Nonlocal Electron Parallel Heat Transport in Divertor Plasmas and Atomic Physics Rates

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In steep parallel electron temperature gradients, such as those found in the divertors of magnetic fusion devices, it has long been known that the electron heat transport is nonlocal, and that the electron velocity distribution function (EVDF) is non-Maxwellian [1,2,3]. In particular, high energy electrons in the tail of the EVDF affect macroscopic properties, such as the rates of ionization and excitation, in the cold region even where there is no steep temperature gradient. We study in this work some effects of a nonlocal transport on the temperature profile and on the atomic processes for atomic hydrogen in an advancing heat front. Our aim is to develop a nonlocal heat flow model that would allow a hydrodynamic code such as “UEDGE” [4] to have not only a correct heat flux along the field lines, as is to be expected from a nonlocal heat flux formula, but also to account for the effects of the non-Maxwellian distributions on the atomic rates with a computational cost comparable to ordinary flux-limited heat flow calculations, as opposed to the prohibitively long computation times required for electron kinetic simulations. Here, these are used only to establish the nonlocal heat flow formulas and to validate the results in some specific cases.

Our new nonlocal model was first developed for laser heated plasmas [5] and we want to apply it here to determine the temperature profile in the divertor region. The nonlocal heat flux is assumed to be written as a convolution between the Spitzer-Härm heat flux and a kernel that takes into account the contribution of supra-thermal electrons ($v > 3v_{th}$). An analogous formula was developed to compute the angle averaged distribution function, $F_0(x,v)$, as a convolution over the local Maxwellians [5]. In Fig.1, we present the electron temperature profiles as given by our model and by the electron kinetic (Fokker-Planck) code FPI [1,2] at two different times. We used a 1D box (length=20m) with the ions treated like a cold immobile fluid and typical divertor parameters, $Z=1$, $n_e = 2.5 \times 10^{13} \text{cm}^{-3}$ (uniform). The left side of our box is maintained at a constant temperature of 25 eV and the right side at 1 eV. Thus the heat front propagates from left to right. The effect of the atomic collisions on the electron distribution function was not accounted for in this work. The temperature profile
at $t=30$ $\mu$s from our model is seen to be in much better agreement with the kinetic simulation than those obtained with flux limited heat diffusion, for any value of the flux limiter.

We now present some of the effects of a non-Mawellian EVDF on the atomic physics. We consider a hydrogen plasma with both ionized and neutral H. The atomic processes taken into account are listed below with their rates. Molecular processes are not taken into account, nor sub-level splitting; only the main quantum number is kept (1-30).

**Ionization:** $e + H(i) \rightarrow e + e + H^+$  
Rate: $S_i$ (cm$^3$ s$^{-1}$)

**Three body recombination**  $e + e + H^+ \rightarrow H(i) + e$  
Rate: $\alpha_i$ (cm$^6$ s$^{-1}$)

**Excitation and deexcitation**  $e + H(i) \rightarrow e + H(j)$  
Rate: $C_{ij}$ (cm$^3$ s$^{-1}$)

**Radiative recombination:**  $e + H^+ \rightarrow H(i) + hv$  
Rate: $\beta_i$ (cm$^3$ s$^{-1}$)

**Radiative desexcitation:**  $H(j) \rightarrow H(i) + hv$  
Rate: $A_{ji}$ (s$^{-1}$)

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**Figure 1:** Temperature profiles during heat front propagation. (•) Fokker-Planck (FPI) results at 5 and 30 $\mu$s; and from the nonlocal model (AM), and flux limited diffusion at 30 $\mu$s.
Note that the plasma does not reabsorb the photons. To compute the elementary rates \( S_i, \alpha_i, C_i, \beta_i, A_{j\beta} \), we use the cross sections given by R.K. Janev and J.J. Smith [6] and for \( f(v) = F_0(x,v,t) \), either a Maxwellian distribution or a non-Maxwellian computed with the kinetic Fokker Planck code FPI, or that computed with our nonlocal model.

\[
\text{Rate} = 4\pi \int_{V_{ef}} dV v^3 \sigma(v)f(v)
\]

(1)

We denote by \( N_i \) the density of neutrals in level \( i \) and by \( N_e \) the density of electrons in the continuum. The time evolution of the population of the level \( i \) is then given by:

\[
\left( \frac{dN_i}{dt} \right)_{j=1,\infty} = -N_e N_i S_i - N_e N_i \sum_{j=1}^{\infty} C_j - N_i \sum_{j=1}^{\infty} A_j + N_e \sum_{j=1}^{\infty} N_j C_j + \sum_{j=1}^{\infty} N_j A_{j\beta} + (\beta_i + \alpha_i N_e)N_e^2
\]

(2)

Our code can either directly integrate (2) (we limit the number of levels to 30) and follow the time evolution of each level; or, use a collisional radiative ansatz, assuming that the time scale for the fundamental level \( n=1 \) is very large compared to the time scales of the excited levels. It can be shown that (2) is then equivalent to:

\[
\frac{dN_i}{dt} = -S_{\text{eff}} N_i N_e + R_{\text{eff}} N_e^2 \quad \text{with} \quad N_e + N_i = N_0
\]

(3)

where \( S_{\text{eff}} \) and \( R_{\text{eff}} \) are the effective ionization and recombination rates. They depend nonlinearly on the elementary rates \( S_i, \alpha_i, C_i, \beta_i, A_{j\beta} \). This simplified method is applied in the code UEDGE, which uses a table of \( S_{\text{eff}} \) and \( R_{\text{eff}} \) as functions of \( N_e \) and \( T_e \), previously computed and tabulated from the detailed model. We have verified that these rates are indeed recovered when the EVDF is Maxwellian. When there is a surplus of energetic electrons in the EVDF, the rate of ionization from the ground state, \( S_1 \), is greatly enhanced, and the excitation rates from the ground state, \( C_{1n} \), are moderately increased, as was shown in [3], but it remained to be seen how the global rate, \( S_{\text{eff}} \), is affected. Rates from excited states and for radiative recombination are negligibly affected. We have not yet computed non-Maxwellian rates for collisional recombination and deexcitation, but these are expected to be very close to the Maxwellian ones, as these processes are due mostly to slower electrons. When a non-Maxwellian EVDF is used, our calculations of elementary rates, (see table I), show that only the ionization and excitation rates from the fundamental \( S_i, C_{ij} \), differ significantly from the Maxwellian ones, and that the nonlocal model can account for much of the difference. We are presently working on improving this aspect of the model. We show on fig 2 the temperature profile \( T_e(x) \) and the effective ionization rate \( S_{\text{eff}} \) computed with the Maxwellian and non-Maxwellian EVDF.
Fig. 2. Temperature and $S_{\text{eff}}$ computed with Maxwellian EVDF and non-Maxwellian EVDF.

<table>
<thead>
<tr>
<th></th>
<th>Maxwellian</th>
<th>Non-Maxwellian</th>
<th>Relative difference with the Maxwellian</th>
<th>Nonlocal model</th>
<th>Relative difference with the Maxwellian</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>0.27258E-13</td>
<td>0.76298E-12</td>
<td>36.8</td>
<td>0.182917E-11</td>
<td>66.1</td>
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<td>$S_2$</td>
<td>0.57196E-08</td>
<td>0.56934E-08</td>
<td>4.5 E-3</td>
<td>0.151262E-09</td>
<td>0.97</td>
</tr>
<tr>
<td>$C_{12}$</td>
<td>0.28403E-11</td>
<td>0.36287E-11</td>
<td>0.27</td>
<td>0.261669E-11</td>
<td>0.08</td>
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<tr>
<td>$C_{20}$</td>
<td>0.71158E-16</td>
<td>0.48813E-15</td>
<td>5.89</td>
<td>0.128811E-14</td>
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<td>$C_{23}$</td>
<td>0.11396E-06</td>
<td>0.11374E-06</td>
<td>2.2 E-3</td>
<td>0.535640E-08</td>
<td>0.95</td>
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<tr>
<td>$S_{\text{eff}}$</td>
<td>0.16215E-12</td>
<td>0.10767E-11</td>
<td>5.6</td>
<td>0.185317E-11</td>
<td>10.4</td>
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<tr>
<td>$R_{\text{eff}}$</td>
<td>0.95680E-12</td>
<td>0.95553E-12</td>
<td>0.001</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table I: Rate calculations at $X=11.2$ m, with Maxwellian and non-Maxwellian EVDF from the FPI simulation, and from the nonlocal model.