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ABOUT THE TOROIDAL MAGNETIC FIELD GENERATION IN THE MAGNETOSPHERE OF CRAB PULSAR

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Abstract

Some aspects of relativistic electron-positron plasma dynamics in the magnetosphere of Crab pulsar are investigated. It is shown that because of pulsar, its magnetic field and magnetospheric relativistic plasma corotation centrifugal force plays important role in the dynamics of latter. Taking into account centrifugal force in the equation of the plasma motion one gets new effect of plasma particle radial braking in the pulsar magnetosphere when the condition $V_r > c/\sqrt{2}$ is fulfilled for the particle radial velocity. Time dependence of plasma particle radial velocity is harmonic. This is similar to the influence of external periodic force on pulsar magnetosphere relativistic plasma, as a result of which parametric type instability development takes place and exponentially growing toroidal magnetic field is generated in the magnetosphere of Crab pulsar.

Introduction

As it is well known pulsar is a rapidly spinning neutron star with very strong magnetic field of the order $10^{12} - 10^{13}$ G. Pulsars are surrounded with magnetospheres which have rather complicated structure and properties. It is widely accepted that pulsar magnetosphere is filed with relativistic electron-positron plasma [1]. Pulsar radiation is generated in its magnetosphere as a result of the development of various plasma processes in the region above its magnetic poles close to the neutron star surface. Generated radiation is emitted in a narrow cone around the magnetic axis of pulsar. The rotation axis and the magnetic axis of pulsar form some angle between them. So, at some moment during pulsar rotation its magnetic axis is directed towards the Earth and we register the radiation pulse. The period between the successive pulses equals to the period of neutron star rotation. This is the way how the neutron stars have got their name - pulsar means pulsating radio source.

From all the above mentioned it becomes evident how extremely important is to study the structure of the pulsar magnetosphere and the dynamical plasma processes which take place there.

In the present paper we discuss the case of Crab pulsar PSR 0531+21. This is very interesting and peculiar one among all discovered pulsars. It radiates simultaneously radio, optical, X-ray and Gamma-pulses. Besides its rotation axis and magnetic axis are nearly perpendicular.

In the pulsar magnetosphere, close to the neutron star surface magnetic field has a dominant role. Its energy exceeds the energy of the magnetospheric relativistic electron-positron plasma by many orders of the magnitude. The matter inside pulsar is in superconductive state and therefore magnetic field is frozen in it. Besides, magnetic field is frozen into the magnetospheric plasma too. So, the solid body type rotation - so called corotation of pulsar, its magnetic field and magnetospheric relativistic electron-positron plasma takes place. Of course, corotation can not take place on large radial distances from the pulsar surface. In fact, corotation is strictly impossible beyond the light cylinder

- cylindrical surface, on which azimuthal velocity for the case of corotation equals to the speed of light - $V_\varphi = \Omega r = c$ (where Ω is the pulsar rotation frequency). Corotation must be violated somehow before the light cylinder. One of the possible scenario for this will be presented below.

As a result of corotation centrifugal force plays the important role in the dynamics of the relativistic electron-positron plasma in the pulsar magnetosphere, which is demonstrated in the forthcoming section.

The Role of Centrifugal force in the Dynamics of pulsar Magnetosphere Relativistic Plasma

First of all about the simplified geometrical model of the pulsar magnetic field which we use. As it has been already mentioned we discuss the case of Crab pulsar the rotation axis and magnetic axis of which are nearly perpendicular. We assume that pulsar magnetic field lines are radial straight lines located in the plane perpendicular to the rotation axis of pulsar. This assumption is justified, because we discuss the physical processes in the thin layer of pulsar magnetosphere close to the star surface and the thickness of this layer is much less than the curvature radius of the pulsar magnetic field lines.

According to the principle of equivalence we can not distinguish gravitation from noninertiality. Thus, for the investigation of pulsar magnetosphere relativistic electron-positron plasma dynamics so-called "3+1" formalism can be used, which is described in [2]. Using this formalism one can find that the equation of the motion for the pulsar magnetosphere relativistic electron-positron plasma has the following form [3,4]:

$$\frac{\partial \vec{p}}{\partial t} + \left(\vec{V}\vec{\nabla}\right)\vec{p} = \vec{F} + e\left(\vec{E} + \left[\vec{V}\times\vec{B}\right]\right), \tag{1}$$

were m and e are the particle rest mass and electric charge respectively, $\vec{V}$ and $\vec{p}$ - particle three-velocity and momentum, $\vec{E}$ and $\vec{B}$ - electric and magnetic fields. Here and below we use so-called geometric units - $c = G = 1$. Besides,

$$\vec{F} = -\gamma m \alpha \vec{\nabla}\alpha = \gamma m \Omega^2 \vec{r}. \tag{2}$$

Here γ is the particle Lorentz-factor and $\alpha = \sqrt{1 - \Omega^2 r^2}$ is the so-called "lapse function" [2]. As for the force $\vec{F}$, it is the relativistic generalization of centrifugal force.

As it has been mentioned above pulsar magnetic field is frozen in the magnetospheric relativistic plasma. Thus, the freezing-in condition

$$\vec{E} + \left[\vec{V}\times\vec{B}\right] = 0 \tag{3}$$

is fulfilled in the pulsar magnetosphere. Taking into account this condition in the equation of plasma motion (1) one can get the following expression for the plasma particle radial acceleration in the pulsar magnetosphere:

$$\frac{d^2 r}{dt^2} = \frac{\Omega^2 r}{1 - \Omega^2 r^2}\left[1 - \Omega^2 r^2 - 2\left(\frac{dr}{dt}\right)^2\right]. \tag{4}$$

From this expression it is evident that when the following condition

$$V_r = \frac{dr}{dt} > \frac{1}{\sqrt{2}} \tag{5}$$

is fulfilled in the pulsar magnetosphere, the radial acceleration of plasma particles changes sign and becomes negative. This means that plasma particles are no longer accelerated radially, but braked.

This effect of plasma particle radial braking is the consequence of corotation - when plasma particle velocity increases, their mass also increase due to the relativistic effect and when the condition (5) is fulfilled the inertia of plasma particles is so large that their further acceleration is impossible and they are braked. This is the point where the necessity of corotation violation and changing of the pulsar magnetic field structure arises. One of the possible ways of this will be discussed below.

The solution of the equation (4) has the form:

$$r\left(t\right)=\frac{V_{0i}}{\Omega}sin\left(\Omega t\right), \tag{6}$$

where V_{0i} is the plasma particle initial velocity. From (6) it follows for plasma particle radial velocity

$$V_r=V_{0i}cos\left(\Omega t\right). \tag{7}$$

So, the time dependence of plasma particle radial velocity is harmonic. The situation is similar to the one when external periodic force is acting on plasma and this can trigger the development of instabilities in the relativistic plasma of the pulsar magnetosphere. This issue is addressed in the following section.

Parametric Type Instability Development and Toroidal Magnetic Field Generation in the Pulsar Magnetosphere

As we have seen time dependence of plasma particle radial velocity in the pulsar magnetosphere is harmonic. This is equivalent to the influence of external periodic force on plasma as a result of which instabilities may develop in it. Thus, we study the stability of the pulsar magnetosphere relativistic electron-positron plasma. Study of the stability of magnetospheric plasma with respect to the radial potential perturbations has been performed in the paper[4]. Now we investigate the stability of the magnetospheric relativistic plasma with respect to the electromagnetic perturbations. For this purpose we need the equation of the motion, continuity equation and Maxwell equations:

$$\frac{\partial\vec{p}}{\partial t}+\left(\vec{V}\vec{\nabla}\right)\vec{p}=\gamma m\Omega^2\vec{r}+e\left(\vec{E}+\left[\vec{V}\times\vec{B}\right]\right), \tag{8}$$

$$\frac{\partial n}{\partial t}+div\left(n\vec{V}\right)=0, \tag{9}$$

$$rot\vec{E}=-\frac{\partial\vec{B}}{\partial t}, \tag{10}$$

$$rot\vec{B}=4\pi\vec{j}+\frac{\partial\vec{E}}{\partial t}. \tag{11}$$

Here n is the particle density and $\vec{j}$ is the current density. We discuss the perturbations wave-vector of which is directed radially outwards - $\vec{k}\left(k_r,0,0\right)$. Perturbed electric field is directed along the z-axis (which coincides with pulsar rotation axis) - $\vec{E_1}\left(0,0,E_{1z}\right)$ and perturbed magnetic field has only the toroidal component - $\vec{B_1}\left(0,B_{1\varphi},0\right)$.

Using Bessel function technique and Fourier transformation one can finally get from the set of equations (8-11) the following dispersion relation:

$$\omega^2 = \frac{{\omega_c}^2}{{\gamma_0}^6\left(1+{\omega_p}^2/{k_r}^2{\gamma_0}^3\right)} - \frac{{k_r}^2{V_{0i}}^2}{2}, \tag{12}$$

where ω_c is the cyclotron frequency and ω_p is the plasma frequency. Time dependence of perturbed toroidal magnetic field is exponential:

$$B_{1\varphi} \sim exp\left(-i\omega t\right). \tag{13}$$

From the expressions (12) and (13) we can conclude that when the following condition

$$\frac{{k_r}^2{V_{0i}}^2}{2} > \frac{{\omega_c}^2}{{\gamma_0}^6\left(1+{\omega_p}^2/{k_r}^2{\gamma_0}^3\right)}$$

is fulfilled parametric type aperiodic instability is developing. As a result of the development of this instability exponentially growing toroidal magnetic field is generated in the pulsar magnetosphere.

Superposition of generated toroidal magnetic field and pulsar magnetic field will give the spiral configuration magnetic field. Since plasma particles follow the magnetic field lines corotation will be violated in the pulsar magnetosphere and instead of it we will have differential rotation or shear flow of magnetospheric plasma.

On large radial distances the step of the magnetic field spiral decreases and beyond the light cylinder magnetic field is practically purely toroidal. On larger radial distance from pulsar - around 10^{17} cm this magnetic field is reconnected with the magnetic field of Crab Nebula, which has also toroidal structure.

Conclusions

So, in the magnetosphere of Crab pulsar close to the star surface corotation of pulsar, its magnetic field and magnetospheric plasma takes place. Because of corotation centrifugal force plays important role in plasma dynamics. Taking into account this force gives new effect of plasma particle radial braking in the magnetosphere. This is a strong perturbation of plasma and it triggers the development of parametric type aperiodic instability. As a result exponentially growing toroidal magnetic field is generated in the magnetosphere and after this magnetic field structure changes to spiral. Since the plasma particles follow the magnetic field lines corotation will be violated. On large radial distances step of the magnetic field spiral decreases and we get practically purely toroidal magnetic field which is finally reconnected to the magnetic field of Crab nebula.

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