29th EPS Conference on Plasma Phys. and Contr. Fusion Montreux, 17-21 June 2002 ECA Vol. ***26B****, P-2.107 (2002)*

Plasma Equilibrium with Stationary Toroidal Flow in Tokamaks

V.D. Pustovitov

Russian Research Centre "Kurchatov Institute", Moscow, 123182, Russia

Introduction. Sometimes tokamaks operate with rotating plasma. For example, with toroidal velocity up to 300 km/s in DIII-D tokamak [1]. However, MHD plasma equilibrium is usually considered as a static problem.

Effects of rotation can be easily incorporated in the equilibrium theory [2,3]. The equilibrium effects due to plasma rotation could be observed in experiments using standard magnetic diagnostics [4]. The lack of interest to the problem has a natural explanation: plasma rotation effects in equilibrium are practically negligible. This can be seen in all theoretical works briefly summarized in the well-known review [5].

On the contrary, in recent publications [6,7] it was declared that plasma rotation can substantially affect the plasma equilibrium in tokamaks. Moreover, it was stated [6] that equilibrium β limit in a tokamak can be raised by factor of 1.6-2.5 by an appropriate choice of the toroidal velocity profile. Here we discuss these optimistic predictions.

Rotation and integral force balance. The force balance equation

$$\rho(\mathbf{v}\nabla)\mathbf{v} = -\nabla p + \mathbf{j}\times\mathbf{B} \tag{1}$$

for stationary purely toroidal rotation, $\mathbf{v} = v_t(r,z)\mathbf{e}_\zeta$, turns into

$$\frac{\rho v_t^2}{r}\mathbf{e}_r - \nabla p + \mathbf{j}\times\mathbf{B} = 0\,. \tag{2}$$

Here ρ is the plasma mass density, $\mathbf{v}$ is the flow velocity, p is the plasma pressure, $\mathbf{B}$ is the magnetic field, $\mathbf{j} = \mathrm{rot}\mathbf{B}$ is the current density, r,ζ,z are the cylindrical coordinates, ζ is the toroidal angle, and $\mathbf{e}_r$ and $\mathbf{e}_\zeta$ are the unit vectors along ∇r and $\nabla\zeta$.

Multiplying (2) by $\mathbf{e}_r$ and integrating over the plasma volume (with $p = 0$ at the boundary), we obtain the integral condition of the plasma equilibrium along the major radius:

$$\int\frac{p+\rho v_t^2}{r}dV + \int\mathbf{e}_r(\mathbf{j}\times\mathbf{B})dV = 0\,. \tag{3}$$

Here the first term describes the expansion force in $\mathbf{e}_r$ direction. Larger plasma pressure gives larger force. The pressure can be increased till the electromagnetic force $\mathbf{j}\times\mathbf{B}$ is able to counteract the ballooning force and keep the plasma in the desired position. The maximum pressure, when the equilibrium condition is still satisfied, is called an equilibrium limit. One can see from (3) that the toroidal rotation must *lower* this limit. This is quite natural since the rotation results in a centrifugal force [the first term in (2)] pushing the plasma outward.

This conclusion is incompatible with statements [6,7] about a possibility of increasing the equilibrium limit by means of the plasma toroidal rotation. Before analysing this inconsistency let us estimate the role of the rotation term in Eqs. (1)-(3).

By definition, $p = n_i T_i + n_e T_e$, $\rho = m_i n_i$, therefore

$$p/\rho = v_{T_i}^2 \left[1 + (n_e T_e)/(n_i T_i)\right], \tag{4}$$

where n_i and T_i (n_e and T_e) are the ion (electron) density and temperature, m_i is the ion mass, and $v_{T_i} \equiv \sqrt{T_i / m_i}$ is the ion thermal velocity. Thus,

$$\rho v_t^2 / p < v_t^2 / v_{T_i}^2 . \tag{5}$$

For hydrogen plasma $v_{T_i} = 979\sqrt{T_i / T_0}$ km/s, where T_0=10 keV. This velocity is rather high. Even for the rotation with v_t of the order of 300 km/s the ratio (5) is small, and influence of the rotation on the equilibrium of hot plasmas (T_i>10 keV) can be disregarded.

Rotation and Shafranov shift. The effect of plasma flow on the Shafranov shift [4,8] was studied, for the first time, in [2,3]. In [3] the formula was obtained

$$\Delta' = \Delta'_S - \frac{a}{R}\frac{\overline{\rho v_t^2} - \rho v_t^2}{B_\theta^2}, \tag{6}$$

where Δ'_S is a 'static' part of Δ', the prime denotes d/da, a is the minor radius of magnetic surfaces, R is the major radius, $B_\theta(a) = J/(2\pi a)$ is the field due to a longitudinal current flowing through the tube $a = \mathrm{const}$, the bar denotes a cross-section averaging, see [3,4,8]. The magnetic pressure is $\mathbf{B}^2/2$ in the units used here.

The value Δ'_S was calculated by Shafranov [8,4]:

$$\Delta'_S = -\frac{a}{R}\left[\frac{l_i}{2} + 2\frac{\bar{p} - p}{B_\theta^2}\right]. \tag{7}$$

Here $l_i \equiv \overline{B_\theta^2} / B_\theta^2$ is the internal inductance of the plasma tube $a = \mathrm{const}$ of unit length.

It is clear that Δ'_S is negative for radially decreasing p. Equation (6) shows that the rotation with decreasing profile of ρv_t^2 increases $|\Delta'|$ and, as a result, the magnetic surface shift becomes larger. On the other hand, one can reduce $|\Delta'|$ by making ρv_t^2 a growing function of a, so that $\overline{\rho v_t^2} < \rho v_t^2$. These both conclusions were clearly formulated in [3]. In [6] the last conclusion about the 'positive' influence of the toroidal rotation is presented as an original result, without mentioning [3], though the analysis [6] almost replicates that of [3]. Really new in [6] is the statement that the plasma shift can be substantially suppressed or even completely eliminated "by an appropriate choice of the toroidal velocity profile, no matter how high the absolute value of the rotation velocity." Formula (6) shows that this statement is incorrect. It can be seen even in the starting equation (2) that the effect of the toroidal rotation on plasma equilibrium must depend on the absolute value of ρv_t^2.

The conclusion about suppression or even elimination of Shafranov shift is incorrect too. At least, due to the weakness of the effect, see (5). Besides, the reduction of $|\Delta'|$ in a central part of the plasma column does not yet guarantee an increase in the equilibrium pressure limit discovered in [6]. Complete solution of an equilibrium problem must include integration of Δ' with proper boundary conditions, that was not performed in [6]. Possible increase of ρv_t^2 near the axis cannot be separated from the decrease at the plasma periphery. The total effect of the rotation must be negative because of the centrifugal force. This is clearly seen in the integral force balance (3). To show this total negative effect using traditional description based on expressions (6) and (7), we apply (6) to the plasma boundary:

$$\Delta'(b) = -\frac{b}{R}\left[\frac{l_i}{2} + \beta_J + \frac{\overline{\rho v_t^2}}{B_J^2}\right]. \tag{8}$$

Here all values are evaluated at the plasma boundary, $a = b$, $\beta_J = 2\overline{p}/B_J^2$ is the poloidal beta, $B_J = B_\theta(b)$. It is supposed that $p = 0$, $\rho v_t^2 = 0$ on the boundary.

Expression (9) differs from the well known one for a static case [4] by the term with toroidal velocity v_t only. Its contribution is always negative, makes $|\Delta'(b)|$ larger.

Rotation and equilibrium limit. Increase of $|\Delta'(b)|$ is directly related to the decrease of the equilibrium beta limit. This can be shown explicitly using the formula [9,10]

$$B_\perp = -B_J \frac{b}{2R}\left[\ln\frac{8R}{b} - \frac{3}{2} - \frac{R}{b}\Delta'(b)\right] \tag{9}$$

for the magnitude of the external vertical field necessary for keeping the plasma column in a tokamak in a given equilibrium position. Substitution of (8) gives

$$B_{\perp} = -B_J \frac{b}{2R}\left[\ln\frac{8R}{b} - \frac{3}{2} + \frac{l_i}{2} + \beta_J + \frac{\overline{\rho v_t^2}}{B_J^2}\right]. \tag{10}$$

When $v_t = 0$, this formula reproduces the classical result of Shafranov [4,8]. Formula (10) shows that larger vertical field is needed for rotating plasma. It was explained in [8] that, for circular plasma, the equilibrium limit is set by appearance of the separatrix X-point on the inner side of the plasma column. This means that there is an upper bound for $|B_{\perp}|$. From (10) it follows that for given $|B_{\perp}|$ larger β_J are achieved when the rotation is absent.

One can see that Eqs. (6), (8), (10) are the same as known equations for static equilibrium [4,8] with only difference: $2p$ is replaced by $2p + \rho v_t^2$. This allows concluding that equilibrium limits with and without rotation are related by the formula

$$\beta_{rot} = \beta_{static} - \frac{\overline{\rho v_t^2}}{B_0^2}, \tag{11}$$

if boundary conditions for p and ρv_t^2 are the same. Here B_0 is the toroidal field.

Conclusions. Despite a possible local improvement of the equilibrium conditions due to a special choice of the rotation velocity profile, first predicted in [3] and rediscovered in [6], the resulting integral effect must be negative. However, the decrease in the equilibrium β limit because of the toroidal plasma flow is negligible for typical plasma parameters.

References

[1] Garofalo A.M., et al., Nucl. Fusion **41** (2001) 1171.

[2] Zehrfeld H.P., Green B.J., Nucl. Fusion **12** (1972) 569.

[3] Zehrfeld H.P., Green B.J., Nucl. Fusion **13** (1973) 750.

[4] Zakharov L.E. and Shafranov V.D., in: Reviews of Plasma Physics (edited by M.A. Leontovich and B.B. Kadomtsev), Vol. 11, Consultants Bureau, New York (1986) 153.

[5] Takeda T., Tokuda S., J. Comput. Phys. **93** (1991) 1.

[6] Il'gisonis V.I., Pozdnyakov Yu.I., JETP Letters **71** (2000) 314.

[7] Ilgisonis V.I., Plasma Phys. Control. Fusion **43** (2001) 1255.

[8] Shafranov V.D., in Reviews of Plasma Physics (edited by M.A. Leontovich), Vol. 2, Consultants Bureau, New York (1966) 103.

[9] Greene J.M., Johnson J.L., Weimer K.E., Phys. Fluids **14** (1971) 671.

[10] Pustovitov V.D., in Reviews of Plasma Physics (edited by B.B. Kadomtsev and V.D. Shafranov), Vol. 21. Consultants Bureau, New York (2000) 1.