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BEHAVIOUR OF A DUST CLOUD IN THE PLASMA SHEATH ADJACENT TO A CONDUCTING WALL

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We investigate the behaviour of a dust cloud in the sheath electric field and the gravitational field, in particular its position of localisation, i.e., the distance from the wall. The charge of the dust grains is negative. For the negatively biased wall considered ($\varphi_w < 0$), the potential of the electric field in the plasma is also negative and tends monotonically to zero far from the wall. In the plasma sheath, with a thickness of several Debye lengths, the decrease in potential is rather sharp, while in the pre-sheath the potential changes smoothly. The gravitational force is assumed to be directed towards the wall. Thus, the repulsive electric field (driving dust grains away from the wall) and the gravitational field (driving the grains towards the wall) can form a potential well for the negatively charged grains. The plasma sheath is assumed to be collisionless [1]. For the absorbing wall considered, there are no electrons with positive velocities ($v_z > 0$) and energies higher than $e|\varphi_w|$ (with e the positive elementary charge), and electrons with energies lower than $e|\varphi_w|$ reflected in the sheath in front of the wall. Assuming the incoming electrons at the sheath edge ($x \to \infty$) to have a half Maxwellian velocity distribution, the solution of Vlasov's equation for these conditions can be integrated to yield the following expression for the electron density:

$$n_e = n_{e0} \frac{1 + \mathrm{erf}\left(\sqrt{\frac{e}{T_e}\{|\varphi_w| - |\varphi(z)|\}}\right)}{1 + \mathrm{erf}\left(\sqrt{\frac{e}{T_e}|\varphi_w|}\right)} \exp\left(-\frac{e}{T_e}|\varphi(z)|\right), \tag{1}$$

where $\mathrm{erf}(x)$ is the error function. Assuming the ions to be cold ($T_i \ll T_e$), we can use for the ion density $n_i(z)$ and the ion velocity $u_i(z)$ the relations

$$n_i(z)\,|u_i(z)| = n_{i0}\,|u_{i0}|, \quad |u_i(z)| = \left[u_{i0}^2 + + \frac{2e}{m_i}|\varphi(z)|\right]^{1/2}, \tag{2}$$

where n_{i0} and u_{i0} are the ion density and velocity, respectively, at the sheath edge. The potential energy of a dust grain in the sheath electric field and the gravitational field is

$$U(z) = Z_d e\,|\varphi(z)| + m_d g z, \tag{3}$$

where Z_d is the charge number and m_d is the mass of the grain. The potential energy (3) forms a potential well for the grains. Usually the limiting energy of the trapped grains in the well, $U_{\lim} = Z_d e|\varphi_w|$, is much larger than their termal energy, $Z_d e|\varphi_w|/T_d \gg 1$, so that the theory of trapping proposed in [2] is not applicable here and we can assume the trapped dust grains to be mainly localised at the bottom of the well [3]. According to (3), the single dust grain is at rest if

$$\frac{\partial}{\partial z}U(z) = Z_d e \frac{\partial}{\partial z}|\varphi(z)| + m_d g = 0. \tag{4}$$

To take the sum over all dust particles, we multiply this equation by $n_d(z)\,dz$ (with $n_d(z)$ the dust-grain number density) and after partial integration obtain

$$Z_d e \int_{z_0}^{z_0+\Delta_c} dz\,|\varphi(z)|\frac{\partial}{\partial z}n_d(z) = m_d g \mathcal{N}, \quad \mathcal{N} = \int_{z_0}^{z_0+\Delta_c} dz\,n_d(z), \tag{5}$$

where N is the number of dust grains in a cylinder elongated along the z axis and with cross section area $1cm^2$, z_0 is the position of the wall-side edge of the dust cloud, and Δ_c is the thickness of the cloud along the z axis. The distribution of grains in the plane perpendicular to the z axis is assumed to be uniform. From the balance of forces (5) it follows that this condition essentialy depends on the non-uniformity of the density distribution. The cloud thickness Δ_c is, in fact, determined by the way of filling the well with the dust particles and the transversal (lateral) non-uniformity of the electric field. The potential distribution in the sheath is to be found from Poisson's equation:

$$\frac{\partial^2}{\partial z^2}\left|\varphi(z)\right| = 4\pi e\left\{n_i(\varphi) - n_e(\varphi)\right\} - 4\pi Z_d e n_d(z). \tag{6}$$

Recent experiments show that the dust cloud ("dust condensate") is located at a distance from the wall several times larger than the Debye length, the thickness of the cloud is also of this order [4], and the shape of the cloud is trapezoidal with sharp boundaries [3]. Accordingly, we choose our model dust-grain density distribution in the form

$$n_d(z) = \left[n_{c1} + \frac{z - z_0}{\Delta} n_{c2}\right]\left[\Theta(z - z_0) - \Theta(z - (z_0 + \Delta))\right], \tag{7}$$

where $\Theta(x)$ is the step function, n_{c1} is the dust-grain number density at the wall-side boundary of the cloud, and n_{c2} is the density at the plasma-side boundary. In the dimensionless quantities $\chi(x) = e\left|\varphi(x)\right|/T_e$, $N_d = N/\left(\lambda_d n_0\right)$, $U_{eff} = U/\left(Z_d T_e\right)$, $x = z/\lambda_D$, $\Delta = \Delta_c/\lambda_D$, $n_{1,2} = n_{c1,2}/n_0$, $n(z) = n_d(z)/n_0$ Eqs. (3), (5) and (6) can be written as

$$\frac{\partial^2\chi}{\partial x^2} = \left\{1 + 2\frac{c_s^2}{u_{i0}^2}\chi\right\}^{1/2} - \frac{1 + \mathrm{erf}\left(\sqrt{\chi_w - \chi}\right)}{1 + \mathrm{erf}\left(\sqrt{\chi_w}\right)}\exp\left(-\chi\right) - Z_d n(z), \tag{8}$$

$$\frac{n_2 - n_1}{\Delta}\int_{x_0}^{x_0+\Delta} dx\chi(x) + n_1\chi(x_0) - n_2\chi(x_0 + \Delta) = PN_d, \quad N_d = \frac{1}{2}(n_1 + n_2)\Delta \tag{9}$$

$$N_d = \frac{1}{2}(n_1 + n_2)\Delta \tag{10}$$

and

$$U_{eff} = \chi(x) + Px, \quad P = \frac{m_d g\lambda_D}{Z_D T_e}, \tag{11}$$

where $n_0 = n_{i0} = n_{eo}$, $\lambda_D = \left(T_e/4\pi e^2 n_0\right)^{1/2}$ is the electron Debye length, and $c_s = \left(T_e/m_i\right)^{1/2}$ is the ion-sound velocity. For a given total grain number N_d, this system of equations contains the parameters x_0, n_1, n_2 and Δ, which are connected with each other by the relations (9) and (10). Hence, our model contains two arbitrary parameters. For completeness of the description, we would have to know the character of dust trapping in the well and the lateral forces acting on the dust grains. Up to now, the quantitative connection between these factors and the form of the dust cloud is not known. To be specific, we choose n_1 and Δ as free parameters. The first term on the left-hand side of (9) is connected with the non-uniformity of the dust density within the cloud, whereas the second and third terms appear due to the sharp changes in the dust density at the borders of the cloud. Because of the monotonous decrease of the electric potential (and depending on the character of this decrease) for the appropriate x_0 and Δ the balance of forces (9) can be fulfilled both for $n_1 > n_2$ and $n_1 < n_2$. The system of Eqs. (8)-(10) must be solved self-consistently. In what follows are given the results of our numerical solution.

We choose the folloiwng parameters $Z_d = 10^3$, $m_d = 10^{-10}g$, $n_0 = 1.6 \times 10^9 cm^{-3}$, and $T_e = 1.2eV$ [5]. For this case $\lambda_D = 2 \times 10^{-2}cm$ and $P = 1$. In Fig. 1 is shown the dependence of the position of the wall-side dust-cloud boundary x_0 on the thickness of the cloud for a small number of dust grains, $N_d = 2 \times 10^{-3}$ (or $N = 6.4 \times 10^4$) and for a wall-side dust-grain density $n_1 = 1.5 \times 10^{-3}$. The non-monotonous dependence of x_0 on the dust-cloud thickness indicates that the dust cloud itself changes the profile of the well. Figure 2 shows the shape of the potential well (11) without a dust cloud (solid line) and for the two different cloud thicknesses $\Delta = 1$ (dashed line) and $\Delta = 0.2$ (dash-dotted line). Of course, the spatial distribution of the potential in the plasma sheath is also subject to essential change. Figure 3 shows the change in this distribution comparing the solutions of Poisson's equation (8) without the dust cloud (solid line) and for the different cloud thicknesses $\Delta = 1$ (dashed line) and $\Delta = 0.2$ (dash-dotted line). For Bohm's criterion to be fulfilled we chose $u_{i0} = 1.5c_s$.

If the gravitational force acts parallel to the wall then, due to their heavy mass, we can assume the dust grains to be immobile (in the z-direction) as compared with the ions. The motion of the latter is again described by (2). The role of the dust grains is reduced to the compensation of the volume charge at the sheath edge: $n_{i0} - n_{e0} - Z_d n_{d0} = 0$. The standard calculations [6] result in the Bohm criterion

$$u_{i0}^2 > \frac{n_{i0}}{n_{e0}} c_s^2 = \left(1 + \frac{Z_d n_{d0}}{n_{e0}}\right) c_s^2.$$

Consequently, the existence of dust particles makes the Bohm criterion more strict.

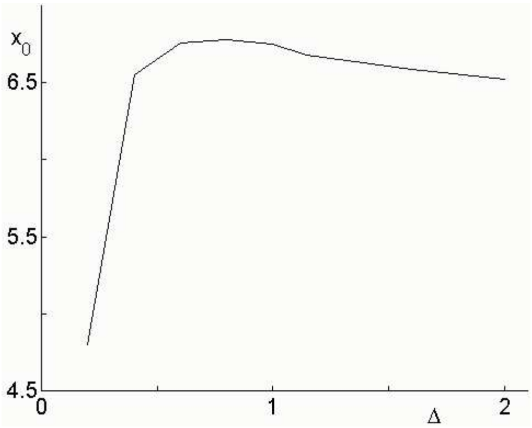

Fig. 1

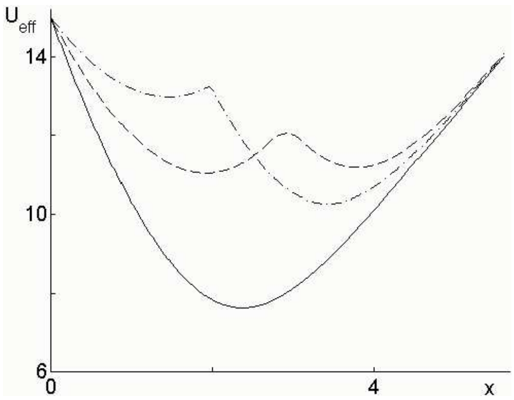

Fig. 2

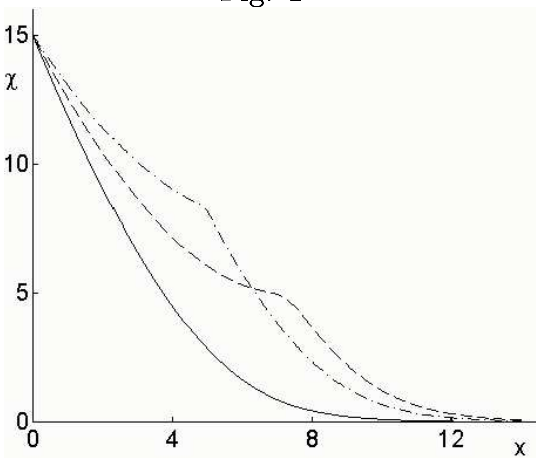

Fig. 3

From Figs. 1-3 one can draw the conclusion that the dust cloud is localized at a distance from the wall of the order of several Debye lengths. This result is experimentally confirmed. Even a small density of dust particles can, due to the high charge number Z_d of the dust grains, substantially change the potential distribution in the plasma sheath. The force balance (9), together with Poisson's equation (8), allows dust clouds with both decreasing ($n_1 > n_2$) and increasing ($n_1 > n_2$) shapes. To make the curves more discernible we chose the value $\chi_w = 15$ for the dimensionless electric potential at the wall. The results presented above are also valid for smaller χ_w.

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