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Calculation of Self-consistent Radial Electric Field in Presence of Convective Electron Transport in a Stellarator*

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Introduction

Convective transport of supra-thermal electrons can play a significant role in the energy balance of stellarators in case of high power electron cyclotron heating. Here, together with neoclassical thermal particle fluxes also the supra-thermal electron flux should be taken into account in the flux ambipolarity condition, which defines the self-consistent radial electric field. Since neoclassical particle fluxes are non-linear functions of the radial electric field, one needs an iterative procedure to solve the ambipolarity condition, where the supra-thermal electron flux has to be calculated for each iteration. A conventional Monte-Carlo method used earlier for evaluation of supra-thermal electron fluxes [1] is rather slow for performing the iterations in reasonable computer time. In the present report, the Stochastic Mapping Technique [2, 3] (SMT), which is more effective than the conventional Monte Carlo method, is used instead. Here, the problem with a local mono-energetic supra-thermal particle source is considered and the effect of supra-thermal electron fluxes on both, the self-consistent radial electric field and the formation of different roots of the ambipolarity condition are studied.

Flux balance and neoclassical particle fluxes

In a stellarator the constraint that the ion and electron fluxes be equal determines the radial electric field. Thus, the equation for the flux balance, $\Gamma_e^{nc} + \Gamma_e^s = Z_i \Gamma_i^{nc}$, has do be fulfilled on each flux surface. Here, Γ_α^{nc} with $\alpha = e, i$ are the neoclassical particle fluxes [1],

$$\Gamma_\alpha^{nc} = -n_\alpha \left\{ D_{11}^\alpha \left(\frac{n'_\alpha}{n_\alpha} - \frac{q_\alpha E_r}{T_\alpha} \right) + D_{12}^\alpha \frac{T'_\alpha}{T_\alpha} \right\} ,$$

with q_α, n_α, T_α, E_r being the particle charge, density, temperature and the radial electric field, respectively, and prime denotes a derivative with respect to a formal radius.

The neoclassical diffusion coefficients D_{11}^α and D_{12}^α are computed according to the Shaing-Houlberg-model [4], where instead of ϵ_h the effective ripple ϵ_{eff} [5] is used [6]. The balance equation is a non-linear equation in the radial electric field which might have multiple roots.

Supra-thermal particle fluxes

The supra-thermal particle flux, Γ_e^s, is of particular importance for the confinement since it can influence the radial electric field through the ambipolarity condition [1]. Following

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the SMT approach [2, 3], the usual expression for particle flux through the magnetic surface $\hat{\psi} = \hat{\psi}_0$ defined in guiding center variables and flux coordinates $(\hat{\psi}, \theta, \varphi)$ can be written as an average over Poincaré cuts of the phase space flux density,

$$\Gamma_e^s = 2\pi \sum_{\mathbf{m}} \int \mathrm{d}^5 u \Gamma_{\mathbf{m}}(\mathbf{u}) \delta(t - u^5) \left[\Theta\left(\hat{\psi}(\mathbf{Z}(\mathbf{z_m}, \tau_{b\mathbf{m}})) - \hat{\psi}_0\right) - \Theta\left(\hat{\psi}(\mathbf{Z}(\mathbf{z_m}, 0)) - \hat{\psi}_0\right)\right] .$$

Here, $\Gamma_{\mathbf{m}}(\mathbf{u})$ is the pseudo-scalar particle flux density through those Poincaré cuts, $\mathbf{u}$ denotes the five variables x^1, x^2, p, λ, t, where x^1, x^2 are contravariant coordinates in a local magnetic coordinate system, and p, λ, t are the momentum modulus, the particle pitch and the time, respectively. The summation over $\mathbf{m}$ is a summation over contributions from different Poincaré cuts and Θ is the Heaviside step function. In SMT, $\mathbf{Z}(\mathbf{z_m}, \tau)$ is the solution to equations of particle drift motion with $\mathbf{z_m}$ being the initial value of phase space variables on the Poincaré cut with index $\mathbf{m}$, and with $\tau_{b\mathbf{m}}$ being a transition time between cuts. All details of SMT can be found in Ref. [2, 3]. The energy flux is obtained in the same way and it differs from Γ_e^s by the factor $w_k(\mathbf{u})$ in the sub-integrand, where w_k is the value of the kinetic energy of the particle at the phase space point $\mathbf{u}$ on the cut.

Computational results

For numerical computations, the magnetic field from the W7-AS stellarator [7] was used in its real space representation. In Figure 1, particle and energy fluxes of supra-thermal particles are shown, respectively. The particle source is on the magnetic axis in the magnetic field minimum located at the elliptic cross section of W7-AS. Trapped particles with a pitch value $\lambda_0 = 0.1$ and fixed energies w_0 ranging from $w_0 = 2T_0$ to $w_0 = 9T_0$ are generated there. The source rate in these computation was $\nu_{\mathrm{stat}} = P_{source}/w_0$ where $P_{source} = 400$ kW is the source power. The profiles of the equilibrium parameters were the following, $T_\alpha(\hat{\psi}) = T_0(1.2 - \hat{\psi})$, $n_\alpha(\hat{\psi}) = n_0(1.2 - \hat{\psi})^2$, $\Phi(\hat{\psi}) = T_0\hat{\psi}/e$, where $T_0 = 3$ keV, $n_0 = 3 \cdot 10^{13}$ cm^{-3}, and $a = 17.4$ cm, respectively. The quantity $\psi/\psi_b = \hat{\psi} = (r/a)^2$ was chosen as a formal flux label, where the radius $r = R - R_0$ is computed in the mid plane of the symmetric cross section and R_0 is the radius of the magnetic axis. It can be seen that the energy of the source particles has a significant influence on the profiles of supra-thermal fluxes.

The results of self-consistent modeling are presented in Figures 2 and 3, where a modified density profile $n_0(1.2 - \hat{\psi}^2)$ is used. Figure 2 shows the self-consistent E_r-profile with neoclassical fluxes only. Supra-thermal electron fluxes are given for $\lambda_0 = 0.1$ and two energies, in each case with its respective E_r-profile. One can clearly see that particles with lower energy ($4T_0$) have time to slow down, whereas particles with higher energy ($9T_0$) quickly drift out of the plasma.

Figure 3 shows the formation of the “electron root” in a rather narrow region near the magnetic axis. Figure 3 also shows the dependence of fluxes on E_r in two radial positions. One can see that at $r = 5.5$cm two stable solutions exist, which finally result in the formation of the “electron root”. The decision which root has to be chosen is based on the minimization of a generalized heat production rate [1]. Following that approach, the position of the poloidal shear layer can be determined from

$$P(r) = \int_{E_r^i}^{E_r^e} \left(Z_i \Gamma_i^{nc} - \Gamma_e^{nc} - \Gamma_i^s\right) dE_r = 0 ,$$

where E_r^i and E_r^e are the stable solutions for E_r in the "ion" or "electron root", respectively. The "ion root" is then realized for $P > 0$ and the "electron root" for $P < 0$ [1]. Basically, the ion root is realized almost everywhere. When approaching the axis, the neoclassical fluxes are decreasing together with the magnetic surface area, but at the same time the supra-thermal flux is increasing. Finally, the neoclassical bifurcation occurs and the root is changed from "ion" to "electron". Further inward, the "electron root" disappears which can be seen in Figure 3 where the pertinent root vanishes. This event is an artifact of the neoclassical transport model used in the present computation where the ion flux is decreasing with increasing E_r and cannot balance the supra-thermal flux anymore. As discussed in Ref. [1], the validity of the neoclassical theory may be violated in such a case of a very strong radial electric field.

Summary

The application of SMT to a "global" computation of supra-thermal particle fluxes in a stellarator shows that this method is fast enough to allow for iterations of the radial electric field using the ambipolarity condition taking into account fluxes from supra-thermal particles. For this purpose, SMT is the ideal tool, because the computation of one self-consistent profile requires only tens of minutes on a DEC Alphastation 500 depending on accuracy. Therefore, SMT combined with a neoclassical balance code permits the self-consistent modeling of particle and energy balance in a stellarator with strong electron or ion cyclotron heating where the convective transport of supra-thermal particles plays a significant role. It is also shown that convective fluxes are very sensitive to the detailed structure of the supra-thermal particle source. In the case of ECRH, non-linear effects of wave-particle interaction are dominant in the formation of such a source [8]. The method for modeling this effects has been recently developed and will be included in future models based on SMT.

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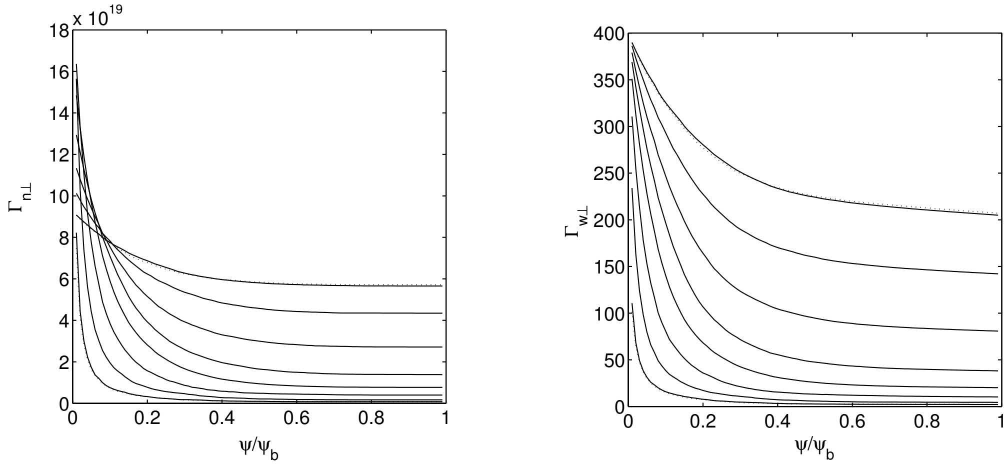

Figure 1: Particle flux (left) and energy flux (right) versus normalized flux label $\hat{\psi}$, respectively. The particle energy ranges from $w_0 = 2T_0$ to $w_0 = 9T_0$ from bottom to top.

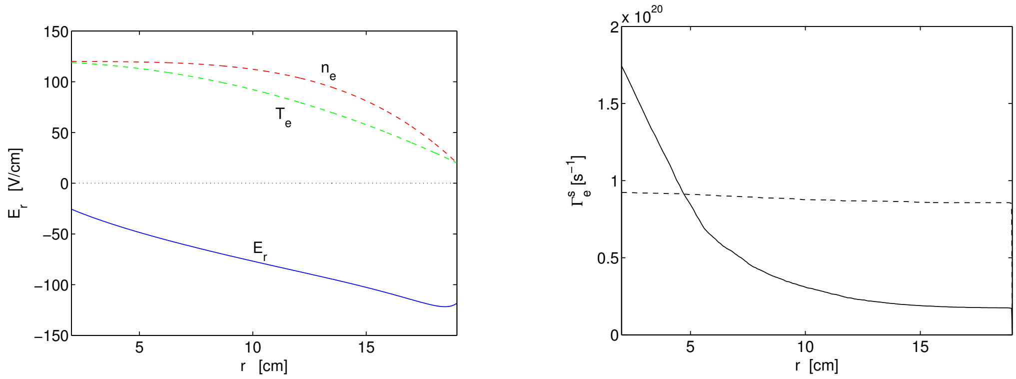

Figure 2: Radial electric field E_r resulting from neoclassical fluxes only (left) and supra-thermal particle flux Γ_e^s (right) versus radius r, respectively. The supra-thermal fluxes are computed for $w_0 = 4T_0$ (full) and $w_0 = 9T_0$ (dashed) in each case with a selfconsistent E_r. In addition, the profiles of n_e and T_e in dimensionless units are given (left plot).

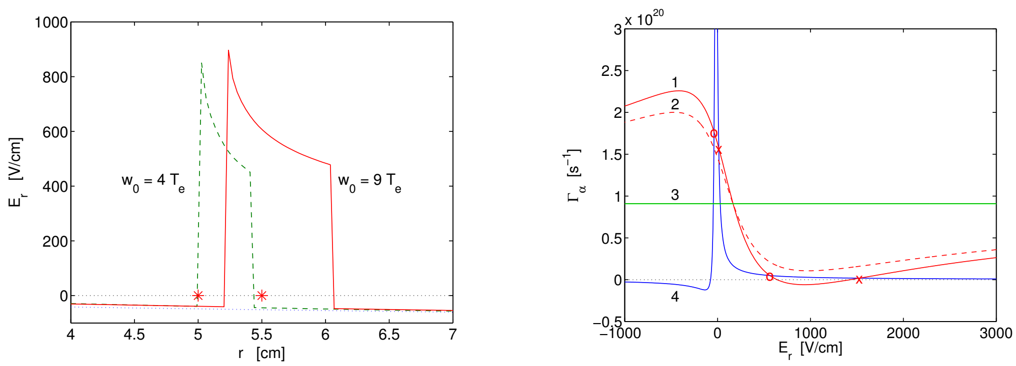

Figure 3: Left: Radial electric field E_r versus radius r for two energies of source particles and for the neoclassical equilibrium (blue dots). Right: Total electron flux at $r = 5.5$cm (1) and at $r = 5.0$cm (2), supra-thermal electron flux (3) and ion flux (4) versus radial electric field E_r, respectively, for the $w_0 = 9T_0$-case. Circles mark stable roots, whereas crosses mark unstable ones.