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Axial Magnetic Field Generated from Photon Spin

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Photons have spin $\hbar$, and in circularly polarised light these spins are aligned, so that a beam of finite radius has angular momentum. The light beam can be represented as a number density n_ν of photons of energy $\hbar\omega$ moving in the z direction with the speed of light c. Thus in vacuum the intensity I is simply $n_\nu \hbar\omega c$, while the spin density is $\pm n_\nu \hbar$ or $\pm I/(\omega c)$ for right (+) and left (-) hand polarisation and is independent of the energy of each photon. The local density of angular momentum is related to spin density in an analogous way as diamagnetic velocity is related to the density of magnetic moments. Thus for a quasi-axisymmetric beam of circularly polarised light, the mean axial component of angular momentum per unit volume $\overline{M}_z$ is given by

$$\overline{M}_z = -\frac{1}{\omega c}\frac{r}{2}\frac{\partial I}{\partial r} \qquad (1)$$

and is usually concentrated near the edge of the beam. The total angular momentum per unit length of the beam represents the total spin, however, as can be seen from the integral

$$\int_o^{r_o} \overline{M}_z 2\pi r dr = -\frac{2\pi}{\omega c}\left\{\left[\frac{r^2 I}{2}\right]_{r=o}^{r_o} - \int_o^{r_o} I r dr\right\} = \frac{1}{\omega c}\int_o^{r_o} I 2\pi r dr \qquad (2)$$

where I is 0 at $r \geq r_o$, the beam radius.

When the absorbing medium is a plasma the angular momentum of the absorbed photons is transferred primarily to the electron species, i.e. the electrons experience a torque. If this were simply translated into a change of angular momentum of the electrons it would constitute a very large azimuthal current and associated axial magnetic field. The resulting induced azimuthal electric field E_θ opposing this generation of magnetic field would however be a much larger term, especially in the typical case considered here of a short (~1 ps) laser pulse of intensity $>10^{18}$Wcm^{-2} propagating through an underdense plasma. Indeed to a good approximation the torque from the absorbed laser light is instead largely balanced by an equal and opposite torque associated with E_θ, and the inertia of the electrons plays only a small role. There is a reversed torque by E_θ on the ion species and it is through this and collisions with electrons that the ions also acquire angular momentum, which after the laser pulse is over will represent most of the absorbed angular momentum.

The equation for the mean rate of change of angular momentum of the electrons per unit volume can be written as

$$n_e m_e r\frac{dv_{e\theta}}{dt} = -n_e erE_\theta - n_e er\left(v_{ez}B_r - v_{er}B_z\right) + \frac{\alpha_{ab}\overline{M}_z c}{L} - n_e m_e \nu_{ei} r v_{e\theta} \qquad (3)$$

where α_{ab} is the fraction of the laser intensity absorbed over an axial distance L; n_e, $\underline{v}_e$ and ν_{ei} are the electron number density, velocity, and collision frequency with the ions and $\overline{M}_z$ is the density of angular momentum in the z direction averaged over a wavelength . The magnetic field terms are not important in the early stage. It will be verified later that the inertial and collisional terms are small so that equation (3) together with Faraday's law give

$$rE_\theta \simeq -\frac{\alpha_{ab} r}{2n_e e\omega L}\frac{\partial I}{\partial r} \qquad (4) \qquad\qquad \frac{\partial B_z}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}\left(rE_\theta\right). \qquad (5)$$

Thus the axial magnetic field B_z can be expressed as

$$B_z = -\frac{2}{\omega Ler_0^{\,2}}\int\frac{\alpha_{ab} I_0}{n_e}dt \qquad (6)$$

assuming that B_z is uniform within the beam radius, i.e. I is $I_0(1 - r^2/r_0^2)$, a parabolic intensity profile.

The inverse dependence of B_z on n_e should be noted, as this contrasts with earlier theories of the inverse Faraday effect. This is discussed in ref. 5. Of course as n_e goes to zero, α_{ab} also tends to zero faster and there is no singularity. Of interest in this formula is that if this axial magnetic field is measured by Faraday rotation of a probing laser beam the angle of rotation is proportional to n_eBL which is a direct factor of eq. (6). An experimental measurement should therefore be quite robust provided the absorption is measured. In Najmudin et al (4) a straightforward substitution of the experimental parameters $t = 10^{-12}$s, $I_0 = 7.3 \times 10^{22}$ W/m^2, $r_0 = 10^{-5}$m, $\omega = 1.79 \times 10^{15}$ s^{-1}, $n_e = 2.1 \times 10^{25}$ m^{-3}, $L = 10^{-3}$m and assuming $\alpha_{ab} = 1$ gives B_z = 240 tesla (2.4 MG) compared to the experimental measurement for this density of 4 MG. However the laser power of 2.2×10^{13} W greatly exceeds that for self focusing (5,6). This effect will increase the intensity I and reduce the beam radius r_0, leading to a very sensitive increase in B_z in eq. (6). Indeed the critical value of laser power for self focusing, calculated assuming only weak relativistic effects, is 8.5×10^{11}W compared to the laser power of 2×10^{13}W in the experiment. A radial expansion of the heated plasma in the focal spot of the laser will also occur, lowering the electron density. This can be estimated as follows, assuming that the electron pressure in the focal spot increases as $\frac{2}{3}\alpha_{ab} It/L$. The ions as well as the electrons will have time to expand radially as can be seen from integrating twice.

$$\frac{n_e m_i}{Z}\frac{dv_r}{dt} = -\frac{\partial}{\partial r}\left(\frac{2}{3}\frac{\alpha_{ab} It}{L}\right) \qquad (7)$$

to give the time for cavitation t_c

$$t_c = \left(\frac{9 n_e m_i r_0^2 L}{2 Z \alpha_{ab} I_o}\right)^{1/3} \qquad (8)$$

which for Z = 2, $m_I = 4m_p$ (helium) and the above parameters gives 10^{-12}s. Ponderomotive forces will also cause plasma expansion and later also the axial magnetic field pressure itself. There could also be a pinching azimuthal magnetic field set up by absorption of photon momentum and ponderomotive forces, not considered here. That quasi-neutrality is a good approximation (i.e. electron cavitation with no ion motion is a poor assumption on these time scales) can be shown by calculating $\frac{\delta n}{n} = \frac{\varepsilon_o T}{e r_o^2 n_e}$ for an electron temperature T_e of 1 MeV and r_0 and n_e as above. Then $\delta n / n$ is 0.03. The other assumptions in the model, namely that the electron inertia and collisions are small can be tested by comparing their values to the source term in eq. (3); their relative values are 0.03 and 5 x 10^{-8} respectively, the latter for a classical collision frequency at $T_e = 1MeV$. If an anomalous collision frequency is triggered this argument may be modified.

Generally there is reasonable agreement within the uncertainties of the experiment and the simplifications of the model. Of importance is the trend for higher magnetic fields at higher intensities and lower electron densities, which is seen in the experiment.

It should be noted that, though the photon spin is $\hbar$, the angular momentum density in the circular polarised laser beam is $-\frac{1}{2} r \partial I / \partial r / \omega c$ and is a classical expression, which should be described by the electric and magnetic fields of a laser beam propagating in vacuo. It is instructive to consider first a plane-polarised beam of circular cross-section πr_0^2. If the electromagnetic wave is represented solely by

$E_x = E_0(r)\cos(\omega t - kz)$ (9) $\qquad$ $B_y = B_o(r)\cos(\omega t - kz)$, (10)

there is an obvious problem of how to satisfy $\nabla.\underline{B} = 0$ and $\nabla.\underline{E} = 0$ for fields that appear to originate and end at the edges of the beam. It is clear that axial components of both fields of the form and magnitude

$$E_z \sim \mp \frac{E_0}{k r_0}\sin(\omega t - kz), x > 0 / x < 0 \quad (11) \qquad B_z \sim \mp \frac{B_0}{k r_0}\sin(\omega t - kz), x > 0 / x < 0 \quad (12)$$

must exist to satisfy $\partial E_x / \partial x = -\partial E_z / \partial z$ and $\partial B_y / \partial y = -\partial B_z / \partial z$. At the same time Faraday's and Ampere's laws $\partial E_x / \partial y = \partial B_z / \partial t$ and $\partial B_y / \partial x = \mu_0 \varepsilon_0 \partial E_z / \partial t$ are satisfied with this ordering, the precise functional form depending on the beam profile. The field lines are thus propagating as closed .

When a second but orthogonal plane-polarised wave, 90^o out of phase is added, with fields

$$E_y = E_0(r)\sin(\omega t - kz) \qquad (13) \qquad\qquad B_x = -B_0(r)\sin(\omega t - kz) \qquad (14)$$

together with axial fields

$$E_z \sim \pm\frac{E_0}{kr_0}\cos(\omega t - kz), y > 0 / y < 0 \quad (15) \qquad B_z \sim \mp\frac{B_0}{kr_0}\cos(\omega t - kz), y > 0 / y < 0 \quad (16)$$

a rotating component of the Poynting vector is obtained. This is because the E_z field of the first plane wave given by eq. (11) is exactly in phase with the B_x field (eq.14) of the second wave, and leads to a $\pm y$ components of the Poynting vector above and below the y axis respectively. Similarly there is a contribution in the $\pm x$ direction to the left and right of the x axis. Similar arguments can be made for the axial components of the second wave interacting with the transverse components of the first wave. In total there is a local density of angular momentum <u>M</u> given by

$$\underline{r} \times \underline{p} = \varepsilon_o \underline{r} \times (\underline{E} \times \underline{B}) = \varepsilon_o\left[(\underline{r}.\underline{B})\underline{E} - (\underline{r}.\underline{E})\underline{B}\right] = \underline{M} \qquad (17)$$

the component of interest being axial.

We thus find that the mean azimuthal component of the Poynting vector is ~ $I/(kr_0)$ or rather $-\frac{1}{2}\partial I / \partial r / k$ and the mean angular momentum density $\overline{M}_z$ is $-\frac{1}{2} r \partial I / \partial r / \omega c$ as stated earlier. There is thus complete equivalence between the photon spin picture and classical fields provided the longitudinal field components are included.

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