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To the Theory of Parametric Instability of the Cyclotron Radiation in Reactive Electron Medium.

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I. Noticeable attention has been recently paid to the investigation of different processes of parametric interaction of bichromatic radiation with modulated electron ensembles, that is mainly caused by revision of traditional notion of classes of charged particles that are capable of stimulated emission. The first effect that is worth noting here is so-called maser without inversion (MWI) [1]. It is the generation of cyclotron radiation at two harmonics of gyrofrequency by a modulated at difference harmonic ensemble of electrons with "inversionless" energy distribution (stable against monochromatic generation). The amplification mechanism in such scheme corresponds to the parametric interaction of the modes in the medium with modulated active susceptibility, i.e. modulated distribution function of particles resonant to both partial HF waves (and not only their beats, as ordinarily happens in the standard induced scattering).

The present paper is devoted to the investigation of another novel effect of parametric generation of cyclotron radiation, which was recently discovered as result of developing of the theory of MWI in another, more realistic scheme [2]. This effect is the instability of the bichromatic radiation due to the modulation of the reactive susceptibility of the electron ensemble. In this scheme two Brilluin components of the waveguide mode with the same transverse structure (with respect to the constant magnetic field $\vec{B} = \vec{z}_0 B_0$) and different frequencies $\vec{E} = \sum_{j=1}^{2} \vec{E}_j \exp(ik_\perp x + ik_{\parallel j} z - i\omega_j t)$, that are resonant to the electrons with momentum components $p_\parallel^R, p_\perp^R$ at the first harmonic of the cyclotron frequency $\omega_j = \frac{eB_0}{mc\gamma_R} + k_{\parallel j} v_\parallel^R$, interact with ensemble of electrons with momentum components close (*but not equal*) to the resonant values. The electron ensemble is described by the unperturbed distribution function, modulated in time and space providing the parametric coupling of HF waves $f = f_0\left(p_\parallel, p_\perp^2/2\right) + f_M\left(p_\parallel, p_\perp^2/2\right)\cos\left(\varphi_M + \left(k_{\parallel 1} - k_{\parallel 2}\right)z - (\omega_1 - \omega_2)t\right)$. The analysis in [2] is based on solving in linear approximation of kinetic equation in truncated variables, which submit under the resonant approximation to the equations of "nonlinear pendulum", and wave excitation equations with resonant harmonics of the current, expressed in terms of the distribution function. It is shown that in this system the simultaneous amplification of two waves is possible in the absence of particles resonant to these waves. Due to the specific dependence of cyclotron frequency detuning on momentum allowed for both relativistic dependence of gyrofrequency on energy and Doppler shift: $\Delta_j = \omega_j - eB_0/mc\gamma - k_{\parallel j}v_{\parallel}$ it is possible to set such modulation of distribution function in vicinity of common resonant point $p_{\parallel}^R$, $p_{\perp}^R$, that particle density in momentum space expressed via Δ_1 and Δ_2 will oscillate in opposite phase. The dependence $f_M\left(p_{\parallel}, p_{\perp}^2/2\right)$ must be like this $f_M\left(p_{\parallel} - p_{\parallel}^R, p_{\perp}^2/2\right) = -f_M\left(p_{\parallel}^R - p_{\parallel}, p_{\perp}^2/2\right)$. As consequence the oscillations of corresponding susceptibilities (medium responses on the first and second field) will be “antiphase”. Then if all particles are out of the resonance with waves, i.e. $\Delta_j\left(p_{\parallel}, p_{\perp}\right)\tau >> 1$ where τ -is the interaction time, but $\left(k_{\parallel 1} - k_{\parallel 2}\right)\left(V_{\parallel} - V_{\parallel}^R\right)\tau << 1$ the parametric coupling of these two waves will assume their simultaneous amplification. The linear increment of such amplification is found in [2].

II. This result does not make clear the mechanism of energy exchange between the bichromatic field and medium, which is accompanied by amplification (or absorption) of two waves simultaneously but not by scattering from the field of one frequency to another, as in standard induced scattering, when the energy exchange is connected with the frequency transformation in scattering process. Let us now to clarify this effect, solving nonlinear equations of particle motion in the field of constant amplitude, going over to the accompanying frame of reference, where the frequencies of two waves are equal, thus excluding the interpretation of this energy exchange as change of photon energy in scattering process. Consider two circulary polarized waves propagating for simplicity along the constant magnetic wave. So the electric and magnetic field can be written as: $\vec{E} = \mathrm{Re}\,\vec{e}_+ \mathrm{E}(z) e^{-i\omega t}$, $\vec{B} = \mathrm{Re}\,\vec{e}_+ \mathrm{B}(z) e^{-i\omega t} + \mathrm{B}_0 \vec{z}_0$, $\mathrm{E}(z) = \mathrm{E}_0\left(e^{ikz} + e^{-i\varphi_0 - ikz}\right)$, $\mathrm{B} = -\left(c/\omega\right)\partial \mathrm{E}(z)/\partial z$, $\vec{e}_+ = \vec{x}_0 + i\vec{y}_0$. Consider the relativistic equations of particle motion in this field with initial conditions: $p_{\parallel}\left(t=0\right) = P_{\parallel}^0 = m\gamma_0 V_{\parallel}^0$, $\vec{p}_{\perp}\left(t=0\right) = \vec{P}_{\perp}^0 = m\gamma_0 \vec{V}_{\perp}^0$, $z\left(t=0\right) = Z_0$. Suppose the following resonant conditions to be fulfilled: $\left(\left|p_{\perp}\right|^2 - \left|p_{\perp R}\right|^2\right)/\left|p_{\perp R}\right|^2 << 1$, $\left|p_{\parallel}\right| << \left(\left|p_{\perp}\right|^2 - \left|p_{\perp R}\right|^2\right)/mc\gamma_R$ signifying that Doppler shift is much smaller that cyclotron detuning: $|kV_{\|}| << |\Delta_0|$, where $\Delta_0 = \omega - eB_0/mc\gamma$, and that there is large resonant parameter $R = (|V_\perp|/c)^2 (\omega/\Delta_0) >> 1$. Suppose that the electron is not resonant to the field, i.e. $t\Delta >> 1$, then its motion can be presented as superposition of slow part and term oscillating in the wave field: $\vec{p} = (P_{\|} + \tilde{p}_{\|})\vec{z}_0 +$ $+ \mathrm{Re}(P_\perp + \tilde{p}_\perp)\vec{e}_+ \exp(-i\, eB_0 t/mc\gamma_0)$. Setting the wave amplitude to be rather small so that $\frac{1}{\gamma_0}\frac{e|\mathrm{E}_0|}{mc}\frac{\omega}{\Delta_0^2}\frac{|V_\perp^0|}{c} << 1$, we can write $\mathrm{E}(z) = \mathrm{E}(Z) + \xi\frac{\partial \mathrm{E}(Z)}{\partial Z}$, $Z = V_{\|}^0 t + z_0$, $z = Z + \xi$. Developing the theory of successive approximations we find in linear approximation the averaged values for squared oscillating parts of momentum: $\langle \tilde{p}_{\|}^2 \rangle = e^2 |\mathrm{E}_0|^2 \frac{|V_\perp|^2}{c^2}\frac{1}{\Delta_0^2}(1-\cos(2kZ+\varphi_0))$, $\langle |\tilde{p}_\perp|^2 \rangle = e^2|\mathrm{E}_0|^2 \frac{1}{\Delta_0^2}(2 + R^2 - 2R)(1-\cos(2kZ+\varphi_0))$

In the square approximation we derive the equation for slow longitudinal motion: $dP_{\|}/dt = -R\,2e^2|\mathrm{E}_0|^2/\gamma_0 mc\Delta_0 \sin(2kZ+\varphi_0)$, and for the slow variation of the amplitude of gyrorotations: $\frac{d}{dt}|P_\perp|^2 = 2kV_{\|}e^2|\mathrm{E}_0|^2\frac{1}{\Delta_0^2}\sin(2kZ+\varphi_0)(R^2+2R)$. Then it becomes possible to find the expression for the evolution of electron energy averaged over large number of oscillations in wave field in the square approximation in the wave amplitude: $\frac{d}{dt}\langle w \rangle \approx \frac{1}{2m\gamma_R}\left\{\frac{d}{dt}|P_\perp|^2 + \frac{d}{dt}P_{\|}^2 + \frac{d}{dt}\langle|\tilde{p}_\perp|^2\rangle + \frac{d}{dt}\langle\tilde{p}_{\|}^2\rangle\right\}$. We obtain that alteration of the mean electron energy under action of the standing wave is not zero if the time of interaction is limited: $kV_{\|}t << 1$. The change of transverse energy is $4\omega/\Delta_0 >> 1$ times larger than change of longitudinal energy, so that $d\langle w \rangle/dt \approx d\langle w_\perp \rangle/dt = dw_\perp^{sl}/dt + d\langle \tilde{w}_\perp \rangle/dt \approx$ $4\left(kV_{\|}e^2|\mathrm{E}_0|^2/m\gamma_0\Delta_0^2\right)R\sin(2kZ+\varphi_0)$. Since $d\langle w \rangle/dt \propto -1/\Delta_0^3\left(V_{\|}^0\,\partial|E|^2/\partial z\right)$ the energy of particle decreases if it moves in direction of stronger field in case $\Delta_0 > 0$, and in direction of weaker field if $\Delta_0 < 0$. It becomes obvious, that setting the initial modulation on z of longitudinal velocity of electrons in ensemble in correspondence with spatial dependence of field amplitude, so that $\varphi_0 = \varphi_M + \pi/2 + Sign\Delta_0$ (φ_M -phase of modulation) the decrease of the energy of electron ensemble due to its interaction with nonresonant wave field is obtained. So the calculation of the evolution of energy of electron ensemble by the formula

$$\frac{d\langle W\rangle}{dt} = \int \frac{d\langle w\rangle\left(P_\parallel^0, P_\perp^0, z_0, t\right)}{dt}\left(f_0\left(P_\parallel^0, \frac{\left|P_\perp^0\right|^2}{2} \right) + f_M\left(P_\parallel^0, \frac{\left|P_\perp^0\right|^2}{2} \right)\cos\left(\varphi_M + 2kz_0\right) \right) dP_\parallel^0 d\frac{\left|P_\perp^0\right|^2}{2} dz_0 =$$

$= -\mu\left|E_0\right|^2$ gives the same result for increment $\mu = \frac{2e^2\omega}{m\gamma_R^2}\frac{\left|V_\perp^R\right|^2}{c^2}\left|\int \frac{p_\parallel}{mc} f_M\left(p_\parallel, \frac{\left|p_\perp\right|^2}{2} \right)\frac{1}{\Delta_0^3} dp_\parallel d\frac{\left|p_\perp\right|^2}{2}\right|$ as that obtained in [2] from linear theory. It was also important to find out, might the energy extracted from the medium during parametric "nonresonant" interaction $\Delta\langle W\rangle$ be larger than energy exchange in short time period while $t\Delta_0 < 1$ during "quasi resonant" interaction $\delta\langle W\rangle = \langle W\rangle_{t=0} - W_0$. If not, the effect is not so interesting. We have shown that in the analyzed situation (so cold autoresonance) when $k_\perp = 0$, $\omega/k_\parallel = c$ the ratio $\Delta\langle W\rangle/\delta\langle W\rangle \div R\ (V_\parallel kt)$, so it can be larger than 1. It can de shown that if $\frac{\omega}{k_\parallel} \neq c$ the ratio $\frac{\Delta\langle W\rangle}{\delta\langle W\rangle} \div \frac{n_\parallel^2+1}{n_\parallel^2-1}(V_\parallel kt)$ also can be large. It is also worth noting, that resonant interaction results in the redistribution of energy between the energy of gyrorotations $W_\perp^{sl}$ and averaged oscillatory transverse energy $\langle\tilde{W}_\perp\rangle$. The source of field amplification for $\Delta_0 > 0$ is $W_\perp^{sl}$ and for $\Delta_0 < 0$ is $\langle\tilde{W}_\perp\rangle$.

III. From the standpoint of the quantum analogy the elementary acts of electron-bichromatic field interaction in the absence of partial resonances may be only scattering processes from one photon to another. Then the Manley-Rowe relation seems to be fulfilled, that forbids the amplification of both waves. But two notes can make things clear. The time of interaction in this effect must be limited by condition, $\left(k_{\parallel 1} - k_{\parallel 2}\right)\left(V_\parallel - V_\parallel^R\right)t << 1$ which means that particles are resonant to the beat wave. The restriction of time means that the number of photons is the undefined quantity. Besides single act of scattering can not be considered as elementary one in this effect. Every scattering act "field1→field2" must be supplemented with reversed one "field2→field1" with emission prevailed over absorption in both these acts. Such combination results in amplification of two waves simultaneously.

References

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2 M. A. Erukhimova and M. D. Tokman, JETP 91 (2), 2000, 255-264