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Study of the Accuracy of Mach Probes to Measure the Parallel and Perpendicular Flow in the Plasma Edge

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1. Introduction

Different methods have been developed to measure plasma flow parallel and perpendicular to the magnetic field [1,2,3]. These measurements are relevant to study flows induced by biasing experiments and the suppression of turbulence due to flow shear [4]. We have developed a method based on an inclined Mach probe [5] in which we show that it is possible to deduce the flow velocities from the measurement of the ion saturation currents flowing to the probe's surface. The underlying theoretical model is an extension of the fluid approach by Hutchinson [6] valid in strong magnetic fields. As the resulting differential equations are impossible to solve analytically, an approximate analytical solution was proposed. In this paper we will compare the exact solution of the differential equation with the results of the analytical approximation in order to assess the accuracy of our approach. In the second part we discuss and support our choice of collectors for a recently designed probe. In the literature different types of probe designs are proposed. As for the collectors, one can distinguish two types: a Mach probe with flat collectors [7] and the so-called ideal Gundestrup probe (IGP) with curved (round) collectors [8]. We will study the possible involved errors on the measurement of parallel and perpendicular Mach numbers using those different collector shapes.

2. Accuracy study of Mach probe measurement

The density at the probe surface, which is needed to compute the ion saturation current can be related to the unperturbed plasma flow (the quantity to be determined) in the following way [5]:

$$\frac{\partial n}{\partial M_{//}} = n\frac{\left(M_{//}-\frac{M_{\perp}}{\tan(\theta)}\right)(1-n)-\left(M_{//,\infty}-M_{//}\right)}{-(1-n)+\left[\left(M_{//}-\frac{M_{\perp}}{\tan(\theta)}\right)\left(M_{//,\infty}-M_{//}\right)\right]} \quad , (1) \qquad M_{//,MPSE}=\frac{M_{\perp}}{\tan(\theta)}\pm 1 \quad . (2)$$

The dimensionless parallel and perpendicular Mach numbers are defined as $M_{//,\perp} = v_{//,\perp}/c_s$ with v the velocity of the ions and c_s the sound speed. The angle between the collecting area and the magnetic field is θ. The density evolution in the pre-sheath $n_{pre-sheath}$ is normalized with respect to the unperturbed density n_∞. Eq. (1) is integrated from the unperturbed region ($M_{//,\infty}$=1, $n_\infty = 1$) towards the probes' up- and downstream surfaces, determined by the Bohm-Chodura boundary condition (2). The resulting densities at the surfaces n_{up} and n_{down} are then used to compute the ratio R of the ion saturation currents, from which we will determine the Mach numbers. As an example, Fig. 1 shows the evolution of n as a function of $M_{//}$ for $M_{//,\infty} = 0.2$ and different values for $M_\perp$ and θ. The curves end at the magnetic pre-sheath entrance (MPSE), thereby defining the values $M_{//,MPSE}$ and n_{sheath} fulfilling the boundary condition (2) implemented in the model. Varying $0 < M_{//,\infty} < 1$ for a

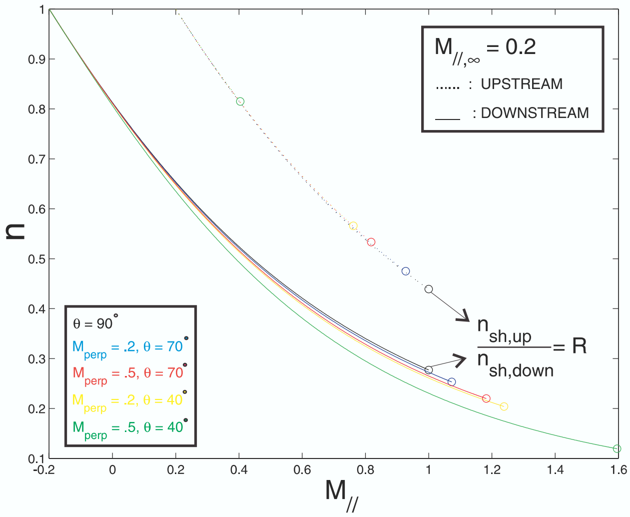

Figure 1: Numerical solutions for the normalized density as a function of Mach number in the pre-sheath.

given perpendicular Mach number and probe angle and retaining the associated MPSE densities, a direct relation between n_{sh} and $M_{//,}$ is obtained. An approximated analytical solution of equation (1) has been proposed by Hutchinson for parallel flow [6] and was extended by Van Goubergen [5] for perpendicular flow, resulting in

$$n_{sh,up,down} = \exp\left[c_{up,down}\left(M_{//,\infty}, M_{\perp}, \theta\right)\left(\pm\left|M_{//,\infty}\right| \pm \frac{\left|M_{\perp}\right|}{\tan(\theta)}\right) - c_0\right] . \tag{3}$$

The parameters $c_{up,down,}$ and c_0 are introduced, to obtain an acceptable agreement with the exact numerical solutions. The ratio of the up– and downstream saturation currents, which will be used in the experiment, with $c=c_{up}+c_{down}$, reads:

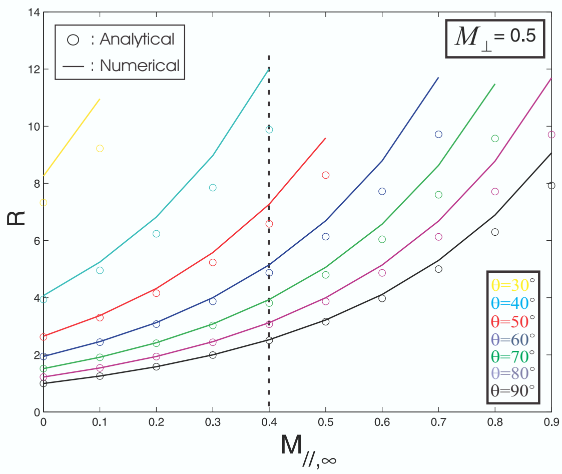

Figure 2: The ratio R as a function of $M_{//,\infty}$ for different inclination angles of the probe at fixed $M_{\perp} = 0.4$.

$$R = \exp\left[c\left(M_{//,\infty}, M_{\perp}, \theta\right)\left(\left|M_{//,\infty}\right| + \frac{\left|M_{\perp}\right|}{\tan(\theta)}\right)\right]. \tag{4}$$

To achieve a perfect agreement of (4) with the numerical solution, the parameter c has to be slightly varied with $M_{//,\infty}$. As this approach is impossible in practice we keep for the following a constant c=2.3, which is the best choice for very small flows. In Fig. 2 we are comparing the numerical and analytical solutions for a constant $M_{\perp} = 0.5$. The disagreement between both approaches is due to the fact that in the analytical solution c has been taken constant, since its dependence on $M_{//,\infty}$ is unknown. However, c changes with $M_{//,\infty}$ and θ for a fixed $M_{\perp} = 0.5$. For $M_{//,\infty} = 0.4$, we have found a weak dependency of c on θ with a small disagreement for θ =90° (due to the dependency on $M_{//,\infty}$) and an error of 6% for θ=40°. Analogously we can take $M_{//,\infty} = 0$ and θ=40° constant and study the influence of $M_{\perp}$ on c. Again c is slightly increasing with $M_{\perp}$. For $M_{\perp} = 0.5$ we find an error of 3%. The discrepancy of the numerical solutions from the analytical ones increases almost linearly with $M_{//,\infty}$. For θ=40° and $M_{\perp} = 0.5$ we find 15% deviation from the numerical solutions. To minimize the error we applied an exponential least square fit. The results give a c_o=-1,07 and $c_{up,down}$=1.105 and hence a c of 2.21, which is the best choice to obtain the most accurate determination of the Mach numbers.

3. Competitive study on the collector shape

From the manufacturing point of view a probe with round (curved) collectors is more recommended than one with flat collectors. However, when considering round collectors the flux tubes change along the probe surface and so does the contribution of the perpendicular flow (Fig. 3).

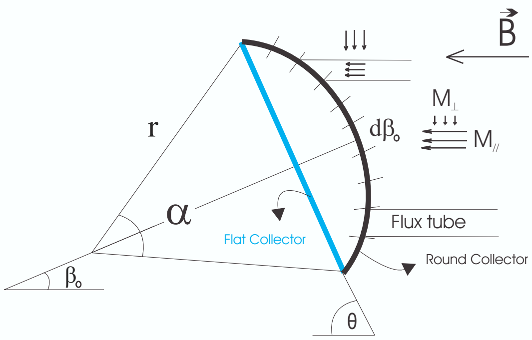

Figure 3: Geometry of a flat and round collector.

This means that in the case of a round collector the current of each collector is proportional to the integral of the density over the changing angle.
Using flat collectors the angle θ between the collector surface and the magnetic field, thus the contribution of $M_\perp$, stays constant along the collecting area and the current is directly related to $M_\perp$. To investigate the effect of the curved surface we use in both cases the analytical relation between the sheath edge density and the ion saturation current according to Eq. (3) with the new values for $c_{up,down}$. Using the convention of Fig. 3 we find for the flat collectors

$$I_{i,sat,up,down} = n_{sh,up,down} e c_s A \cos\beta_0 \ , \qquad (5)$$

$$R_{flat} = \exp\left[c\left(M_{//,\infty} - M_\perp \tan\left(\beta_0\right)\right)\right], \qquad (6)$$

while for the curved collector we find

$$I_{i,sat,up} = b r n_\infty e c_s \, \mathrm{e}^{-c_0} \, \mathrm{e}^{c_{up}\left|M_{//,\infty}\right|} \int_{\beta_0-\frac{\alpha}{2}}^{\beta_0+\frac{\alpha}{2}} \cos\left(\beta\right) \mathrm{e}^{-c_{up}\left|M_\perp\right|\tan(\beta)} \, d\beta \qquad (7)$$

$$R_{round} = -\mathrm{e}^{c\left|M_{//,\infty}\right|} \left(\int_{\beta_0-\alpha/2}^{\beta_0+\alpha/2} \cos\left(\beta\right) \mathrm{e}^{-c_{up}\left|M_\perp\right|\tan(\beta)} \, d\beta \Big/ \int_{\beta_0-\alpha/2+\pi}^{\beta_0+\alpha/2+\pi} \cos\left(\beta\right) \mathrm{e}^{-c_{down}\left|M_\perp\right|\tan(\beta)} \, d\beta \right) \qquad (8)$$

with b as the radial extend of the probe surface. In Fig. 4 we compare the logarithm of R for both cases. As expected the error on the logarithm of R for the round collector increases with increasing angle β or the perpendicular flow. For $\beta = 0$, the probe is orientated perpendicularly to the magnetic field and the perpendicular flow does not contribute to the ion saturation current. Therefore both probes will measure the same parallel plasma flow. With increasing angle the contribution of the perpendicular flow increases and hence a systematic deviation occurs. From Eq. (6) and Eq. (8) it is clear that the comparison is independent of $M_{//,\infty}$, which therefore has been chosen to be zero. Fitting the result of

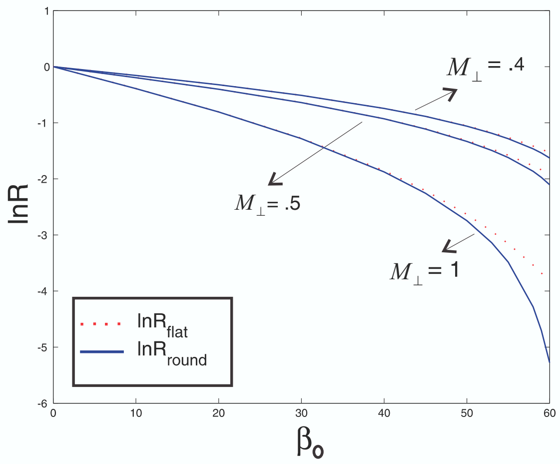

Figure 4: lnR as a function of β for different conditions of $M_\perp$.

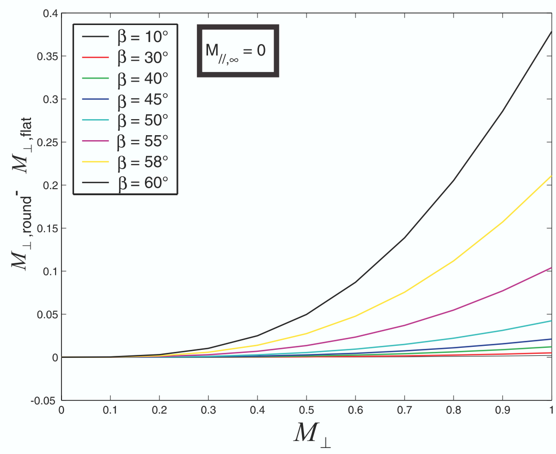

Figure 5: Comparison of $M_\perp$ between flat and round.

$\ln R_{\text{round}}$ using Eq. (6) gives $M_{\perp,round}$ which is plotted as a function of $M_{\perp,flat}$ in Fig. 5. For negative angles the results are equivalent. As shown in the figure $M_\perp$ is significantly

overestimated. For $\beta = 60°$ and $M_\perp = 1$, an error of the order of 40% on the measurement of $M_\perp$ is observed. We have chosen four angles, as defined by the ideal Gundestrup probe, to simulate the measurements. Due to the increasing divergence of the lnR from the flat collector, we found that the perpendicular Mach number will be systematically overestimated by 50% up to 100% for a $M_\perp$ of 0.5 and 1.0.

4. Conclusions

For the range of the parameter domain investigated, we conclude that errors of the order of 5-15 % can be introduced because of the use of the analytical approximation. Moreover the use of Eq. (6) to evaluate the data of a probe with round collectors leads to systematic errors up to at least 50%.

Because Eq. (6) is more convenient to use for the interpretation of data, we have chosen flat collectors for a new probe design [9]. As the model (Eq. (1)) fails for a probe orientation parallel to the magnetic field, we also note that possible orientations of round collectors necessarily are more limited than those of flat collectors. This can reduce the efficiency of the so-called ideal Gundestrup probe.

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