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Relaxation times for magnetized plasma - a parametric study

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Abstract

A previously derived Fokker-Planck-type collision integral for a test-particle in magnetized plasma is explicitly evaluated. Explicit new formulae are obtained for the diffusion coefficients and the interaction-force correlations; their dependence on physical parameters, including the magnitude of the (uniform) field, is briefly studied and commented upon.

In earlier work we have undertaken a study of the dynamics of a charged test-particle (t.p.) moving inside a magnetized background plasma in equilibrium. Starting from first microscopic principles, a *Fokker-Planck-type kinetic equation (FPE)* was derived and analytical expressions for the coefficients were obtained [1]. Emphasis was made on the magnetic field dependence of the collision integral - as compared to the standard *unmagnetized* Landau description [2] - as well as the effect of non space-uniformity of the t.p. distribution function $f(\mathbf{x}, \mathbf{v}; t)$. The diffusion coefficients were explicitly evaluated for a Maxwellian background state and a Debye-type interaction law [3]. The aim of this brief report is to summarize those results and present a set of exact computable expressions for the diffusion coefficients, pointing out their explicit dependence on t.p. microscopic variables (velocity) *and* the magnetic field $\mathbf{B}$. A detailed numerical study will be reported in a forthcoming report.

The FP - type kinetic equation obtained for the above system reads [1], [4]:

$$
\begin{aligned}
\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{x}} + \frac{e}{mc}(\mathbf{v}\times\mathbf{B})\frac{\partial f}{\partial \mathbf{v}} = &\left(\frac{\partial^2}{\partial v_x^2} + \frac{\partial^2}{\partial v_y^2}\right)\Big[D_\perp(\mathbf{v}) f\Big] + \frac{\partial^2}{\partial v_z^2}[D_\parallel(\mathbf{v}) f] \\
+2\, s\,\Omega^{-1}\Big[\frac{\partial^2}{\partial v_x \partial y} - \frac{\partial^2}{\partial v_y \partial x}\Big]\Big[D_\perp(\mathbf{v}) f\Big] &+ \Omega^{-2}\,[D_\perp^{(XX)}(\mathbf{v})](\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}) f \\
-\frac{\partial}{\partial v_x}\Big[\mathcal{F}_x(\mathbf{v})\, f\Big] &- \frac{\partial}{\partial v_y}\Big[\mathcal{F}_y(\mathbf{v})\, f\Big] - \frac{\partial}{\partial v_z}\Big[\mathcal{F}_z(\mathbf{v})\, f\Big] \\
&+ s\,\Omega^{-1}\,\mathcal{F}_y(\mathbf{v})\,\frac{\partial}{\partial x} f - s\,\Omega^{-1}\,\mathcal{F}_x(\mathbf{v})\,\frac{\partial}{\partial y} f \qquad (1)
\end{aligned}
$$

where $\mathbf{B}$ is the external magnetic field (taken to be uniform for simplicity); obviously, $m = m_\alpha$ and $e = e_\alpha \equiv s|e_\alpha|$ denote the mass and charge, respectively, of the t.p. (of species $\alpha = e, i, ...$) and $\Omega = |e_\alpha|B/m_\alpha c$ is the cyclotron frequency. By integrating over position $\{\mathbf{x}\}$, one obtains a reduced FPE, describing the evolution of $f_{local}(\mathbf{v}; t) = \int d\mathbf{x}\, f$; this is a 'linearized' version of a kinetic equation which has appeared in earlier works [5].

The *diffusion coefficients* in (1) are functions of $\{v_\perp, v_{||};\ \Omega;\ t\}$; they are given by:

$$\left\{ \begin{array}{c} \left\{ \begin{array}{c} D_\perp \\ D_\angle \\ D_\perp^{(XX)} \end{array} \right\} \\ D_{||} \end{array} \right\} = D_0\, \lambda \int_0^{\Omega t \to \infty} d\tau' \int_1^\infty dx\, e^{\lambda^2 (1-x^2) \sin^2 \frac{\tau'}{2}} \left(1 - \frac{1}{x^2}\right)^{\{1,0\}} e^{-\tilde{v}_{||}^2}$$

$$J_O\!\left(2\lambda \sqrt{x^2 - 1}\, \tilde{v}_\perp \sin \frac{\tau'}{2}\right) \left\{ \begin{array}{c} \left\{ \tilde{F}_\perp \right\} \\ \tilde{F}_{||} \end{array} \right\} \left\{ \begin{array}{c} \left\{ \begin{array}{c} \frac{1}{2} \cos \tau' \\ \frac{-s}{2} \sin \tau' \\ 1 + \frac{1}{2} \cos \tau' \end{array} \right\} \\ 1 \end{array} \right\} \qquad (2)$$

where all quantities are non-dimensional[1] except $D_0 = \frac{2\sqrt{2}\, n\, e^4}{m^{3/2} \sqrt{k_B T}}$. Remember that the *dynamical friction* vector $\mathcal{F}_i$ is defined via velocity derivatives of D_{ij}; both D_{ij}, $\mathcal{F}_i$ are thus related to the (interaction) force-correlation matrix $C_{ij}(\tau) = \langle F_{int,i}(t)\ F_{int,j}(t-\tau) \rangle_R$ [3]. The functions $\tilde{F}_* = \tilde{F}_*(\phi(x, \tau'), \tilde{v}_{||})$ $(* = \perp, ||)$ are:

$$\tilde{F}_{\{\perp,||\}}(\phi, \tilde{v}_{||}) = \pm\sqrt{\pi}\, \phi + \frac{\pi}{4} \sum_{s=+1,-1} \left[e^{(\phi + s\tilde{v}_{||})^2} \left(1 \mp 2\,\phi^2 \mp s2\,\phi\, \tilde{v}_{||}\right) Erfc(\phi + s\, \tilde{v}_{||}) \right] \qquad (3)$$

where $\phi = \frac{\lambda}{2}\tau' x$, $\tilde{v}_{\{\perp,||\}} = v_{\{\perp,||\}}/v_{th}\sqrt{2}$ and $\lambda = \sqrt{2}\,\frac{k_D}{\Omega}\, v_{th} = \sqrt{2}\,\frac{\omega_p}{\Omega} = \sqrt{2}\,\frac{\rho_L}{r_D}$. As obvious, $v_{th} = (k_B T/m)^{1/2}$ is the thermal velocity, $k_D = \frac{4\pi e_\alpha^2 n_\alpha}{k_B T_\alpha} \equiv r_D^{-1}$ is the Debye wave-number, $\omega_{p,\alpha} = (\frac{4\pi e_\alpha^2 n_\alpha}{m_\alpha})^{1/2}$ is the plasma (Langmuir) frequency and $\rho_L = v_{th}/\Omega$ is the Larmor radius. Notice the competition between collision and gyration scales via λ.

We have chosen a set of typical values, i.e. a temperature of $T = 10\, KeV$ and a particle density of $n = 10^{14}\, cm^{-3} = 10^{20}\, m^{-3}$, implying a plasma frequency $\omega_{p,e} = 5.64 \cdot 10^{11}\, s^{-1}$ and a (gyro-)frequency of: $\Omega_e = 1.76\, 10^{11} \times B \quad s^{-1}$ (B expressed in Tesla).

In figure 1, we have represented all coefficients against time t (measured in cyclotron periods), for $B = 1T$. The diffusion coefficients increase fast in time, practically attaining their asymptotic value within a few periods.

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[1]Relations (2), (3) actually correspond to the formulae presented in [3], rescaling the integration variables therein as: $\tau' = \Omega\tau$, $x = (1 + \frac{k_\perp^2}{k_D^2})^{1/2}$; however, notice that we limit ourselves in the single species case (e.g. electron plasma) here (i.e. $\alpha' = \alpha$, cf. [3]).

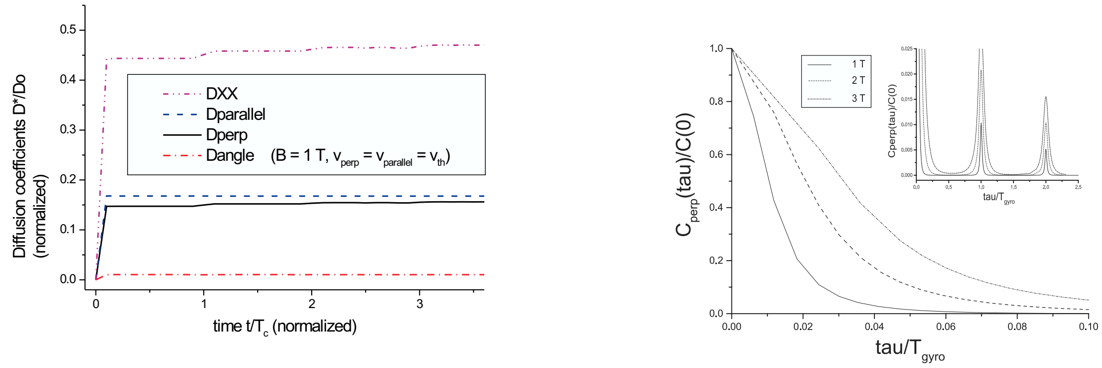

Figure 1: (a) Diffusion coefficients plotted against time t for $B = 1T$. (b) The correlation function $C_{\perp}(t)$ for $B = 1, 2, 3T$ (in ascending order); notice the tiny peaks every period.

The velocity dependence of the coefficients qualitatively reproduces the unmagnetized result [2]: see figure 2; the diffusion coefficients take lower values for faster particles.

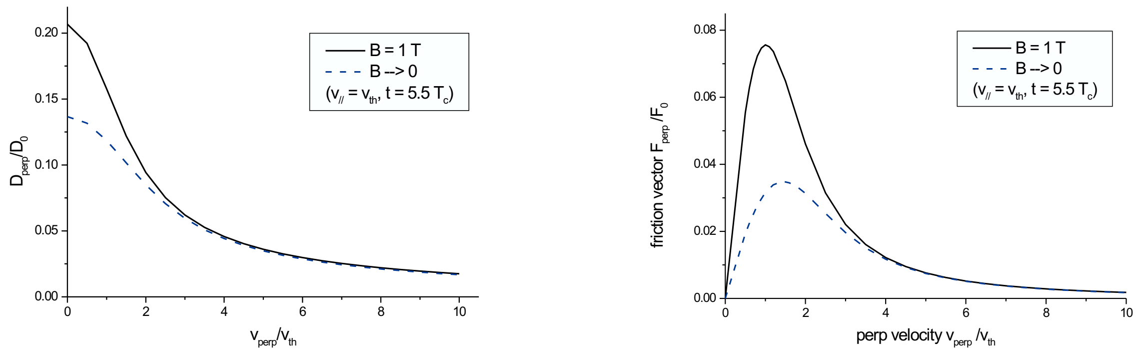

Figure 2: The diffusion coefficient $D_{\perp}$ and the corresponding friction vector $\mathcal{F}_{\perp}$, plotted against velocity $v_{\perp}$, for $B = 1T$ (solid line) and $B = 0$ (dashed line).

In figure 3a we have depicted $D_{\perp}$ versus λ. Above $\lambda \approx 1$ (i.e. for $\rho_L \cong r_D$ or higher) the field slightly enhances relaxation [6]: the higher its magnitude B, the higher the value of $D_{\perp}(\tau)$. Physically speaking, this fact reflects particle confinement by the magnetic field: particles stick to their helicoidal trajectory around the field lines and thus interact longer. The friction vector $\mathcal{F}_{\perp} \sim \partial D_{\perp}/\partial v_{\perp}$ behaves in a similar way (fig. 3b) . However, their $\| -$ counterparts (fig. 3c, d) are practically time- (and field-) independent.

In conclusion, we have reported a set of new exact formulae for the diffusion coefficients in magnetized plasma. These formulae suggest an explicit dependence on particle velocity and physical parameters (plasma temperature, density) and - most important - the magnitude of the magnetic field. A more extended study will be reported soon.

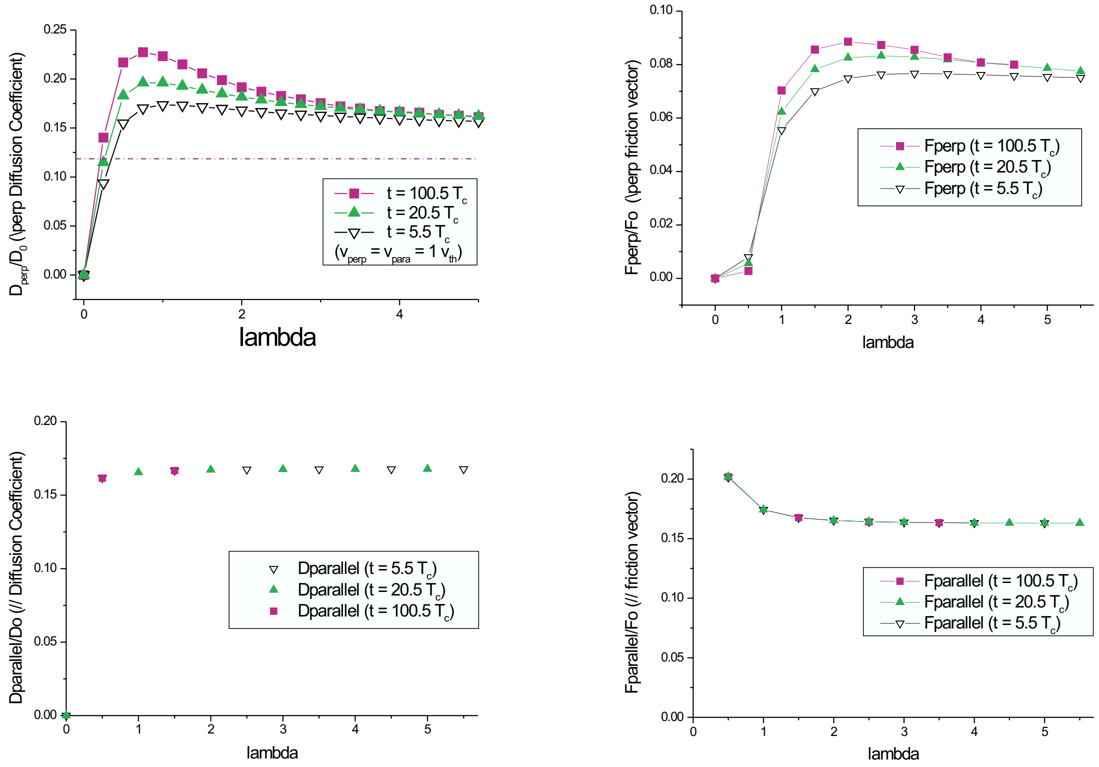

Figure 3: The perpendicular diffusion coefficient $D_\perp$ and the friction vector (norm) $\mathcal{F}_\perp$ (top), and their $\|$ −counterparts (bottom) plotted against λ ($\sim 1/B$), at different instants of t. $D_\perp$ slightly increases in time, yet only around $\lambda \approx 1$ (i.e. $\rho_L \approx r_D$), above which it practically remains constant. The field-dependence is smoothed out, as $D_\perp$ approaches the asymptotic value for $\Omega \to 0$ (dash-dot line). $D_{||}$ comes out to be independent of the field and so does $\mathcal{F}_{||}$.

References

[1] I. Kourakis, Plasma Phys. Control. Fusion **41** 587 (1999).

[2] see, for instance, in R. Balescu, *Statistical Mechanics of Charged Particles*, Wiley, 1963.

[3] I.Kourakis, D.Carati, B.Weyssow, Proceedings of the International Conference on Plasma Physics 2000 / APS-DPP Conference, Québec 2000, 49 - 53.

[4] f was assumed to be tranlationally invariant along the field, i.e. independent of z.

[5] e.g. N.Rostoker, Phys. Fluids 3 (6), 922 (1960); P.P.J.M.Schram, Physica **45** (1969) 165; D.Montgomery, L.Turner, G.Joyce, Phys.Fluids **17** (5) (1974) 954.

[6] Remember that the $D_{\perp,||}(t)$ are related to the *inverse* of the relaxation time; see e.g. D. C. Montgomery & D. A. Tidman *Plasma Kinetic Theory*, McGraw-Hill 1964.