29th EPS Conference on Plasma Phys. and Contr. Fusion Montreux, 17-21 June 2002 ECA Vol. **26B**, P-1.113 (2002)

Bifurcation of Temperature and Anomalous Transport in the Edge Region of W7-X

P. Bachmann*, J. Kisslinger, D. Sünder*, H. Wobig

Max-Planck-Institut für Plasmaphysik, Garching bei München, EURATOM
* *Bereich Plasmadiagnostik Berlin*

Introduction

The Wendelstein 7-X configuration is an advanced stellarator with a rotational transform 0.84 in the plasma center and close to one at the plasma edge. There are 5 magnetic islands at the plasma boundary, which are scheduled to serve as divertor geometry. Thermal transport in the divertor region is described by the transport equation, which is inherently three-dimensional. An attempt has been made to reduce the three-dimensional heat conduction equation in the boundary region to a two-dimensional one. For this purpose a reference field $\mathbf{B_0}$ has been defined, which has closed field lines everywhere and a rotational transform of unity. Averaging along field lines of the reference field the thermal conduction equation can be reduced to a two-dimensional one describing the temperature distribution on the poloidal plane. Anomalous transport affects the width of the scrape-off layer and the thermal transport. Furthermore, the nonlinearity of the radiation losses in the edge region and the non-linearity of the boundary conditions can lead to a bifurcation of the temperature distribution. Numerical examples of bifurcated solutions and temperature profiles are given for simplified two-dimensional model equations (see also [1]).

Basic equations

The starting point is the 3d,t heat conduction equation for the temperature $T(\mathbf{r}, \mathbf{t})$

$$3\frac{\partial}{\partial t}nT - \nabla \cdot \chi(\mathbf{r}, T) \cdot \nabla T + \frac{5}{2}\nabla \cdot Tn\mathbf{V} = H(\mathbf{r}, T) - Q(\mathbf{r}, T) \tag{1}$$

with the density n, the velocity $\mathbf{V}$ and the thermal conductivity

$$\chi(\mathbf{r}, T) = \chi_\perp \mathbf{I} + \left(\chi_{||} - \chi_\perp\right) \frac{\mathbf{B} : \mathbf{B}}{B^2} \tag{2}$$

We assume

$$\mathbf{B} = \mathbf{B}_{reg} + \delta\tilde{\mathbf{B}}, \quad \mathbf{B}_{reg} = \nabla\chi \times \nabla\alpha \tag{3}$$

where $\mathbf{B}_{reg}(\chi,\alpha)$ is the regular, non-oscillating field - the Clebsch variables $\chi = const$ and $\alpha = const$ describe the field lines - and $\delta\tilde{\mathbf{B}}$ describes the magnetic fluctuations. Since the field lines , in general, are not closed, and magnetic surfaces do not exit, the functions $\chi = const$ and $\alpha = const$ can not be used as coordinate surfaces. For this purpose we introduce the reference field $\mathbf{B}_0 = \nabla\psi \times \nabla\eta$ with closed field lines ($\iota = 1$) and $\mathbf{B}_{reg} = \mathbf{B}_0 + \delta\mathbf{B}$, where the new functions ψ and η have the properties that the surfaces $\psi = const$ are toroidally closed and all lines $\eta = const$ are closed after one poloidal and one toroidal transit.

Taking into account also the electrostatic fluctuations of the density and velocity, $n = \overline{n} + \delta\tilde{n}$ and $\mathbf{V} = \delta\tilde{\mathbf{V}}$, and averaging the transport equation both over the fluctuations and along the field lines of $\mathbf{B_0}$, we obtain finally

$$< 3\frac{\partial}{\partial t}\overline{n}\overline{T} > - < \nabla\cdot\chi_\perp\cdot\nabla_\perp\overline{T} > -\nabla\cdot < \frac{1}{B_0^2}[\delta\mathbf{B}:\delta\mathbf{B} + \overline{\delta\tilde{\mathbf{B}}:\delta\tilde{\mathbf{B}}}]\chi_{||}\cdot\nabla\overline{T} > \qquad (4)$$
$$+\frac{5}{2}\overline{\delta\tilde{n}\delta\tilde{\mathbf{V}}}\cdot\nabla\overline{T} = \ < H(\mathbf{r},\overline{T}) > - < Q(\mathbf{r},\overline{T}) >$$

where the averaging procedures are defined by

$$< g > = \oint g\,\frac{dl}{B_0}, \quad \overline{g} = \frac{1}{\tau}\int_0^\tau g dt \qquad (5)$$

with l - the coordinate along the reference field, $\tau = 2\pi/\omega$, ω - characteristic frequency. The spatial averaging procedure anihilates all terms with $\mathbf{B_0}\cdot\nabla...$

Cylindrical model

Simplifying the equation (4) by introducing cylindrical coordinates ρ, ϕ, assuming $B_0 = B_z = const$, $\partial/\partial z = 0$ and using dimensionless variables [2] we obtain the equation for $u(\rho,\phi) = T(\rho,\phi)/T_r$ (T_r-reference temperature)

$$u_t = \kappa_{\rho\rho}u_{\rho\rho} + \kappa_{\phi\phi}u_{\phi\phi} + 2\kappa_{\rho\phi}u_{\rho\phi} + \kappa_\rho^{(0)}u_\rho + \kappa_\phi^{(0)}u_\phi + \kappa_\rho^{(\rho)}u_\rho^2 + \kappa_\phi^{(\phi)}u_\phi^2 + 2\kappa_\rho^{(\phi)}u_\rho u_\phi + H - Q \quad (6)$$

with heat conduction coefficients

$$\kappa_{\rho\rho} = 1 + \chi_{||0}u^\beta B_\rho^{2*}, \quad \kappa_{\phi\phi} = \frac{1}{\rho^2}\left(1 + \chi_{||0}u^\beta B_\phi^2\right), \quad \kappa_{\rho\phi} = \frac{1}{\rho}\chi_{||0}u^\beta B_\rho B_\phi \qquad (7)$$

$$\kappa_\rho^{(0)} = \frac{1}{\rho}\left[1 + \chi_{||0}u^\beta\left\{\frac{\partial}{\partial\rho}\left(\rho B_\rho^{2*}\right) + \frac{\partial}{\partial\phi}\left(B_\phi B_\rho\right)\right\}\right] \qquad (8)$$

$$\kappa_\phi^{(0)} = \frac{1}{\rho^2}\chi_{||0}u^\beta\left\{\frac{\partial}{\partial\rho}\left(\rho B_\rho B_\phi\right) + \frac{\partial}{\partial\phi}\left(B_\phi^2\right)\right\} + \frac{\xi_1}{\rho}\,\overline{\delta\tilde{n}\delta\tilde{\mathbf{V}}} \qquad (9)$$

$$\kappa_\rho^\rho = \beta\chi_{||0}u^{\beta-1}B_\rho^{2*}, \quad \kappa_\phi^\rho = \frac{1}{\rho^2}\beta\chi_{||0}u^{\beta-1}B_\phi^2, \quad \kappa_\rho^\phi = \frac{1}{\rho}\beta\chi_{||0}u^{\beta-1}B_\rho B_\phi \tag{10}$$

with the magetic field components

$$B_\phi = -\frac{\partial A_z}{\partial \rho}, \quad B_\rho = \frac{1}{\rho}\frac{\partial A_z}{\partial \phi}, \quad B_\rho^{2*} = B_\rho^2 + \xi_2\,\overline{(\delta\tilde{B}_r)^2} \tag{11}$$

$$A_z = A_0[\int_0^\rho d\rho'\,(1-\iota(\rho')) + \epsilon\rho^5\cos(5\phi)] = A_0\rho\left[0.16(1-\rho/2) + \epsilon\rho^4\cos(5\phi)\right] \tag{12}$$

with the approximation $\iota(\rho) = 0.84 + 0.16\rho$. Heating and radiation loss functions are

$$H(\rho,\phi,t) \Rightarrow H_0\rho^{-\eta_1}, \quad Q(u,\rho,\phi,t) \Rightarrow f_q(\rho_w-\rho)^{\eta_2}u\exp\left[\frac{-(u-u_1)^2}{\Delta_{q1}}\right] \tag{13}$$

where $\beta,\ \xi_1,\ \xi_2,\ \eta_1,\ \eta_2,\ \epsilon$ are constant, ρ_w is the wall position.

Numerical results

We present here numerical results of the boundary value problem

$$u_\rho(\rho_0,\phi,t) = -\Gamma_0, \ u(\rho_R,\phi) = T_w, \ \ u_\phi(\rho,0,t) = u_\phi(\rho,2\pi/5,t) = 0, \tag{14}$$

$$u(\rho,\phi,t=0) = const, \ \rho\in[\rho_0,\rho_w], \ \rho_0 = 0.8, \rho_w = 1.2, \ \phi\in[0,2\pi/5] \tag{15}$$

of eq. (6) for parameters $\epsilon = -0.0015,\ \chi_{||0} = 10,\ \beta = 2.5,\ \xi_1 = \xi_2 = 0,\ \eta_1 = \eta_2 = 1,\ H_0 = 3,\ u_1 = 1,\ \Delta_{q1} = 0.1,\ f_q = 98.44,\ \Gamma_0 = -8.75,\ T_w = 0.$

The magnetic vector potential with 5 islands is displayed in Fig. 1 describing the magnetic boundary structure in W7-X. The existence of temperature bifurcation and multiple solutions of the heat transport equation is demonstrated in Fig. 2. The influence of the island structures of the heat conduction coefficients in the edge region on the temperature profiles is shown in Fig. 3.

Summary

A 2d,t heat conduction equation for the temperature is derived, averaging the 3d,t equation over the fluctuations and along the lines of the reference field. Numerical calculations in the frame of a simplified cylindrical modell show both the influence of the island-structures of the heat conduction coefficients on the temperature profiles and the existence of multiple temperature profiles in the boundary plasma of W7-X.

References

[1] P. Bachmann, D. Sünder, H. Wobig, Contrib. Plasma Phys. 40 (2000) 399-404

[2] P. Bachmann, J. Kisslinger, D. Sünder, H. Wobig, Plasma Phys. Control. Fusion 44 (2002) 83-102

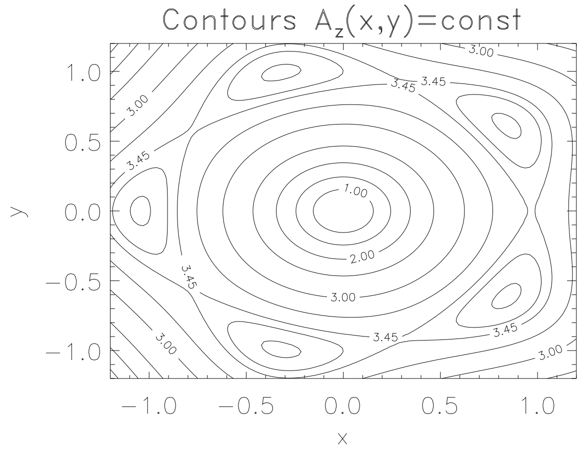

Figure 1: Magnetic vector potential A_z for $A_0 = 35$

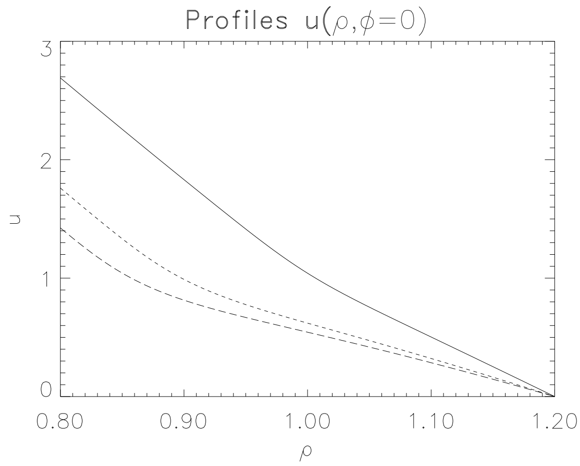

Figure 2: Multiple stationary temperature profiles $u(\rho, \phi = 0)$ for $A_0 = 10^{-5}$; .... - the unstable solution

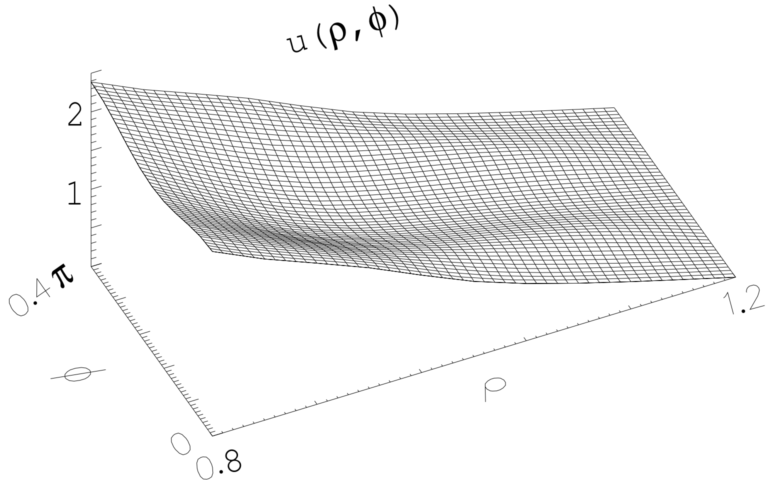

Figure 3: Temperature profile $u(\rho, \phi)$ corresponding to the highest profil in Fig.2 for $A_0 = 35$