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Fast recovery of the q profile on ASDEX Upgrade using a nonlinearly optimized function parameterization model

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Introduction

Accurate determination of the safety factor (q) profile is a critical issue for the successful exploitation of Advanced Scenario configurations in tokamaks. This is true both as a prerequisite for the realtime control of the q profile to minimize the negative effects of MHD activity on the plasma as well as providing results necessary for post-shot stability analysis of experimentally realized equilibrium states. On the ASDEX Ugrade experiment, the ill-conditioned inverse problem of equilibrium identification from diagnostic data is carried out in realtime using the Function Parameterization (FP) technique [1,2]. Up to now, only magnetic data has been used as realtime diagnostic input and this FP model, which can only reliably identify q values near the plasma boundary, consists of a quadratic polynomial whose arguments are the 16 highest variance orthogonal linear combinations of 46 magnetic measurements as determined by principal component analysis (PCA) carried out on a simulated equilibrium database. Identification of the full q profile requires knowledge of the internal magnetic configuration of the plasma which is provided, in the case of ASDEX Upgrade, primarily by the Motional Stark Effect (MSE) diagnostic [3]. The addition of MSE data to the existing equilibrium magnetic signals leads, however, to rather large conventional FP models and consequently to a significant increase in the time required for realtime parameter identification. This has motivated the development of a more compact "FP" model, where the linear combinations of the expanded set of diagnostic inputs are selected using a nonlinear optimization procedure [4]. Such a compact model is needed to provide a computationally viable identification of the q profile for the planned realtime control of the current density profile on ASDEX Upgrade.

Here, we present preliminary results of the application to q profile recovery from experimental ASDEX Upgrade data of nonlinearly optimized FP models trained on an equilibrium database with a realistic variation of the plasma geometry (7.5m^3 $<$ Volume $<$ 15.6m^3, 1.52m $< R_{geo} <$ 1.76m, $-$0.15m $< Z_{geo} <$ 0.25m) and compare these results with interpreted equilibria using the CLISTE code [5,6]. We have found that due to the strongly heteroscedastic nature of the uncertainties in q, it is more correct to recover the iota = 1/q profile and then invert it than to directly recover q itself by unweighted least squares.

Description of Algorithm

The essence of the nonlinear optimization (NLO) is the selection of a small set of iteratively optimized linear combinations LC of the data with the objective of a successful recovery of $q(\rho)$ using a more compact quadratic model than that of the standard FP approach. Thus $LC < NP$ where NP is the number of principal components of the input signals needed to give a good FP fit. The justification for reducing the number of linear combinations below NP lies in the observation that not all the variance of the magnetic measurements is due solely to variations in the current density profile or equivalently the q profile and hence we expect that it will be possible to recover $q(\rho)$ with fewer than the NP transformed measurements which describe all measurement variance. This compact FP model is given by:

$$y_{i,r} = \sum_{\substack{l=0,\\ j=l}}^{LC} \beta_{l,j,r} \left(\sum_{t=1}^{NP} w_{t,l} P_{i,t} \right) \left(\sum_{u=1}^{NP} w_{u,j} P_{i,u} \right) + \epsilon_{i,r} \tag{1}$$

where $y_{i,r}$ is value of the r^{th} of NR recovered parameters for the i^{th} of NC cases in the database, $P_{i,t}$ is the t^{th} principal component for the i^{th} case, $(w_{1,l}, w_{2,l} \ldots, w_{NP,l})$ is the vector of measurement weights for the l^{th} of LC optimized linear combinations and $((\beta_{l,j,r}\,,\ {\scriptstyle l=0,LC}), {\scriptstyle j=l,LC})$ is the set of $(LC+1)(LC+2)/2$ coefficients which recovers the r^{th} output parameter ($l = 0$ corresponds to the intercept where $\mathbf{w}_o \cdot \mathbf{p}_i \equiv 1$)).

Starting with the NP principal component variables, the NLO problem is structured as follows: Choose the matrix of weights $W_{(NP\times LC)}$ (consisting of the $w_{t,l}$'s) which minimizes the cost function defined as

$$Cost = \sum_{i=1}^{NC} \sum_{r=1}^{NR} (y_{i,r} - \hat{y}_{i,r})^2 + \alpha \sum_{l=1}^{LC} \left(1 - (\sum_{t=1}^{NP} w_{t,l}^2)^2 \right)^2 \tag{2}$$

where $\hat{y}_{i,r} = y_{i,r} - \epsilon_{i,r}$ is the predicted value of $y_{i,r}$ and the rightmost term, tuned by the regularization parameter α, favours weight vectors of unit norm: $\mathbf{w}_l \cdot \mathbf{w}_l = 1$. This avoids indeterminacies in the magnitudes of W and the set of β's since essentially only the product $\|W\|\|\beta\|$ is unique where $\|W\|$ and $\|\beta\|$ represent the norms of the weight vectors and the linear regression coefficents, respectively.

The NAG routine E04FCF performs the NLO of the $NP \times LC$ weight coefficients. In a conventional neural network algorithm the β's would also be varied along with the W matrix. A special feature of the present work is that the β's are not explicitly part of the NLO, but rather are determined by linear regression within the user-supplied subroutine called by E04FCF to determine the set of $NC \times NR$ residuals $\{\epsilon_{i,r}\}$. For the present case where the set of parameters to be recovered consists of an array of discrete values from a radial profile, the residuals will be highly correlated. We have improved the convergence of the algorithm by using instead the transformed residuals:

$$\mathbf{z} = \mathbf{S}^{-1/2}\mathbf{r} \tag{3}$$

where $\mathbf{r} = (\epsilon_{1,i}, \ldots, \epsilon_{NR,i})$ is the vector of NR residuals for each case in the database and $\mathbf{S}_{(NR\times NR)}$ is the covariance matrix of the recovery errors which is determined at the outset by "best possible" regressions for each parameter using all NP linear combinations. The advantage of using the transformation (3) is that the covariance matrix for the transformed residual variables $\mathbf{z}$ is now diagonal, i.e. they are uncorrelated.

We have found that direct recovery of the q profile via a standard least squares regression is not optimal for the following reasons: The simplest ansatz for the error structure of the MSE data assumes independent errors of fixed amplitude for each channel (in reality, the electron density profile has a significant bearing on the actual error levels). A fixed angular error $\delta(\gamma)$ is equivalent to a fixed error level in the ratio of the poloidal to the toroidal magnetic field B_θ/B_ϕ. Since errors in B_ϕ are negligible we have $\delta(\gamma) \simeq \delta(B_\theta)/B_\phi$. Now q is defined as

$$q(r) = \frac{1}{2\pi}\oint \frac{B_\phi}{RB_\theta}ds \;\approx\; \frac{rB_\phi}{RB_\theta} \equiv q_{cyl}(r) \tag{4}$$

where $q_{cyl}(r)$ is an approximation valid for circular flux surfaces. Thus the sensitivity of q_{cyl} to measurement errors is given by

$$\delta(q_{cyl}(r)) = -\frac{\delta(B_\theta)}{B_\theta}q_{cyl}(r) \;=\; -\frac{\delta(B_\theta)}{B_\phi}\, q_{cyl}^2\,\frac{R}{r} \tag{5}$$

Thus at a given flux surface radius, the ansatz that the magnitude of the error term $\delta(B_\theta)/B_\phi$ is fixed leads to a heteroscedastic error structure for q_{cyl}, namely $\delta q_{cyl} \sim q_{cyl}^2$. This invalidates one of the conditions under which the usual least squares regression model gives a good solution, namely that the errors be of uniform magnitude or homoscedastic. To overcome this problem we chose to regress iota profile $\iota(r)$. It is noted that the errors go as

$$\delta(\iota) = \delta(1/q) = -\frac{1}{q^2}\delta(q) \tag{6}$$

Results

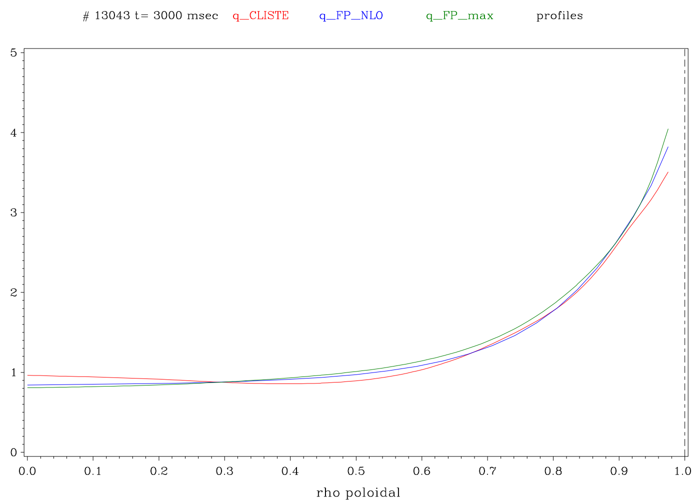

Fig.1 q profile for # 13043, t= 3.00 s.

The red curve in Figure 1 is the q profile from a CLISTE equilibrium fit using MSE and equilibrium magnetic measurements for ASDEX Upgrade discharge 13042 at $t - 3.00\,$s with $I_p = 1\,MA$, $n_e = 4.3 \times 10^{19}$m^{03}, $B_\phi = -2.37\,$T and $q_{95} = 3.5$. The rmse fit error was 1.4 mT for the 46 magnetic signals and 0.18° for the 10 MSE channels. The blue curve is the inverse of the recovered iota profile from 16 nonlinearly optimized linear combinations of the magnetic and MSE data. The green curve corresponds to the best possible FP model, again for the inverted iota profile. The NLO and FP curves are very close, and both deviate somewhat from the CLISTE profile. The discrepancy at the edge is worrying, since q is expected to be accurately recoverable here. The MSE signals cover the region $0.1 < \rho < 0.7$ so that q in the edge region is determined by information in the magnetic signals only. Here the signal error magnitude satsisfies $\delta(B_\theta) \sim Const.$ so that the assumption of homoscedasticity in the uncertainties in $\iota(\rho)$ does not apply in this case and this may explain the edge discrepancy.

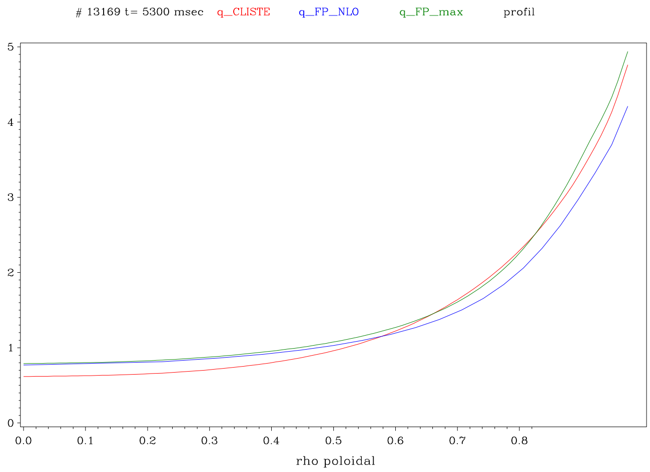

Fig.2 q profiles for # 13169, t= 5.30 s.

Figure 2 presents results for discharge 13169, $t-5.30\,\mathrm{s}$, $Ip = 1MA$, $n_e = 9.2\times10^{19}\mathrm{m}^{03}$, $B_\phi = -2.5\,\mathrm{T}$ and $q_{95} = 4.7$. Here the FP and NLO q profiles agree closely for $\rho < 0.5$ while for $\rho > 0.5$ the FP and CLISTE are reasonable consistent.

Discussion

The present results are preliminary The models presented here were trained with a database where B_ϕ and I_p were randomly varied. Since, as outlined above, it is not possible to simultaneously choose a homoscedastic error structure for $\iota(\rho)$ both in the core and edge regions, more accurate results could be obtained by training separate models for each of the small set of discrete values of B_ϕ which are normally used on the experiment, while taking into account the effect of the aspect ratio on the regression weights.

The new algorithm is planned to be ready for use use by late 2002 or early 2003 for realtime feedback control of ASDEX Upgrade.

References

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