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Modulational Interaction as a Possible Source for Streamers and Zonal Flows

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1. The modulational processes can strongly affect the wave structure and spectra, and, consequently, change qualitatively the turbulent properties of the plasmas. With modulational instability developing under weak turbulence conditions, the correlations between the waves phases have increased and after all a strongly turbulent state with another (as compared to weak turbulence case) wave–wave and wave–plasma interactions is formed [1]. Here we discuss whether the modulational instability of drift waves can produce strongly anisotropic secondary flows, such as streamers and zonal flows.

2. Consider an inhomogeneous collisionless plasma in the presence of an external magnetic field $\mathbf{B}_0$, and suppose that the plasma density varies perpendicular to the direction of $\mathbf{B}_0$. For simplicity, we assume that the electron plasma temperature T_e is constant, and that the ions can be considered as cold ($T_e \gg T_i$). We choose the axes $0x$ and $0z$ of the reference frame in the directions of the plasma inhomogeneity and the external magnetic field, respectively. We also introduce the inverse inhomogeneity length $\varkappa = -d\ln n_0/dx$, where $n_0 = n_0(x)$ is the undisturbed plasma density, and suppose that $\varkappa > 0$. We assume that $k_z v_{T_i}, \nu_i \ll \omega \ll k_z v_{T_e}, \omega_{Li}, \omega_{Bi}$, where v_{T_i} and v_{T_e} are the thermal velocities of the ions and electrons respectively, ω_{Li} is the ion Langmuir frequency, ω_{Bi} is the ion cyclotron frequency, and ν_i is the ion collision frequency. Under these conditions the electric field in a plasma can be considered as the potential one $\mathbf{E} = -\nabla\phi$, while the electron distribution is described by Boltzmann formula. To describe such a non–isothermal plasma we use a set of a single–fluid hydrodynamics equations, namely, the ion continuity equation and the momentum equation describing the motion of cold ions.

3. Let us derive the averaged force and its potential of the low-frequency field using the method presented in [2]. The method assumes the expansion of all the values using the small parameter $\varepsilon = e\phi_0/T_e$, so that for the value f we have $f = \sum_n f_{(n)}$, where $f_{(n)} \sim \varepsilon^n$. Here ϕ_0 is the electrostatic potential. The plasma density is written as $n = n_0(x) + n_{(1)} + n_{(2)} + ...$ We look for all the values of the first–order approximation in following form: $f_{(1)} = \frac{1}{2}(f_1 + f_1^*) = \frac{1}{2} f^0_{(1)} e^{-i\omega t + i\mathbf{kr}} + c.c.$, where *c.c.* stands for the complex conjugate. In addition we separate the longitudinal and transverse parts of all the vectors: $\mathbf{E} = \mathbf{E}_\perp + E_z\mathbf{i}_z$, where $\mathbf{i}_z$ is the unit vector along the z–axis.

In the first–order approximation we obtain the dispersion equation for the drift waves $D(\omega, \mathbf{k}) = \omega^2(1 + k_\perp^2\rho_s^2) - \omega_*\omega - k_z^2 v_s^2 = 0$. Here $\omega_* = k_y v_0$, $v_0 = \varkappa v_s^2/\omega_{Bi}$, $v_s = \sqrt{T_e/m_i}$ is the ion acoustic velocity, $\rho_s = v_s/\omega_{Bi}$ is the effective ion gyroradius.

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To obtain the averaged force of a low–frequency field we average the second–order equations over time. We find for the second–order electric field

$$\langle \mathbf{E}_{(2)} \rangle = \frac{m_i}{4e} \nabla |v^0_{(1)}|^2 = \frac{e}{4m_i} \nabla \left[\frac{\omega^2_{Bi} + \omega^2}{(\omega^2_{Bi} - \omega^2)^2} |\mathbf{E}^0_{(1)\perp}|^2 + \frac{1}{\omega^2} |E^0_{(1)z}|^2 \right], \tag{1}$$

which produce the averaged force $\langle \mathbf{F} \rangle = \mathbf{e} \langle \mathbf{E_{(2)}} \rangle$. The potential of this field is $\langle \phi_{(2)} \rangle = -(e\mu/4T_e)|\phi^0_{(1)}|^2$, where $\mu \approx \rho_s^2 k_\perp^2 + k_z^2 v_s^2/\omega^2$. The averaged force of the low–frequency field has the same effect on a plasma as the Miller force in the high–frequency case: under the influence of such force, the electric field becomes "compressed" and the plasma is ejected from the high–field region, i.e., field focusing occurs [2]. This fact can be demonstrated using the second–order time–averaged expression. The hydrodynamics approach allows us to obtain Bernoulli–like equation

$$2T_e \langle n_{(2)} \rangle + \frac{\rho_i |v^0_{(1)}|^2}{2} - \frac{|\phi^0_{(1)}|^2}{8\pi r^2_{De}} = \text{const}, \tag{2}$$

where the left–hand side is a constant along the lines of external magnetic field $\mathbf{B}_0$. Here $\rho_i = n_0 m_i$, $r_{De} = \sqrt{T_e/(4\pi e^2 n_0)}$. As it can be seen from Eq.(2), the increase in the field amplitude $E^0_{(1)}$ leads to the decrease in plasma density and vise versa. This is the reason for the development of the modulational processes.

4. The compression of the plasma by the action of the averaged force of a low-frequency field can lead to the formation of the long wavelength stationary anisotropic flows in weak drift plasma turbulence. In order to derive the nonlinear equations describing this process we assume the presence in a plasma of a drift wave $\tilde{\phi}$ with the frequency ω_0, the wave number $\mathbf{k}_0$, and the amplitude ψ, so that $\tilde{\phi} = \psi e^{-i\omega_0 t + i\mathbf{k}_0 \mathbf{r}}$. The amplitude $\psi = \psi(\mathbf{r}, t)$ is the slowly varying envelope amplitude of the drift wave field. We assume also that this wave is excited in some linear process and γ_L is the growth rate of this drift wave.

We assume that there is the hierarchy between the values of different time–scales, i.e., distinguish fast and slow processes. The drift wave with the frequency ω_0 is assumed to correspond to the fast process, while the modulational instability results in the excitation of a drift wave which corresponds to the slow one.

Separating slow and fast motions we write equations for the slow processes

$$\partial_t \bar{\eta} - \varkappa \bar{v}_x + \nabla \bar{\mathbf{v}} = \varkappa \langle \tilde{\eta} \tilde{v}_x \rangle - \nabla \langle \tilde{\eta} \tilde{\mathbf{v}} \rangle \tag{3}$$

$$\partial_t \bar{\mathbf{v}} + \langle (\tilde{\mathbf{v}} \nabla) \tilde{\mathbf{v}} \rangle = -v_s^2 \nabla \bar{\eta} + \frac{1}{2} v_s^2 \nabla \langle \tilde{\eta}^2 \rangle + \bar{\mathbf{v}} \times \boldsymbol{\omega}_{Bi} \tag{4}$$

and for fast those

$$\partial_t \tilde{\eta} - \varkappa \tilde{v}_x + \nabla \tilde{\mathbf{v}} = \varkappa \bar{\eta} \tilde{v}_x + \varkappa \tilde{\eta} \bar{v}_x - \nabla (\bar{\eta} \tilde{\mathbf{v}} + \tilde{\eta} \bar{\mathbf{v}}) \tag{5}$$

$$\partial_t \tilde{\mathbf{v}} + \boldsymbol{\sigma} = -v_s^2 \nabla \tilde{\eta} + \tilde{\mathbf{v}} \times \boldsymbol{\omega}_{Bi}. \tag{6}$$

Here and below the values with bar are related to the slow motion, while the values with tilde corresponded to the fast one. The dimensionless densities $\tilde{\eta}$ and $\bar{\eta}$ are determined by $n/n_0 = 1 + \tilde{\eta} + \bar{\eta}$. Under the conditions $k_z v_s \ll \omega \ll \omega_{Bi}$ we can neglect the Reynolds stress $\boldsymbol{\sigma} = (\bar{\mathbf{v}}\nabla)\tilde{\mathbf{v}} + (\tilde{\mathbf{v}}\nabla)\bar{\mathbf{v}}$ in the last equation.

5. From Eqs.(3) and (5) we obtain two–dimentional ($\partial_z = 0$) starting equations describing the soliton–like structures

$$[\partial_t(1 - \rho_s^2\Delta_\perp) + v_0\partial_y]\eta = \frac{1}{4}(1-\mu)\frac{e^2}{T_e^2}[v_0\partial_y - \rho_s^2\Delta_\perp\partial_t]|\psi|^2 - \frac{e^2\rho_s^2}{2T_e^2}\frac{\omega_0}{\omega_{Bi}}(k_{0x}\partial_y - k_{0y}\partial_x + \varkappa k_{0y})\partial_t|\psi|^2, \quad (7)$$

and

$$\partial_t\psi + \mathbf{v}_{g\perp}\nabla_\perp\psi + ia\Delta_\perp\psi = -ib(\eta\psi) - \mathbf{v}_{g\perp}\nabla_\perp(\eta\psi) - a\psi\mathbf{k}_{0\perp}\nabla_\perp\eta, \quad (8)$$

where $a = \omega_0^2\rho_s^2/(v_0 k_{0y})$ and $b = a/\rho_s^2$. The group velocity is $\mathbf{v}_{g\perp} = \partial\omega_0/\partial\mathbf{k}_{0\perp}$. The excitation of the one–dimensional soliton–like structures associated with the streamers and zonal flows corresponds to the particular case, when $v_{gx} = 0$ or $v_{gy} = 0$.

Let us investigate the generation of zonal flows ($\partial_y = 0$). In the limit $\partial_y \to 0$ Eq.(7) describes the soliton–like structures only if $v_{gx} \ll ak_x$, and, consequently, if $k_{0x} \ll k_x$. We obtain the dispersion equation for the modulational instability in the long wavelength case ($\rho_s^2 k_x^2 \ll 1$)

$$\omega^2 - a^2k_x^4 = \left[\frac{1}{2}(1-\mu)k_x^2 - \frac{\omega_0}{\omega_{Bi}}k_{0y}(\varkappa + ik_x)\right]\rho_s^2 k_x^2 ab\frac{e^2}{T_e^2}|\bar{\psi}_0|^2. \quad (9)$$

We seek solutions of this equation in the form $\omega = \pm ak_x^2 + \delta\omega$. The term $\delta\omega$ gives us the growth rate of the instability $\gamma_{mod} = \operatorname{Im}\delta\omega$:

$$\gamma_{mod} = \frac{\omega_0^3}{2\omega_{Bi}^2}\frac{k_x}{\varkappa}\frac{e^2}{T_e^2}|\bar{\psi}_0|^2. \quad (10)$$

Here $\bar{\psi}_0$ is amplitude of the undisturbed initial state. Only the solitons moving to the positive direction of the x–axis are amplified, while the solitons moving in the opposite direction are damped.

Combining Eq. (8) and Eq. (7) we obtain the modified nonlinear Schroedinger equation (MNLS) describing the structure of the quasistationary flows and their evolution. In the case when $\partial_y \ll v\partial_t \ll \partial_x$ we obtain MNLS

$$-i\partial_t\psi + a\partial_x^2\psi - q_0\psi\partial_x^2|\psi|^2 - q_1\psi(\varkappa - \partial_x)|\psi|^2 = 0, \quad (11)$$

where $q_0 = (b/4)(1-\mu)(e^2/T_e^2)\rho_s^2$ and $q_1 = (b/2)(\omega_0/\omega_{Bi})(e^2/T_e^2)\rho_s^2 k_{0y}$. In the case when $v\partial_t \ll \partial_y \ll \partial_x$ we obtain the nonlinear Schroedinger equation (NLS)

$$-i\partial_t\psi + a\partial_x^2\psi + \bar{q}\psi|\psi|^2 = 0, \quad (12)$$

where $\bar{q} = q_0/\rho_s^2$.

Let us consider the streamer generation ($\partial_x \to 0$). In the limit $\partial_x \to 0$ Eq.(8) describes the soliton–like structures only if $v_{gy} = 0$ that leads to the relationship $2\omega_0\rho_s^2 k_{0y} = v_0$. The dispersion equation for the streamer generation due to modulational instability is

$$(\omega - \omega_1)(\omega^2 - \omega_2^2) = (A + iB)\frac{e^2}{T_e^2}|\bar{\psi}_0|^2, \tag{13}$$

where the coefficients A and B are determined by

$$\begin{aligned} A &= -\frac{1}{2}\left[k_{0x}^2(\omega\rho_s^2 k_y - v_0)k_y + \omega\frac{\omega_0}{\omega_{Bi}}\varkappa k_{0y}\right]\frac{\rho_s^2\omega_0}{1+\rho_s^2 k_y^2}\frac{k_y}{k_{0y}}\left(\omega - \frac{\alpha k_y}{\rho_s^2 k_{0y}}\right), \\ B &= -\frac{1}{2}\frac{\omega_0^2}{\omega_{Bi}}\rho_s^2 k_y^2\frac{\omega}{1+\rho_s^2 k_y^2}\frac{k_{0x}}{k_{0y}}\left(\omega - \frac{\alpha k_y}{\rho_s^2 k_{0y}}\right). \end{aligned} \tag{14}$$

The frequency $\omega_1 = v_0 k_y/(1+\rho_s^2 k_y^2)$ corresponds to the drift waves branch, while the frequency $\omega_2 = \alpha k_y^2$ corresponds to the soliton one. We note that the dispersion equation (13) has the structure similar to that of the well–known dispersion equation for the hydrodynamical beam instability. It is well–known, that the growth rate achieves its maximum value for the resonant condition $\omega_1 = \omega_2$. But in this case our approach based on the distinguishing the fast and slow processes doesn't work and we can consider only the nonresonant condition, when $\omega_1 \neq \omega_2$. We seek drift wave solution for small perturbations $\delta\omega$ in the form $\omega = \omega_1 + \delta\omega_1$. From Eq.(13) we obtain the growth rate of the drift wave mode

$$\gamma^{DW} = \mathrm{Im}\,\delta\omega_1 = \frac{B(\omega_1)}{\omega_1^2 - \omega_2^2}\frac{e^2}{T_e^2}|\bar{\psi}_0|^2. \tag{15}$$

In the same manner, for the case of the soliton development we have $\omega = \omega_2 + \delta\omega_2$. The growth rate of the soliton is

$$\gamma^{SW} = \mathrm{Im}\,\delta\omega_2 = \frac{B(\omega_2)}{2\omega_2(\omega_2^2 - \omega_1^2)}\frac{e^2}{T_e^2}|\bar{\psi}_0|^2. \tag{16}$$

We see that if the following inequality is fulfilled $B/(\omega_1 - \omega_2) < 0$, the soliton can be excited. In the opposite case the long–wavelength drift wave mode develops.

Thus we have determined the conditions and the rates of the generation of streamers and zonal flows due to the development of the drift wave modulational instability.

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