Edge Kink/Ballooning Mode Stability in Tokamaks with a Separatrix

S.Yu.Medvedev¹, Y.Martin², O.Sauter², L.Villard², D.Mossessian³

¹Keldysh Institute, Russian Academy of Sciences, Moscow, Russia
²CRPP, Association Euratom-Confédération Suisse, EPFL, Lausanne, Switzerland
³Massachusetts Institute of Technology, Cambridge, Massachusetts, USA

1 Kink/ballooning modes High-n ideal modes are possible candidates to trigger the edge localized modes (ELM) instability [1]. The presence of a magnetic field separatrix at the plasma boundary leads to the stabilization of ideal localized peeling modes which require the presence of a rational magnetic surface just outside the plasma surface. However the external kink modes coupled to the ballooning modes set stability limits for the pressure gradient and current density pedestals at the plasma boundary.

The differences between limiter and separatrix geometries relevant to the peeling mode stability analysis were listed in [2]: because the safety factor goes to infinity at the separatrix, there are rational surfaces inside the plasma for any mode number. Moreover, there is a non negligible variation of q across the mode width. The question on the localized peeling mode criterion [3, 4] applicability to the ideal divertor plasma still needs to be answered. Plasma outside the separatrix and possible transition of the peeling mode into resistive tearing mode should be also taken into account.

The edge kink/ballooning mode [2] is a more robust pressure-driven instability which can set the stability limit lower than the n = ∞ ballooning modes due to coupling with external modes. It decouples from the peeling mode in a limiter plasma when the first rational surface in vacuum is far from the plasma edge so that the corresponding value of m – nq ~ 1 at the plasma boundary. Taking into account the separatrix naturally decouples peeling and ballooning modes and gives kink/ballooning mode stability limits which do not depend on small variations of the safety factor.

2 The KINX code modifications The recent version of the KINX stability code [5] was upgraded to compute high-n mode stability. It uses a new variant of ballooning factor extraction [6] applicable to equilibria with separatrix. The ballooning factor extraction significantly enhances the eigenvalue convergence in case of high toroidal wave numbers especially for large q variations. Another useful option is setting a plasma displacement to zero inside a prescribed magnetic surface and solving only for displacements in the rest of plasma. It provides a possibility to increase the radial resolution near the plasma edge.

The maximal grid dimensions in radial and poloidal directions for the stability calculations were 384 × 384. It allows us to treat mode numbers up to n ≈ 60 thanks to grid packing near the boundary and the X-point.

3 Edge mode stability with separatrix The KINX code was applied to the computations of the kink/ballooning mode stability of the equilibria related to the Alcator C-Mod and TCV tokamaks.

The first series of the Alcator C-Mod equilibria is described in [7], where the KINX results are compared to the ELITE code [8] for plasma limited by the ψ = 0.99ψsx magnetic surface. Concerning the results with separatrix, it should be mentioned that the original equilibrium with vanishing current density at the boundary is stable against modes n = 1 – 60 and is marginally n = ∞ ballooning stable near the boundary. The profiles cut at ψ = 0.99ψsx were used to recompute the equilibrium and produce current density and pressure gradient pedestals at the separatrix (Fig.1).

In the table below the KINX growth rates normalized to Alfvén frequency are compared for the cases with and without separatrix with the same profiles.

<table>
<thead>
<tr>
<th>n</th>
<th>ψ = 0.99ψsx</th>
<th>with separatrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.006</td>
<td>stable</td>
</tr>
<tr>
<td>30</td>
<td>0.056</td>
<td>0.024</td>
</tr>
<tr>
<td>40</td>
<td>0.093</td>
<td>0.076</td>
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</tbody>
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The values of growth rates are not sensitive to small variations of the safety factor despite the fact that the peeling mode stability criterion was not satisfied anywhere in the plasma. In Fig.2 the structure of the $n = 30$ mode is presented with and without separatrix at the boundary.

For a preliminary assessment of the TCV edge mode stability the following procedure was proposed. The reconstructed equilibrium pressure profile was replaced with a profile from Fig.1 scaled to get the same value of $\beta$ at fixed plasma current. The corresponding bootstrap current (not taking collisionality into account) was added to the reconstructed profile of $<\mathbf{J}\mathbf{B}> / <\mathbf{B}\mathbf{V}\phi>$. For the TCV shots #8856 at $t = 0.3s$ and #11410 at $t = 0.5s$ it gives the profiles presented in Fig.3. The equilibrium corresponding to the shot #8856 is unstable against the modes $n > 10$ even without pedestal at the separatrix. Raising the beta value to $\beta_N \sim 2$ in the series with scaled pressure profile drives the $n = 5$ mode unstable.

The equilibrium corresponding to shot #11410 is stable for profiles without pedestal at the separatrix, as was the case for Alcator C-Mod equilibria. However the range of unstable toroidal wave numbers is shifted to lower $n \gtrsim 10$ for the profiles cut at $\psi = 0.99\psi_{sx}$.

The dependence of the edge mode stability on the pedestal height was studied. The pedestals were produced by cutting the original profiles at $\psi = 0.99\psi_{sx}$ and $\psi = 0.98\psi_{sx}$ and recomputing the equilibria with separatrix at the boundary. In the tables below the growth rates are given for equilibria with different pedestals at the separatrix for the TCV cases (#8856 left and #11410 right).

<table>
<thead>
<tr>
<th>$n$</th>
<th>at 0.98</th>
<th>at 0.99</th>
<th>no pedestal</th>
<th>$n$</th>
<th>at 0.98</th>
<th>at 0.99</th>
<th>no pedestal</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>stable</td>
<td>0.054</td>
<td>marginal</td>
<td>10</td>
<td>stable</td>
<td>marginal</td>
<td>stable</td>
</tr>
<tr>
<td>20</td>
<td>0.057</td>
<td>0.178</td>
<td>0.106</td>
<td>20</td>
<td>stable</td>
<td>0.096</td>
<td>stable</td>
</tr>
<tr>
<td>30</td>
<td>0.132</td>
<td>0.249</td>
<td>0.170</td>
<td>30</td>
<td>0.051</td>
<td>0.141</td>
<td>stable</td>
</tr>
<tr>
<td>40</td>
<td>0.187</td>
<td>0.285</td>
<td>0.192</td>
<td>40</td>
<td>0.088</td>
<td>0.161</td>
<td>stable</td>
</tr>
</tbody>
</table>

For the highest values of current density and pressure gradient at the separatrix corresponding to profiles cut at $\psi = 0.98\psi_{sx}$ the modes are more stable than for the lower pedestal. Possible reasons for that are an edge pressure gradient in the second stability region and a decreased pedestal width.

In the Fig.4 the mode structures for the two TCV cases are presented.

4 Discussion Ideal edge mode stability was computed for equilibria with separatrix at the boundary. In the absence of peeling modes kink/ballooning modes can set $\beta_N$ limits below the $n = \infty$ ballooning and low-$n$ external kink mode limits depending on the toroidal wave number, current and pressure gradient pedestal. Current density pedestal at the separatrix significantly affects the edge mode stability through coupling of external kink and ballooning modes.

Despite a large current density pedestal a shift of unstable kink/ballooning mode range in toroidal wave number to higher values was discovered when the pressure gradient near the edge is in the ballooning second stability region. Besides the second stability access such stabilization may be also connected with decreasing pedestal width.

Acknowledgements The CRPP authors are supported in part by the Swiss National Science Foundation.

References
Unstable $n=\infty$ on 24/128

$\beta=0.010177; \ i_n=0.86797; \ g=1.1667$

Unstable $n=\infty$ on 21/128

$\beta=0.010092; \ i_n=0.88507; \ g=1.1402$

Fig. 1 Profiles of the Alcator C-Mod equilibrium: original (left) and cut at $\psi = 0.99\psi_{xx}$ (right). Both equilibria have separatrix at the boundary. The collisionless bootstrap current and marginal ballooning stable pressure gradient are shown by dashed lines.

Fig. 2 Level lines of normal displacement for the Alcator C-Mod equilibria with (right) and without (left) separatrix. Toroidal wave number $n=30$. 

KINX2000: Normal displacement of $n=30$ mode 256x256

$w^2/w_A^2=0.0029923$; kinetic energy norm; unstable eq.; lowest eigenvalue

$w^2/w_A^2=0.00027594$; kinetic energy norm; unstable eq.; lowest eigenvalue
Fig. 3 Profiles of the TCV equilibria with pedestal: #8856 (left) and #11410 (right).

Fig. 4 Level lines of normal displacement for the TCV #8856 equilibrium with pedestal at 0.99, n = 10 (left) and for the TCV #11410 equilibrium with pedestal at 0.98, n = 30 (right).