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# The Upper Hybrid Resonance in the Turbulent Plasma

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The method of enhanced microwave scattering is based on the measurement of the signal, which is Back Scattered (BS) in the Upper Hybrid Resonance (UHR) vicinity. At first this technique was applied to small-scale oscillations and waves investigations in quiet plasma of linear laboratory devices [1]. But now this method is introduced for a study of plasma small-scale turbulence and wave propagation in the tokamak plasma [2]. In this case the incident wave propagation and the scattering take place in the plasma with dominating long-scale density fluctuations in the background, which can modify the BS spectra. The frequency spectrum broadening can be associated not only with the frequency of small-scale fluctuations, but also with spectral broadening of incident and BS waves due to small-angle scattering off the long-scale turbulence component. As it was shown in [3] the small-angle scattering cross-section increases sharply in the vicinity of the UHR. Owing to this cross-section growth a non-linear regime of a multiple small-angle scattering arises. In the present paper the effect of the multiple small-angle scattering due to density fluctuations in the UHR vicinity on the incident and BS wave spectrum is analyzed. The multiple small-angle scattering influence on enhanced scattering correlation analysis [4] is also considered.

**1. The probing wave frequency broadening**

The effect of small-angle scattering has the nature of a phase modulation. So it can be taken into account by the inserting of the turbulent phase variation in the expressions for the wave electric field in the eiconal approximation. The phase variation is an integral of the density fluctuation along the wave ray trajectory with a weight factor, which takes into account the enhancement of the small-angle scattering. For the extraordinary wave in the UHR vicinity it takes form

$$\delta\Psi(x,t) = -\frac{k_c}{2n_{UH}}\int_0^x d\chi \frac{\delta n(\chi,t)}{\left(-\varepsilon(\chi)\right)^{3/2}}, \tag{1}$$

where $k_c = \omega_{ce}(x_{UH})/c$; $\varepsilon = (x - x_{UH})/\ell$ is an element of permittivity tensor, $x = 0$ is an antenna position and $\delta n/n_{UH}$ is the relative density fluctuation in the UHR. This approach allows calculating the averaged electric field extinction due to the multiple small-angle scattering. Assuming a normal distribution of a random value $\delta\Psi$ the averaged electric field is given by:

$$\langle E\rangle \propto \langle e^{i\delta\psi}\rangle = e^{-\langle \delta\psi^2\rangle/2} = \exp\left\{-\frac{\langle \delta n^2\rangle}{n_{UH}^2}R(x)\right\},\ R(x)=\begin{cases} \ell\ell_c k_x^4(x)/16k_c^2, & x_{UH}-x \gg \ell_c \\ \ell^2 k_x^2(x)/2, & x_{UH}-x \ll \ell_c \end{cases} \quad (2)$$

where $k_x(x)=k_c\sqrt{\ell}/\sqrt{x_{UH}-x}$ is the extraordinary wave wavevector projection on the inhomogeneity direction, $\ell_c$ is the fluctuations correlation length. The correlation function

$$K(x,\tau)=\left(\langle A(x,t)A^*(x,t+\tau)\rangle - \langle A(x,t)\rangle\langle A(x,t+\tau)\rangle^*\right)\Big/\left\langle \left|A(x,t)-\langle A(x,t)\rangle\right|^2\right\rangle, \quad (3)$$

where $A(x,t)=e^{i\delta\Psi(x,t)}$, allows us to evaluate the frequency spectrum of the multiple small-angle scattered wave. Assuming the normal distribution of $\delta\Psi$ we get such expression for this correlation function:

$$K(x,t)=\left(e^{\langle \delta\Psi(x,t)\delta\Psi(x,t+\tau)\rangle}-1\right)\Big/\left(e^{\langle \delta\Psi^2\rangle}-1\right) \quad (4)$$

In a non-linear case, when the field correlation time is much less then the fluctuations correlation time $\tau/t_c \ll 1$, the correlation function takes the following simple form:

$$K=\exp\left\{-\frac{\langle \delta n^2\rangle}{n_{UH}^2}R(x)\langle \Omega^2\rangle \tau^2\right\} \quad (5)$$

where $\langle \Omega^2\rangle$ is the averaged square of the long-scale fluctuations frequency. The corresponding probing wave spectrum width is given by

$$\Delta\omega \approx 2\sqrt{R(x)\langle \Omega^2\rangle\langle \delta n^2\rangle/n_{UH}^2} \quad (6)$$

It is well known that in the UHR vicinity, when $k_x \sim \omega/c\cdot\sqrt{c/v_{Te}}$, the extraordinary wave transforms into Bernstein mode, which propagates from the UHR. Near the UHR the wavenumber of Bernstein wave takes form: $k_x=\sqrt{-\varepsilon(x)}/\ell_T$, where $\ell_T$ is associated with the thermal particle motion, $\ell_T \sim \rho_{ce}$, where $\rho_{ce}$ is the electron cyclotron radius. The Bernstein wave spectrum width can be also evaluated with the same method. Near the UHR it is proportional to the wavenumber and equal to the extraordinary wave spectrum width in this region:

$$\Delta\omega \approx \ell k_x(x)\sqrt{2\langle \Omega^2\rangle\langle \delta n^2\rangle/n_{UH}^2},\ x_{UH}-x \ll \ell_c \quad (7)$$

And during the propagation from the UHR the broadening increase practically stops:

$$\Delta\omega \approx \sqrt{\frac{\ell\ell_c}{\ell_T^2}\ln\left(k_x^2(x)\ell_T^2/\ell_c\right)\langle \Omega^2\rangle\langle \delta n^2\rangle/n_{UH}^2},\ x_{UH}-x \gg \ell_c \quad (8)$$

## 2. The correlation technique wavenumber resolution

As we have shown the small-angle scattering effects disturb the probing wave propagation determining the formation of the BS wave frequency spectrum. So we can anticipate that the long-scale turbulence influence on the correlation ES diagnostics is essential too. This method is based on the dependence of the BS signal on the fluctuation phase in the scattering point and allows getting the fluctuations wavenumber spectrum [4]. Taking into account that the main input into BS signal is produced by the linear conversion point, where the eiconal approximation fails, we shall use the accurate expressions for resonant component of the electric field

$$E(\xi) = \sqrt{\frac{8}{\gamma^2 \ell \omega}} \int_0^\infty d\kappa \exp\left[ i\left( \frac{\kappa^3}{3} - \frac{b}{\kappa} + \xi\kappa \right)\right] \qquad (9)$$
$$\gamma = \left(\ell_T/\ell\right)^{2/3},\ \xi = \left(x - x_{UH}(t)\right)/(\gamma\ell),\ b = (k_c\ell)^2\gamma$$

where the long scale fluctuations are accounted for by using a long-scale approximation:

$$x_{UH}(t) = x_{UH} - \ell\,\delta n(t)/n_{UH}\ , \qquad (10)$$

modeling the movement of the UHR due to the long-scale fluctuations but neglecting their influence on the density gradient. Substituting expressions (9), (10), for the probing wave field into the reciprocity theorem in the form proposed by Piliya in [1] we obtain the following formula for the cross-correlation function of BS signals at two probing frequencies

$$\left\langle A_{s1}(t) A_{s2}(t+\tau)\right\rangle = \frac{1}{\gamma^2 n_{UH}^2} \int_{-\infty}^{\infty} \frac{d\vec{Q}}{(2\pi)^3} \left|\delta n_{\vec{Q},\Omega}\right|^2 \left|U(\gamma\ell Q_x)\right|^2 e^{iQ_x\Delta x_{UH}} F\left(Q_y, Q_z\right) K\left(Q_x, \Delta x_{UH}, \tau\right) \qquad (11)$$

Here $\left|U(\gamma\ell Q_x)\right|^2$ is the factor, which describes the scattering enhancement [1], $\Delta x_{UH}$ is the distance between UHR positions for two probing waves, $F\left(Q_y, Q_z\right)$ is the term, which is determined by the antenna diagram, and $K\left(Q_x, \Delta x_{UH}, \tau\right)$ is the additional factor, which is caused by the multiple small-angle scattering. The last term describes the correlation decay due to long-scale turbulence and has a following form:

$$K = \exp\left\{ -\left(\Delta x_{UH}\right)^2 \delta Q_x^2 - \tau^2 \Delta\omega^2 \right\} \qquad (12)$$

where $\delta Q_x$ determines the wavenumber resolution of the diagnostics and $\Delta\omega$ corresponds to the frequency spectrum broadening. For the BS wave in the region of the maximum efficiency of BS we have the BS wave spectrum width:

$$\Delta\omega \approx \frac{1}{\sqrt{2}} \ell Q_x \sqrt{\left\langle \Omega^2 \right\rangle \left\langle \delta n^2 \right\rangle} \Big/ n_{UH}\ , \qquad (13)$$

where $Q_x$ is the radial wavenumber of the fluctuation, which matches expressions (4) and (5) after the substitution of the Bragg condition $2k_x = Q_x$. The prescribed behavior of the broadening agrees with one, which was observed in the experiments on FT-1 tokamak [2]. Using the observed spectrum width $\Delta f = \omega/(2\pi) \approx 5.4\cdot 10^5\, Hz$, $Q_x \sim 100\ cm^{-1}$, assuming the drift nature of the long-scale turbulence: $\bar{\Omega} \approx v_d q_0$ and FT-1 tokamak parameters, which give the characteristic wavenumber of the long-scale turbulence $q_0 \sim 2\ cm^{-1}$, and the drift velocity $v_d \sim 2\cdot 10^5\ \ cm/s$, we get a sensible estimation for the relative amplitude of the long-scale fluctuations: $\delta n/n_{UH} \sim 0.1$

The diagnostics radial wavenumber resolution in the region of the maximum efficiency of BS takes the following simple form

$$\delta Q_x \approx \frac{1}{\sqrt{2}} \ell Q_x \sqrt{\langle q_x^2 \rangle \langle \delta n^2 \rangle} \Big/ n_{UH} \tag{14}$$

where $q_x$ - is a radial wavenumber of the long-scale fluctuations. So the multiple small-angle scattering limits the wavenumber resolution of the diagnostics and must be taken into account for a proper interpretation of the diagnostics results.

**3. Conclusions**

In this paper the influence of the long-scale turbulence on the ES diagnostics is investigated. The unusual behavior of the BS wave frequency spectrum, which was observed on FT-1 tokamak, is explained. The influence of the multiple small-angle scattering on the wavenumber resolution of correlation ES technique is analyzed. The additional possibility to get some information about the long-scale turbulence from the ES diagnostics results arises.

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