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Bootstrap Current for 3D Reactor-Size Configurations

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Introduction

It has been known for more than 30 years that plasma diffusion can be increased through geometrical effects. This fact is very important since it governs the transport of particles and energy, specially in stellarators due to their very complicated geometry [1]. For toroidal systems this is the subject so-called Neoclassical Theory. The magnetic field strength B, in toroidal geometry is inhomogeneous. This implies that some particles do not have enough momentum to go through the zones of higher B and thus are reflected. They become trapped in the low field area. The collisions between passing particles and those trapped give extra contributions to the transport. In equilibrium, the magnetic field configuration is described by a set of nested toroidal flux surfaces. Bootstrap current (BC) arises when the number of particles displaced from their home flux surface moving parallel to the magnetic field line differs from the number displaced moving anti-parallel to it [4]. In tokamaks, this current is in the same direction as the ohmic current and acts to increase the rotational transform , ι. In stellarators the effect of Neoclassical transport is even more significant. The inhomogeneity of the field is not only due to the toroidicity but also to the helicity of the three-dimensional configuration. Depending on the specific geometry of the magnetic field, the BC can even change sign. Thus, the BC strongly affects the rotational transform either increasing it or decreasing it. This is of great importance in stellarators because their magnetic topology, stability and transport properties are very sensitive to the iota and its profile [9, 4]. In particular resonances can occur between rational surfaces (with rational ι) and the natural asymmetries of the configuration giving rise to magnetic islands and stochastic field regions.

Calculation of the bootstrap current

Theoretical models have been developed that contain the full collisionality operator but require significant approximations to be calculated [2, 3, 7]. Others solve the Drift Kinetic Equation giving more precise numerical results but, to our knowledge, only consider pitch angle scattering in the collisionality operator [4, 6, 8]. Since our interest is to perform a comparative study of the effect of the BC on reactor-size configurations, the approximations used in the moment equation approach are well justified, allowing us to use the full collisionality operator. Thus, the BC is evaluated numerically in the collision-less $1/\nu$ regime for a pure electron and ion plasma with $T_i = T_e$, as described in *'Johnson et al. Phys. Plasmas 6 (1999) 2513'* and implemented in the TERPSICHORE code. VMEC equilibria are computed iteratively until the BC converges. Departing from a zero current specification, the equilibrium calculated together with an associated, but inconsistent, BC. This BC is included in the specification of the next equilibrium. Convergence is achieved when the BC from a particular equilibrium differs from the previous one within a given tolerance (in our case: $(\mu_0 j_B(i) - \mu_0 j_B(i-1)) \leq 0.02[HA/m] \sim 16kA$). The BC is given by

$$j_{Bt} = 2\pi \int d\psi \frac{\langle \mathbf{J_b} \cdot \mathbf{B} \rangle}{\langle \mathbf{B}^2 \rangle} \quad , \quad \langle \mathbf{J_b} \cdot \mathbf{B} \rangle = -G_b \left(L_1 \frac{dP}{d\psi} + L_2 N \frac{dT}{d\psi} \right) \tag{1}$$

where P is the plasma pressure, $2\pi\psi$ is the toroidal magnetic flux,N is the plasma density, $\mathbf{B}$ is the magnetic field and $\mathbf{J_b}$ is the BC density. L_1 and L_2 are transport coefficients associated with viscosity and friction. G_b is the so-called geometrical factor, which depends on the collisionality and includes all the information about the geometry of the configuration. It is the flexibility of this term that allows us to use this method for very varied geometries. In the low collisionality regime it takes the form

$$G_b^{1/\nu} = \frac{1}{f_t} \left\{ \langle g_2 \rangle - \frac{3}{4} \frac{\langle B^2 \rangle}{B_{max}^2} \int_0^1 \frac{\langle g_4 \rangle}{\langle g_1 \rangle} \lambda d\lambda \right\} \tag{2}$$

where

$$g_1 = \sqrt{1 - \lambda \frac{B}{B_{max}}} \quad , \quad f_t = 1 - \frac{3}{4} \frac{\langle B^2 \rangle}{B_{max}^2} \int_0^1 \frac{1}{\langle g_1 \rangle} \lambda d\lambda \tag{3}$$

f_t is the fraction of trapped particles. The averaging operator $\langle \ldots \rangle$ is over the surfaces. Variables g_2 and g_4 satisfy the differential equations

$$\mathbf{B} \cdot \nabla \left(\frac{g_2}{B^2} \right) = \mathbf{B} \times \nabla\psi \cdot \nabla \left(\frac{1}{B^2} \right) \quad , \quad \mathbf{B} \cdot \nabla \left(\frac{g_4}{g_1} \right) = \mathbf{B} \times \nabla\psi \cdot \nabla \left(\frac{1}{g_1} \right) \tag{4}$$

with $g_2(B_{max}) = 0$ $g_4(B_{max}) = 0$. This method has been applied to three different 3-dimensional reactor-size configurations: a Quasi-Axisymmetric Stellarator (QAS), a HELIAS type stellarator based on the WENDELSTEIN 7-X device at Greifswald and a Quasi-Helically Symmetric Stellarator. All systems considered have a volume $\sim 1000m^3$ and a magnetic field strength of about $\sim 5T$. It is known that toroidicity is responsible for the counter-BC, while helicity is responsible for the co-BC. Employing this idea QAS has been chosen for its clear toroidicity and QHS for its helicity. W7-X was in fact, designed to geometrically combine these two characteristics [11] aiming to compensate both contributions, and hence reduce the bootstrap current as much as possible.
A nearly parabolic pressure profile

$$P(s) = P_0[(1-s) - 0.1(1-s^{10})] \tag{5}$$

which provides a vanishing pressure gradient at the edge is prescribed. The dependence of the BC on $\beta^* = 2\mu_0 \sqrt{V \langle P^2 \rangle} / \langle B^2 \rangle$ and on density and temperature profile variations is investigated, along with its effect on the rotational transform ι. The density profile is taken to be

$$N(s) = (1 - s^{\ell})^q \tag{6}$$

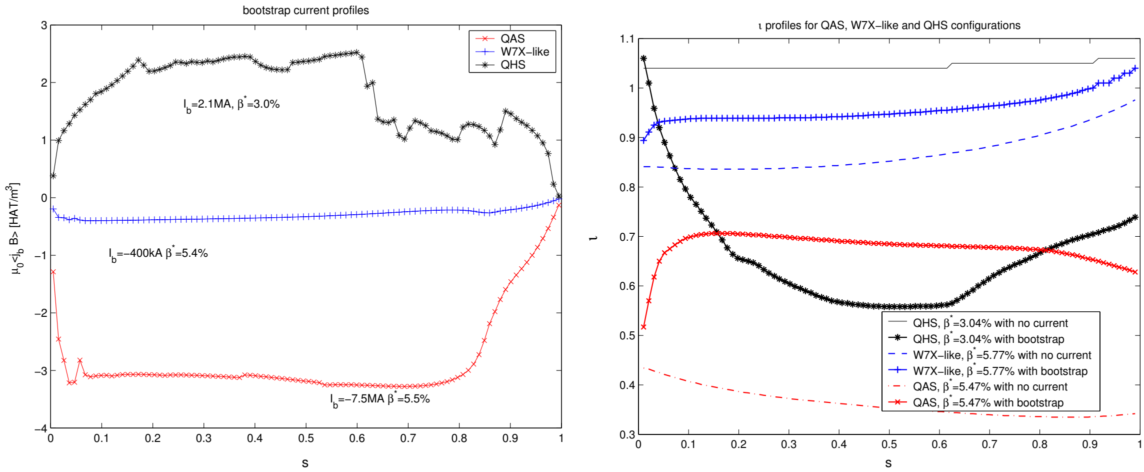

Figure 1: Bootstrap current (a) and ι profiles (b) for the three configurations studied. The maximum β^* achieved in each of the is considered

which can steepened or flattened by varying the parameters ℓ and q.

Results

Since our interest is the study of reactor-like configurations we wish to reach $\beta^* \sim 5\%$ or more. To achieve this values we re-scale the pressure profile through varying P_0. Convergence of the iteration cycle has proven to be more difficult to achieve with high β^*. The converged results of highestβ^* are plotted in figure 1 (a). QHS failed to converge for $\beta^* > 3\%$. In all cases the BC obtained behaves as expected. Its effect on the ι profile is represented in figure 1 (b). Different density and temperature profiles have been also studied when varying the parameters ℓ and q in the density profile (equation 6), and through: $T(s) = P(s)/N(s)$. The absolute value of the BC of the three configurations exhibits a very similar behaviour when varying $N(s)$ and $T(s)$. They show similar relative scaling. The total BC for the QAS configuration shows a difference of around 1.5MA, while the variation of the small BC of the W7X-like reactor is, in comparison, nearly negligible . In general it can be seen that peaked temperature, for fixed pressure profile, appears to increase the absolute value of the BC in the three configurations, while peaked density profiles would tend to decrease it .

Conclusions

For QAS the high bootstrap gives a remarkable increase on the rotational transform. This could trigger external $m/n = 2/1$ kink modes. In order to avoid the instability, a counter current drive would be necessary [10].

The positive BC dramatically affects $\iota(s)$ in QHS which could lead to equilibrium stability problems. In addition, the fact that no consistency between the equilibrium and its associated BC is achieved for $\beta^* > 3\%$ shows that this model implies that such equilibria with reactor-relevant β^* would not be physically realisable.

The W7X-like configuration shows as expected a small bootstrap with a comparably small effect on $\iota(s)$. However, accurate calculations for the real experiment (using the

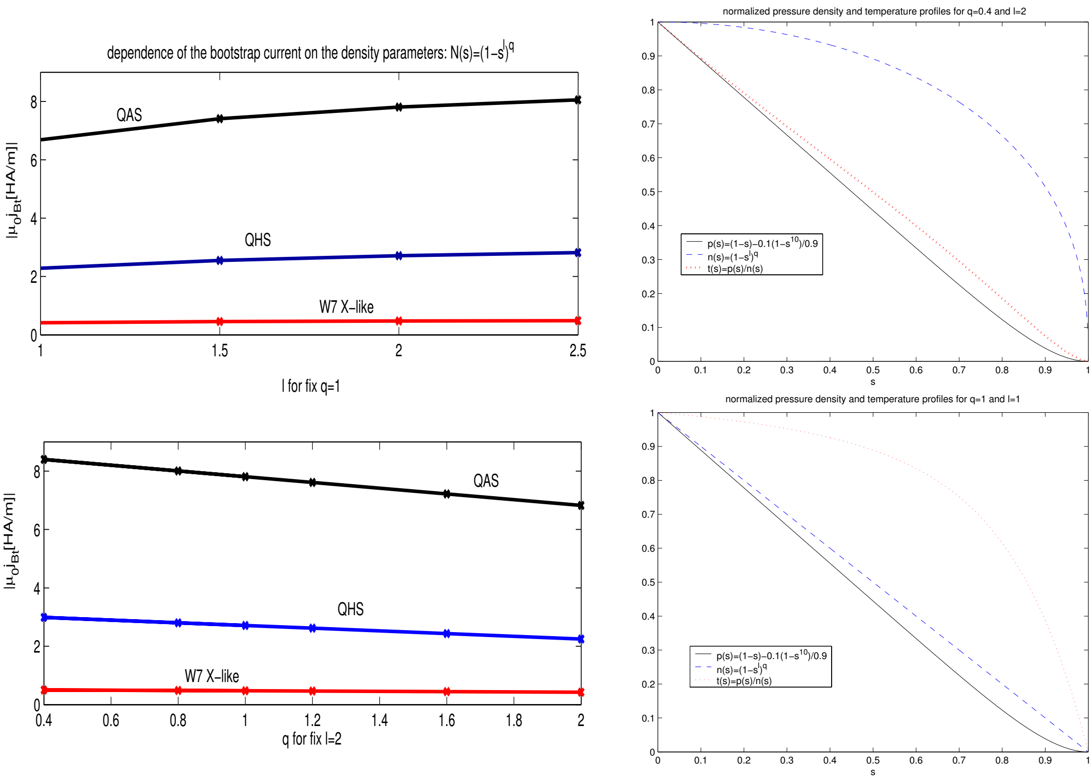

Figure 2: Absolute value of the bootstrap current for different density and temperature profiles. In the three configurations peaked temperature ($\ell = 2$ and q=0.4) seems to increase BC while peaked density ($\ell = 1$ and q=1) seems to decrease it

Drift Kinetic Equation approach) seem to obtain smaller BC than the model used in this work. This difference may be caused by the approximations assumed which may not be as applicable in the experiment as in the reactor-like configuration.

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