > 0$, in the case of islands rotating in the direction of the electron diamagnetic drift, $\omega/\omega_{\star\Lambda} < 0$.

It should be noted also that expression (9) has been derived in the limiting case $w \ll \Lambda$. To extend qualitatively this result on the arbitrary value of the ratio w/Λ one should multiply Δ^{Λ} by the ratio $\Lambda^2/(w^2 + \Lambda^2)$. With this correction, as one can naturally expect, the Λ - effect reduces to the polarization current effect for sufficiently large islands, i.e. at $w/\Lambda \gg 1$.

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