

Anomalous ion diffusion and radial electric field generation in edge tokamak plasma turbulence of the Hasegawa-Wakatani model

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Using a simple, spatially periodic and time-independent model of electrostatic tokamak edge-plasma turbulence, we have recently found [1] a new type of anomalous impurity diffusion in this regime. In the present contribution, we estimate this diffusion for a more general spatio-temporal form of the potential, obtained as a solution of the Hasegawa-Wakatani (H-W) equations [2]. Moreover, we discuss whether this diffusion can still have (as the previous model) the property of both random-walk and Lévy-walk dynamics [3]. Finally, we have found the difference between the drift approximation and the full description of the dynamics (including the effect of the finite Larmor radius). As an interesting consequence of the dynamics discussed, we are looking for the possibility of radial electric field generation in the turbulence regime described by H-W equations, which was shown recently to result from the simple model [1] mentioned above by means of particle-in-cell (PIC) simulation [4] and which had been put forward by Tendler [5].

The low-frequency turbulence in the edge-plasma region of a tokamak and the generation of low-frequency electrostatic potential fluctuations is usually considered as a key effect, leading to anomalous diffusion of ions from this region. There already exist attempts to model this diffusion by means of numerical simulations [6]. In our recent paper [4] we started to study this problem, using a very simple model of the potential (spatially periodic and time-independent potential). There, we found two interesting effects: anomalous diffusion of impurities and, as a consequence thereof generation of a radial electric field.

The aim of the present contribution is to extrapolate our discussion to a more complicated form of the potential. As a natural extension, we take the potential has recently been calculated as a solution of the Hasegawa-Wakatani equations [2], for parameters of the CASTOR tokamak [7]. Moreover, since our recent results for the time-independent potential are basically connected with the full description of the dynamics (i.e. with the effects of finite Larmor radius), we compare the results following from the full description of the dynamics with those from the drift description (both obtained from the code BIT 2 as an exact calculation or a gyro-averaged $\mathbf{E} \times \mathbf{B}$ drift approximation).

The theoretical studies are usually based on the test-particle drift approximation [6],[8],[9] and on the electrostatic field obtained from the Hasegawa-Mima or Hasegawa-Wakatani equations. In our foregoing papers, we used for a simulation of anomalous $\mathbf{E} \times \mathbf{B}$ impurity diffusion in potential fluctuations a very simple time-independent, spatially periodical potential field given as $U = U_0[2 + \cos kx \cos ky]$. This diffusion is caused by chaotization of particle motion due to the non-integrability of the related Hamiltonian and represents a typical finite-Larmor-radius effect. Two types of the chaotic motion have been found, viz. random walk and Lévy walk.

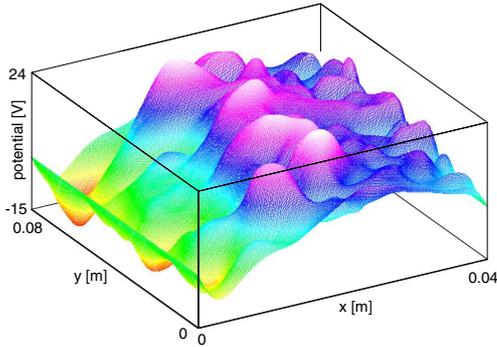


Figure 1. H-W potential at time $20\mu s$.

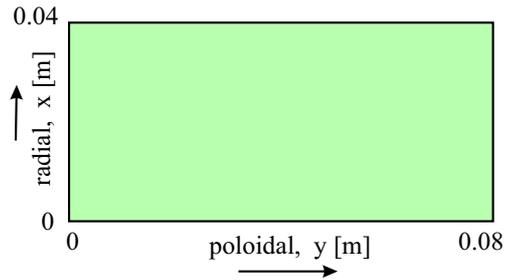


Figure 2. Model geometry.

To be closer to reality, we used in our simulation instead of the time-independent, spatially periodical potential field a potential representing a solution of the Hasegawa-Wakatani equations mentioned above. This time-dependent solution was obtained for CASTOR tokamak parameters from a code developed by Dyablin [7]. Contrary to the usually used drift approximation [8], we also consider the cyclotron motion of ions (finite-Larmor radius). We performed our simulation using the two-dimensional PIC code BIT 2. In this code, the said H-W potential (Fig. 1) was implemented and kept as a background potential in which only the ions (C^+) are moving. The simulation follows the dynamics in a poloidal plane in a region of $4 \times 8 \text{ cm}^2$ (radial \times poloidal direction, Fig. 2). In the poloidal direction, close to the boundary, the H-W potential was slightly adapted to enable full periodicity in this direction. This means that all parameters are set equal at the poloidal boundaries and particles crossing one of them are re-injected from the other one with the same velocity. In the radial direction the system is not periodical. Particles crossing radial boundaries are re-injected with Maxwellian-distributed velocities. The right-hand side boundary ($r = L_r = 4 \text{ cm}$) represents the edge-plasma boundary. The left-hand side boundary is closer to the plasma centre. At both of these boundaries the radial electric field is assumed to be zero. The magnetic field \mathbf{B} is perpendicular to this poloidal plane, i.e. $\mathbf{B} = (0, 0, B)$. At the beginning, the impurity ion density has Gaussian shape (Fig. 3) with the maximum in the middle of the simulation region. The following plasma parameters have been chosen: Magnetic field $B = 1 \text{ T}$, the maximum of the Gaussian impurity distribution is chosen as 1% of the plasma density, e.g., $n_i = 10^{15} \text{ m}^{-3}$ with 10% of maximum at the radial boundaries, impurity temperature $T = 1 \text{ eV}$. The average number of particles per cell is $N = 35$ (256×512 cells).

The electrons and ions are fixed on the background and only the impurity ions and the additional field generated by them are treated self-consistently. This approach enables to calculate e.g. the potential, the radial electric field, the ion temperature, the averaged

poloidal velocity of the impurities, etc.

In the first simulation we were interested especially in the influence of the H-W potential on the motion of the impurity ions and the generation of the radial electric field. The latter can cause a poloidal rotation of plasma due to the $\mathbf{E} \times \mathbf{B}$ drift, which we have found in the foregoing PIC simulation based on the time-independent, spatially periodical potential. The Gaussian profile chosen for the radial density is generally accepted to occur in tokamak plasmas. The total simulation time is $20 \mu\text{s}$.

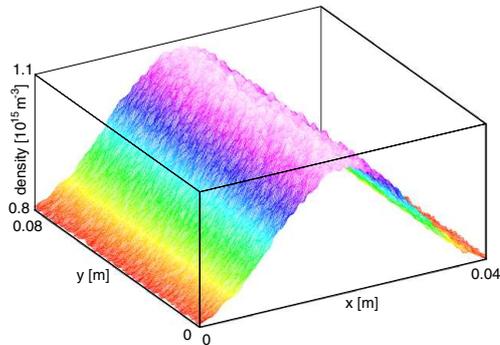


Figure 3. Initial Gaussian density profile of the impurities.

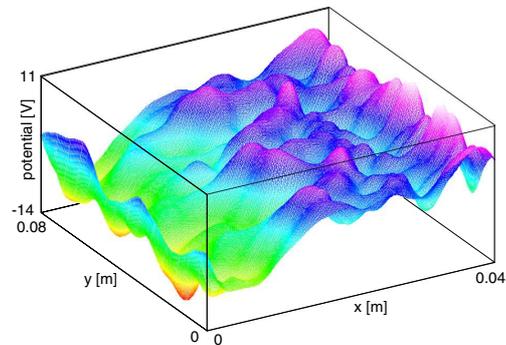


Figure 4. H-W potential plus self-consistent potential.

In Figs. 4 - 7 we summarise the results. Fig. 4 shows, at the end of the simulation, the total potential, e.g., the self-consistent potential of the impurities added to the "external" H-W potential. The total potential is considerably different from the original "external" potential. This is caused by the fact that the impurity ions have a tendency to spend more time in low-potential regions and this effect appears already shortly after the start of the simulation.

The dynamics of the impurities are strongly influenced by the H-W potential and change the initial radial density profile, as seen in Fig. 5. Moreover, this anomalous impurity diffusion leads to charge separation of the impurity ions in the radial direction, which results in the generation of radial a electric field (Fig. 6). This mechanism has also been found in the case of time-independent, spatially periodical potential [4]. As a consequence of this radial electric field, a poloidal $\mathbf{E} \times \mathbf{B}$ fluid velocity is generated (Fig. 7).

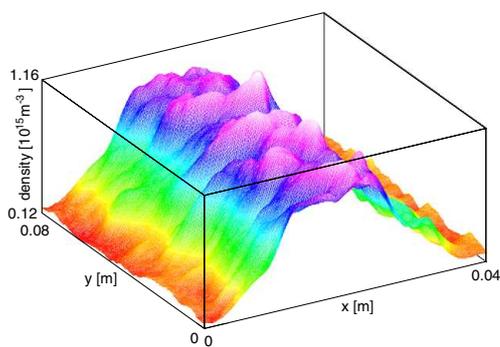


Figure 5. Impurity-density profile at the end of the simulation ($t = 20 \mu\text{s}$).

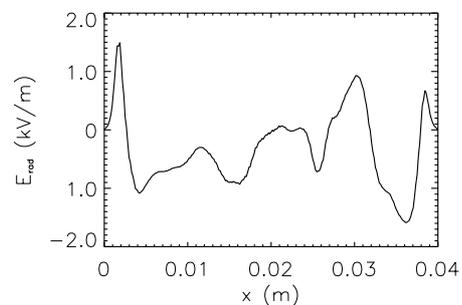


Figure 6. Poloidally-averaged radial electric field profile (at $t = 20 \mu\text{s}$).

In order to show the differences between the usually applied drift approximation and our more exact approach, we started to compare the diffusion coefficients for both cases.

The first results show significant differences these two approaches. The origin of these differences can be easily found from the quite different dynamics of the ions for both cases as seen in Fig. 9 (mentioned also in [7]). Here, the self-consistent field is not considered.

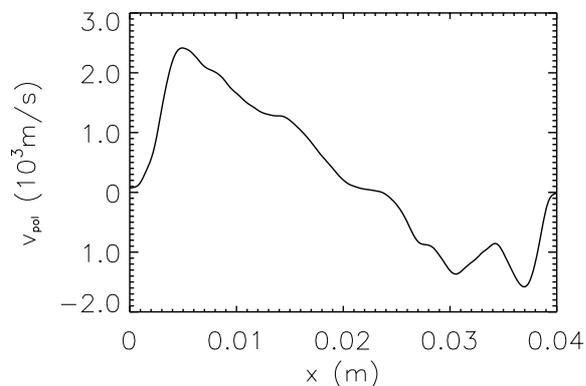


Figure 7. Poloidal fluid velocity.

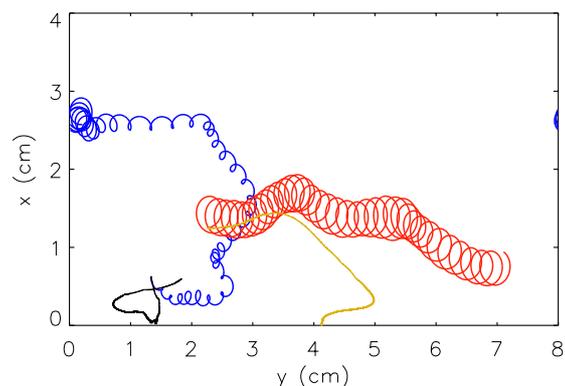


Figure 8. Example of impurity-ion trajectories in the drift approximation (yellow and black) and with inclusion of the finite Larmor radius (red and blue).

Summary

In this contribution, we have shown that the generation of a radial electric field and a poloidal $\mathbf{E} \times \mathbf{B}$ fluid velocity, as recently found in our simple spatially periodical and time-independent potential, persists also in a more realistic form of the fluctuating potential described by the H-W equations. Moreover, we have shown that the drift approximation can give considerably different results than the more exact PIC approach, which includes finite-Larmor radius effects as well.

Acknowledgement: We acknowledge the support of the grant of the Czech Acad. Sci. A1043201 and of the Ministry of Education of CR No: MSM210000018. This work was also partly supported by Austrian Science Fund (FWF) contract P15013 and performed within the Association EURATOM-ÖAW. Its content is the sole responsibility of the authors and does not necessarily represent the view of the EU Commission or its Services.

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