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Effect of a Cylindrical Density Enhancement on Čerenkov Radiation from a Modulated Electron Beam in a Magnetoplasma

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1. Introduction

Cylindrically symmetric magnetic-field-aligned plasma nonuniformities, the so-called density ducts, are known to affect significantly the features of radiation from given sources immersed in such plasma structures [1]. Recent "active" space experiments and model laboratory experiments [1, 2] provide evidence that ducts with enhanced density can be formed in a magnetoplasma due to various nonlinear effects. In this paper, we examine the influence of a cylindrical density enhancement aligned with an external magnetic field and surrounded by a uniform background plasma on the coherent spontaneous emission from a thin modulated electron beam injected along the enhancement axis. Since the study of nonlinear effects connected with the beam–plasma interaction in the presence of plasma nonuniformities seems impossible without a detailed preliminary analysis of the beam-excited electromagnetic field in the linear approximation, we assume that the beam is given and consider the simplest case of an axisymmetric cylindrical duct with a step-shaped radial density profile. A major aim has been to compare the average power radiated from a modulated electron beam of finite length at the modulation frequency belonging to the whistler band in the presence of an enhanced-density duct and in the case where the beam is injected in a homogeneous magnetoplasma [3].

2. Theoretical Model

Let an infinitely long cylindrical density duct of radius a be immersed in a uniform cold magnetoplasma. The duct axis is taken as the z axis in a cylindrical coordinate system (ρ, ϕ, z). Parallel to this axis is an external static magnetic field $\mathbf{B}_0 = B_0\hat{z}_0$. The plasma density N is a function only of distance ρ from the axis, and is defined by

$$N(\rho) = N_a + (\tilde{N} - N_a)[1 - H(\rho - a)], \tag{1}$$

where $\tilde{N}$ and N_a are constant plasma densities inside the duct $(\rho < a)$ and in an outer region $(\rho > a)$, respectively, and H is the Heaviside step function.

The medium is assumed to be described by a dielectric tensor having the following form for a monochromatic signal with a time dependence of $\exp(i\omega t)$:

$$\hat{\varepsilon} = \epsilon_0(\varepsilon\hat{\rho}_0\hat{\rho}_0 - ig\hat{\rho}_0\hat{\phi}_0 + ig\hat{\phi}_0\hat{\rho}_0 + \varepsilon\hat{\phi}_0\hat{\phi}_0 + \eta\hat{z}_0\hat{z}_0), \tag{2}$$

where ϵ_0 is the permittivity of free space and the tensor elements ε, g, and η are functions of ω. Expressions for the tensor elements can be found elsewhere.

We consider a modulated electron beam of finite length and radius b injected along the duct axis ($b < a$). The beam current density can be specified as

$$\begin{aligned}\mathbf{j}(\rho, z, t) &= \hat{z}_0\, j_z(\rho, z, t) = \hat{z}_0 I_0(\pi b^2)^{-1}[1 - H(\rho - b)] \\ &\times H(z)\, H\left(t - \frac{z}{v_b}\right)\left[1 + \sin\omega_0\left(t - \frac{z}{v_b}\right)\right]\exp(-\beta z),\end{aligned} \tag{3}$$

where $I_0 = -en_b v_b \pi b^2$ is the total beam current, ω_0 is the modulation frequency of the beam current, v_b is the beam velocity, n_b is the beam density (constant for $\rho < b$), e is the magnitude of the electron charge, and β is the inverse coherence length of the beam [4].

To derive expressions for the beam-excited field, we use Laplace – Fourier transform of the beam current and the field with respect to the time t and the coordinate z. The solutions of the Laplace – Fourier transformed Maxwell equations in the regions $\rho > a$, $b < \rho < a$, and $\rho < b$ are represented up to coefficients independent of ρ in terms of Hankel functions, Bessel functions of the first and second kinds, and Bessel functions of the first kind, respectively. For example, the components $E_\phi(\rho, k_z, s)$ and $B_\phi(\rho, k_z, s)$ in the region $\rho < b$ are given by the formulas

$$E_\phi(\rho, k_z, s) = i\sum_{k=1}^{2} B_k J_1(k_{\perp k}\rho), \quad B_\phi(\rho, k_z, s) = -c^{-1}\sum_{k=1}^{2} n_k B_k J_1(k_{\perp k}\rho), \tag{4}$$

where

$$n_{1,2} = -\frac{ic}{s}\,\frac{\varepsilon}{k_z g}\left[k_{\perp 1,2}^2 + k_z^2 - \frac{s^2}{c^2}\left(\frac{g^2}{\varepsilon} - \varepsilon\right)\right], \tag{5}$$

J_1 is a Bessel function of the first kind of order unity, B_k are unknown coefficients, c is the velocity of light in free space, $k_{\perp 1}$ and $k_{\perp 2}$ are two branches of the transverse wavenumber corresponding to the axial wavenumber k_z in a magnetoplasma, and s is a complex variable related to the wave frequency, $s = i\omega$.

Application of the boundary conditions at $\rho = a$ and $\rho = b$ and the radiation condition at infinity yields the coefficients in the field expressions. Next, using inverse Laplace – Fourier transform, one arrives at the resulting integral representation for the beam-excited field.

3. Power Radiated

The power lost by beam is

$$P = -2\pi\int j_z(\rho, z, t)\, E_z(\rho, z, t)\, \rho\, d\rho\, dz, \tag{6}$$

where the integration is performed over the beam volume. Substituting the integral representation for the E_z component, using the Maxwell equation relating this component to the B_ϕ component, and performing averaging over a time period long compared to the modulation period, one obtains the following expression for the case of a collisional magnetoplasma in the limit $\beta \to 0$ considered here:

$$\begin{aligned}\langle P\rangle \;=\; & Z_0 I_0^2 \,\frac{L}{2\pi k_0 b^2}\left\{\mathrm{Im}\left(\tilde{\eta}^{-1}\hat{B}_\phi(b,\omega_0/v_b,i\omega_0)\right)\right.\\ & + \int_{-\infty}^{+\infty}\mathrm{Im}\left(\tilde{\eta}^{-1}\hat{B}_\phi(b,\kappa_z+\omega_0/v_b,i\omega_0)\right)\frac{\sin^2(\kappa_z L/2)}{\pi\kappa_z^2 L/2}\,d\kappa_z\\ & \left. + ⨍_{-\infty}^{+\infty}\frac{1}{\kappa_z}\,\mathrm{Re}\left(\tilde{\eta}^{-1}\hat{B}_\phi(b,\kappa_z+\omega_0/v_b,i\omega_0)\right)\frac{\sin\kappa_z L}{\pi\kappa_z L}\,d\kappa_z\right\},\end{aligned} \tag{7}$$

where $\hat{B}_\phi(\rho,k_z,s) = -B_\phi(\rho,k_z,s)/[\mu_0 b\, j_z(0,k_z,s)]$, μ_0 is the permeability of free space, $k_0 = \omega_0/c$, $Z_0 = (\mu_0/\epsilon_0)^{1/2}$, $L = v_b t$ is the instantaneous beam length, $\tilde{\eta}$ denotes the dielectric-tensor element η inside the duct, and the bar on the integral sign indicates the Cauchy principal value.

Equation (7) is a general representation for the average power loss of the modulated beam injected parallel to the axis of a cylindrical channel in a collisional magnetoplasma. It is to be emphasized that Eq. (7) is valid for channels with both enhanced and depressed plasma density. Note that an expression for $\langle P\rangle$ in the special case of a collisionless magnetoplasma can be obtained immediately from Eq. (7) (see [5]).

We now present the results of calculations of the average radiated power for the case of an enhanced-density duct in a magnetoplasma modeled upon the Earth's ionosphere. We assume that the modulation frequency ω_0 belongs to the whistler band, i.e., lies between the lower-hybrid frequency and the electron gyrofrequency which is less than the electron plasma frequency. We note that waves of the whistler band play an important role in many physical phenomena in the ionosphere. In this band, a magnetic-field-aligned duct with enhanced density is capable of guiding improper leaky modes [1] whose attenuation can be very small under certain conditions. The influence of these slightly attenuated leaky modes on the quantity $\langle P\rangle$ is well pronounced if the ratio ω_0/v_b is close enough to the real part of their complex axial wavenumbers $k_{z,\nu}$, where ν is the leaky-mode order ($\nu = 1,\, 2,\, \ldots$) [1].

Figure 1 shows the normalized (to I_0^2) quantity $\langle P\rangle$ as a function of the parameter c/v_b and the beam length L for typical conditions of the active experiments on the formation of artificial enhanced-density ducts in the Earth's ionosphere [2]. Also presented is the quantity $\langle P_a\rangle$ for the beam injection in an ambient uniform plasma. With the parameters used for plot (a), the least-attenuated leaky mode in the duct has the axial wavenumber $k_{z,1} = k_0 \times (24.39 - i0.074)$.

It is evident from Fig. 1 that the presence of a duct affects significantly the dependence of the emitted power on the beam velocity. At $c/v_b = \mathrm{Re}\,(k_{z,\nu}/k_0)$, one can observe resonance peaks of $\langle P\rangle$ related to Čerenkov excitation of guided modes with the axial wavenumbers $k_{z,\nu}$. On the contrary, in the absence of a duct, the power $\langle P_a\rangle$ is a smooth function of v_b and L, except for the region where the parameter c/v_b approaches the point $\mathrm{Re}\,P_c = 12.9$ corresponding to conical-refraction ("double-pole") whistler-mode waves in the surrounding medium [1, 3]. Similar results were also obtained for conditions of the laboratory experiments modeling the beam radiation in the ionospheric plasma.

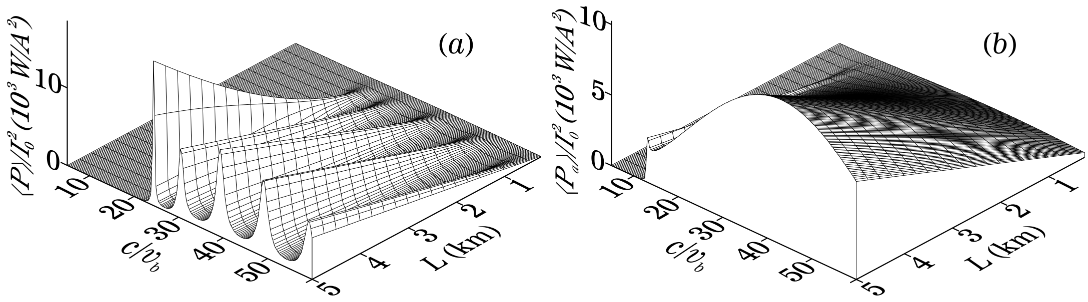

Figure 1: Average power lost by a modulated beam at the modulation frequency as a function of the parameters c/v_b and L (a) in the presence of a duct and (b) for an ambient uniform plasma. $\tilde{N} = 3 \times 10^6$ cm^{-3}, $N_a = 10^6$ cm^{-3}, $B_0 = 0.5$ G, $\omega_0/2\pi = 120$ kHz, $a = 10$ m, and $b = 1.5$ m. For both plots, the electron collision frequency $\nu_{\rm e} = 10^{-3}\omega_0$.

4. Conclusions

In the present paper, we have considered the electromagnetic radiation from a modulated electron beam injected in a magnetic-field-aligned cylindrical density enhancement in a collisional magnetoplasma. Calculations performed for the case of excitation of whistler waves by the beam reveal that the power lost by the beam can increase noticeably due to Čerenkov resonance excitation of whistler modes guided by the density enhancement. It is evident from the results obtained that the resonance features of the excitation of guided modes make it possible to change significantly the beam power loss by the variation of beam energy and to select different regimes of the power loss in much narrower energy intervals than in the case of beam injection in a uniform magnetoplasma.

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