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Evaluation of the Bootstrap Current in Stellarators*

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Introduction

An investigation of the bootstrap current in stellarators is very important since it can negatively affect the plasma equilibrium and the magnetic configuration. In the present work, the bootstrap current is numerically analyzed for configurations with quasi-helical and quasi-axial symmetry as well as for the standard stellarator with different ratio of the toroidal and the helical inhomogeneity of the magnetic field. For this purpose, the technique of integration along magnetic field lines [1,2,3] is used. The results are benchmarked to the W7-X configuration which has been optimized in order to reduce the bootstrap current. In addition, results are given for two configurations in Boozer coordinates, where the influence of Coulomb collisions on the resonant behavior is discussed.

Computational technique and basic parameters

The method to calculate the bootstrap current is based on an integration procedure along the magnetic field lines [1,2,3] and is similar to the one proposed in [4]. However, the contribution of trapped particles which has been neglected in [4] is recovered. Also, in contrast to [4] this method allows one to do the computation in real-space coordinates and to analyze the bootstrap current for the magnetic fields given in real-space as well as in Boozer coordinates. The evaluation of the bootstrap current is performed using the geometrical factor λ_b which corresponds to λ_{b1} in [1,2,3] and which is similar to the corresponding factor in [4]. The dimensionless factor λ_b is obtained from the drift kinetic equation in the long mean-free-path-regime with a Lorentz collision operator which describes pitch-angle scattering but does not conserve momentum. However, it has been shown in [4] that if the fraction of trapped particles is small, the geometrical factor is also valid in case of momentum conservation. In this case, the general expression for the bootstrap current is represented by λ_b multiplied by some combination of the density and the temperature gradients with factors that do not depend directly on the magnetic field geometry. As an example, the following equation for the total bootstrap current can be used, which represents Eq.(4) of [4]:

$$\frac{j_b}{B} = -c\,\lambda_b\,\frac{1}{B_0^2}\left[1.67\,(T_e+T_i)\,\frac{dn}{dr} + 0.47n\frac{dT_e}{dr} - 0.29n\frac{dT_i}{dr}\right]. \tag{1}$$

In the present work, λ_b is calculated for the quasi-helically symmetric (QHS) configuration [5], for the quasi-axially symmetric CHS-qa configuration [6], and for the W7-X configuration [7], all in real space with β=0. To complement this, results are also given for a finite-β W7-X configuration [9] and a slightly different CHS-qa vacuum configuration,

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both in Boozer coordinates. Additionally, two cases of a simplified l=3 configuration with different ratios of the toroidal and the helical magnetic field inhomogeneity are considered in order to analyze the influence of the helical field inhomogeneity, ϵ_h, on the bootstrap current. The first case corresponds to the idealized Uragan-3M torsatron (β=0) with only one toroidal harmonic function. The second case corresponds to a strongly decreased toroidal inhomogeneity, ϵ_t, of the magnetic field with approximately the same ϵ_h.

Bootstrap resonances

It had been shown in [1] that bootstrap current resonances arise from a considerable increase of particle displacement of passing particles from the magnetic surface. They are caused by the fact that on almost rational surfaces particles frequently come close to the global maximum of B where for particles at the trapped-passing boundary $v_{||}$ approaches zero. In the presence of collisions, this situation changes due to collisional particle displacements. This fact can be simulated by introducing the parameter Δ into the expression for $v_{||}$

$$v_{||} = v\sqrt{1 - y\hat{B} + \Delta}\,, \qquad (2)$$

where $\hat{B}$=B/B_0 is the normalized magnetic field module, y=$J_\perp B_0/v^2$ with $J_\perp$ the perpendicular adiabatic invariant, and B_0=B^{abs}_{max} is the reference magnetic field. The parameter Δ can be calculated from

$$\Delta = y\hat{B}\sqrt{-\gamma\ln[-y^2\gamma\ln(y^2\gamma)]}\,, \qquad (3)$$

with γ=$\nu L/(2v_{th})$, L=$(-\hat{B}''_0)^{-1/2}$, and $\hat{B}''_0$=$d^2\hat{B}/ds^2$ at the global maximum of $\hat{B}$, where s is the distance measured along the field line, ν is the collision frequency, and v_{th} is the thermal velocity.

Computational results for the bootstrap current

Computational results for λ_b are shown in Fig. 1 and 2 as function of the mean magnetic surface radius r in units of the mean boundary surface radius a. To simplify the comparison for the CHS-qa and for the l=3 configuration, the normalized quantity λ_{bn}=$\lambda_b\iota\!\!\!-\sqrt{r/R}$ is introduced, where $\iota\!\!\!-$ is the rotational transform and R is the big radius of the torus. This quantity is equal to 1.46 for a tokamak with a large aspect ratio (see, e.g., [4]).
The calculations are performed for non-resonant magnetic surfaces. The behavior of λ_b in the vicinity of some resonant magnetic surfaces is also shown in Fig. 1. For the QHS configuration $\iota\!\!\!-$ reaches its minimum near $r/a \approx 0.5$ and is somewhat smaller than 24/17. The quantity λ_{bn}, changes from approximately 0.6 to 1.36 near the boundary. For the CHS-qa configuration $|\lambda_b|$ is bigger than for QHS but smaller than for an equivalent tokamak. For non-resonant magnetic surfaces $|\lambda_{bn}|$ changes from 0.47 to 0.83 near the boundary. The results related to W7-X were already partly considered in [1,2]. Here, in more detail the tendency of the λ_b behavior in the vicinity of the island surfaces corresponding to $\iota\!\!\!-$=10/11 and $\iota\!\!\!-$=1 is studied. Rather close to the boundary $|\lambda_b|$ is very small. Over the whole r region $|\lambda_{bn}|$ does not exceed 0.25. Note that for all three configurations in Fig. 1 the rotational transform $\iota\!\!\!-$ has an anti-clockwise direction (with increasing toroidal angle). In this case, a negative λ_b corresponds to a tokamak-like direction of bootstrap current since it produces the poloidal magnetic field in an anti-clockwise direction (CHS-qa, W7-X).

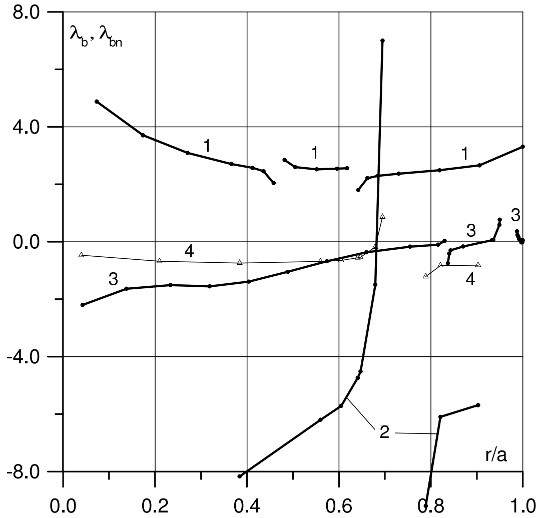

Fig. 1. Parameter λ_b for QHS (curve 1), for CHS-qa (curve 2) and for W7-X (curve 3); curve 4 shows the λ_{bn} parameter for CHS-qa; the discontinuities in the curves correspond to the resonant magnetic surfaces for $\iota\!\!-$ close to 24/17 for QHS, 4/11 for CHS-qa and 10/11 and unity for W7-X.

For comparison, Fig. 2 gives λ_b for W7-X [9]. The results are rather close to the results in Fig. 1 (line 3) with a slightly different position of the 10/11 rational surface. It can easily be seen that resonances resulting from resonances at high order rational surfaces are smoothed by collisional effects. In contrast to this, it becomes evident that the main resonance at 10/11 (2/11 per field period) survives even in the presence of collisions.

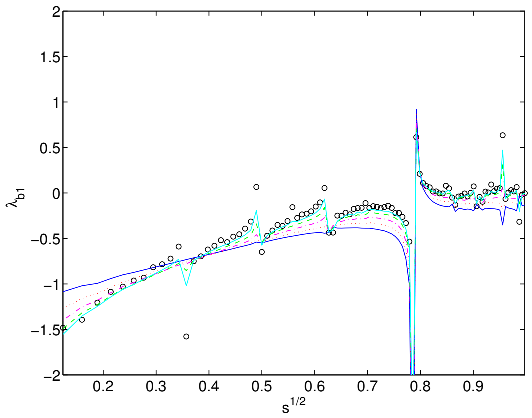

Fig. 2. Parameter λ_b for W7-X. Black circles show results on given flux surfaces in the collisionless limit. The curves correspond to the following values of the collision parameter $\nu/(2v_{th})$: 10^{-6} (blue), 10^{-7} (red), 10^{-8} (magenta), 10^{-9} (green), and 10^{-10} (cyan), respectively. The dominant resonance is at the $\iota\!\!-$=10/11 rational surface.

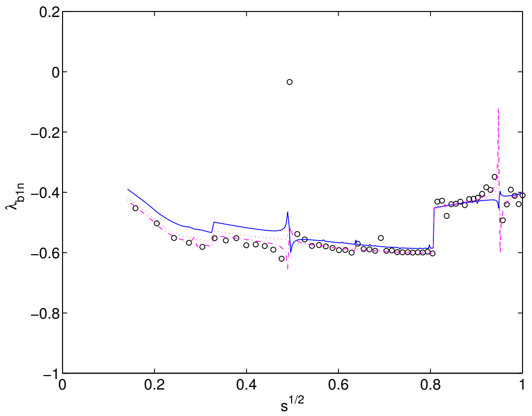

Fig. 3. Parameter λ_{bn} for CHS-qa. Black circles mark results in collisionless limit. The curves correspond to the following values of $\nu/(2v_{th})$: 10^{-6} (blue), 10^{-7} (red), and 10^{-8} (magenta).

For CHS-qa the normalized quantity λ_{bn} is given in Fig. 3. Again, results are comparable with line 4 in Fig. 1. The 4/11 resonance is less pronounced in the Boozer equilibrium from VMEC. Since $\iota\!\!-$ has a non-monotonic behavior with s, the 4/11 resonance appears twice and it can be seen that collisions smooth the effect of this resonance but don't remove it completely. The rather steep change at $s^{1/2}$=0.8 corresponds roughly to the minimum value of $|\iota\!\!-|$=0.35.

The results related to the l=3 configurations are presented in Fig. 4. From this it follows that in case of decreased ϵ_t the quantity λ_b changes its sign for magnetic surfaces rather close to the boundary (curve 1). The reason is that for these values of r, ϵ_h becomes essentially bigger than ϵ_t and the bootstrap current is produced mainly due to the helical field inhomogeneity. In this case, λ_{bn} reaches the value of -1.93 near the boundary. The role of the helical field inhomogeneity is also seen from curves 3 and 4 for the ordinary l=3 configuration. In this case λ_{bn} becomes smaller than unity near the boundary.

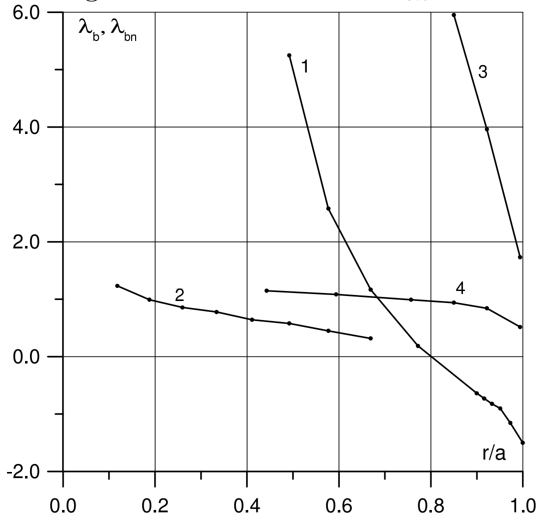

Fig. 4. Parameters λ_b and λ_{bn} for the l=3 magnetic field with a strongly decreased (ten times) value of the toroidal ripple ϵ_t (curves 1 and 2, respectively) and with the original relation between the toroidal and the helical magnetic field ripples ϵ_t and ϵ_h (curves 3 and 4, respectively). Here the rotational transform ι has a clockwise direction. Therefore, the positive values of λ_b correspond to a tokamak-like direction of the bootstrap current.

Summary

Employing a newly developed technique [1,2,3] which is based on an integration along magnetic field lines, the geometric factor in the expression for the bootstrap current is studied numerically for various stellarator configurations. The method is valid in the long mean-free path limit. In addition, the influence of collisions on bootstrap resonances which are observed in all configurations (both, given in real-space variables and in Boozer coordinates) is discussed in two cases. For that purpose an approximate analysis of this effect is presented, clearly showing that low order resonances are not easily destroyed by collisional effects.

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