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Shape Effects on the Stability Limit of the Ideal Internal Kink Mode

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The internal ideal kink mode is one of the possible triggers of sawtooth oscillations [1] and it is important to know the stability of the internal ideal kink to be able to predict the behavior of sawteeth. In this work we have studied the behavior of the stability limit on the poloidal beta inside q=1 surface ("beta Bussac") in TCV-like equilibria, varying plasma shape and current profile using numerical MHD codes.

Analytic formulae for internal kink mode

There are several theoretical works giving analytical approximations of dependence of the ideal internal kink growth rate on different plasma parameters [3, 4, 5]. The summary of several of them is given in [1]. The theoretical analysis gives formulae for the normalized variation of the potential energy of perturbation δW and the growth rate of instability can then be obtained as $\gamma = \delta W/\tau_A$ where $\tau_A = 3^{1/2}R/\upsilon_A$ is the Alfven time, R is the major radius and υ_A the Alfven speed. The main parameter which determines the marginal stability limit of the ideal internal kink mode is the "beta Bussac" $\beta_{bu} = 8\pi / B_{pl}^2 (\langle p\rangle_1 - p(r_1))$, where B_{pl} is the averaged poloidal magnetic field on the q=1 surface, $\langle p\rangle_1$ is the volume averaged total pressure inside q=1 surface and $p(r_1)$ is the total pressure on q=1 surface. The mode becomes unstable, when β_{bu} is larger than some value which we will call β_{bu}^{crit}. According to the analytical theory [1], the value of β_{bu}^{crit} is close to

$$\beta^{crit} = 0.3(1\text{-}5/3(r_1 k_1^{1/2}/(ab)^{1/2}), \qquad (1)$$

where κ_1 is the elongation on q=1 surface, $(ab)^{1/2}$ is the average plasma minor radius.

Parameter range and methods of calculations

TCV (Tokamak à Configuration Variable) is a tokamak, designed especially to explore various plasma configurations. The basic feature of TCV is the unique flexibility, allowing creation of plasmas in a very wide range of shape parameters: elongation up to 2.8 and triangularity between -0.7 and +0.9. The flexible system of EC heating and current drive allows the creation of various current profiles. Both shape and current profiles in TCV plasmas are beyond the range of parameters, for which the analytic expressions for ideal internal kink are obtained.

The ideal MHD code KINX [6] was used for the numerical analysis of the stability of shaped plasmas with various current profiles. The important feature of this finite element code is that the domain decomposition into several subdomains with nested flux surfaces allows the analysis of a broad range of tokamak plasma configurations, including very elongated and triangular plasmas, plasmas with separatrix, doublets etc. This code is an adequate tool for such an analysis.

For a given TCV-like equilibrium produced by the code CHEASE [7] the value of β_{bu}^{crit} is calculated by convergence studies with the code KINX. Thus we ensure that at β_{bu}^{crit} the growth rate of the ideal internal kink mode converges to zero or to a small value.

The scans on plasma elongation ($1.0 \le \kappa_{edge} \le 2.8$) and triangularity ($-1.0 \le \delta_{edge} \le 1.0$) were performed for the TCV aspect ratio $A \approx 3.6$. In most calculations a quadratic parabolic current profile was used (and in one case a cubic parabolic profile).

Results of calculations and analysis

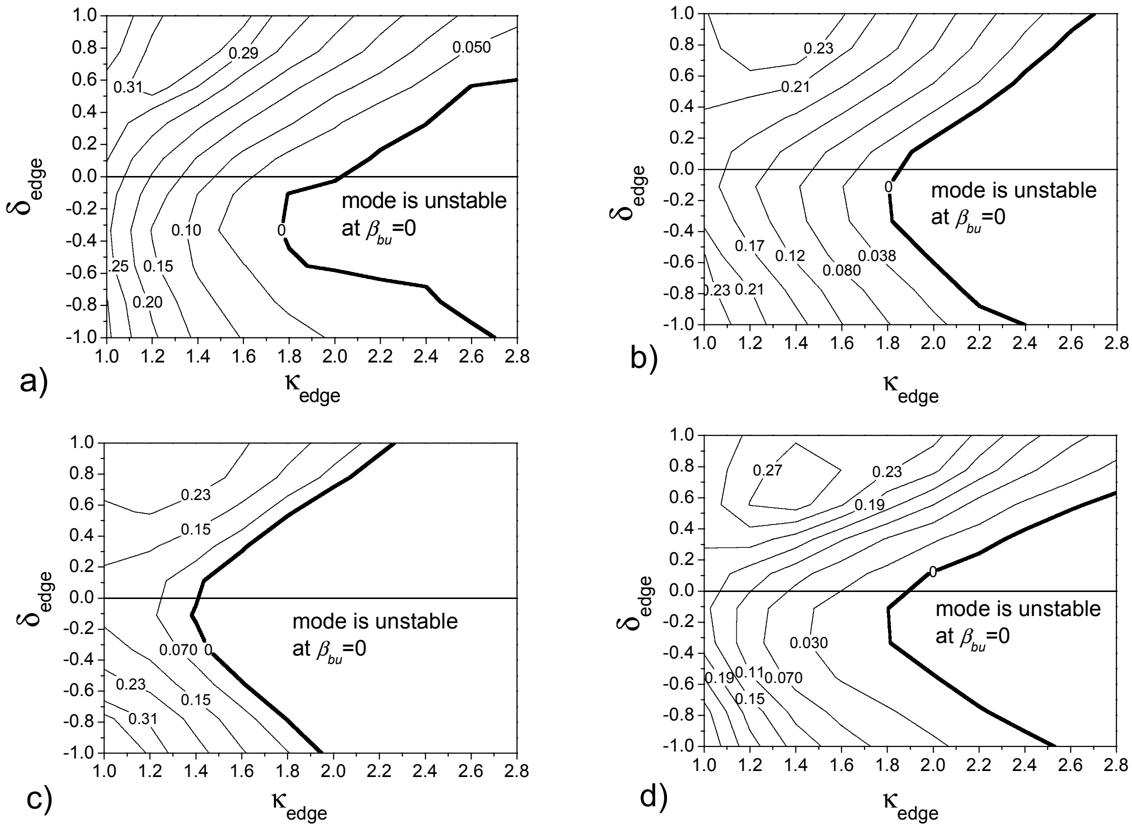

Figure 1. β_{bu}^{crit}: a) $\rho_{q=1}$=0.25 and quadratic current profile b) $\rho_{q=1}$=0.5 and quadratic current profile c) $\rho_{q=1}$=0.7 and quadratic current profile d) $\rho_{q=1}$=0.5 and cubic current profile

In the geometry scans presented here the current profile and radius of q=1 surface $\rho_{q=1}$ were kept fixed. In this case the profile of q inside q=1 surface remains almost unchanged (variation of q on axis is less than 10%) in the whole range of geometry variations.

The general character of the stabilization by an increase in triangularity is the same for different equilibria. There is a pronounced effect of elongation which leads to the destabilization of the mode at zero β_{bu}. From Figure 2 we can see that the stability limit is obtained at almost constant elongation on the q=1 surface in case of quadratic type profiles:

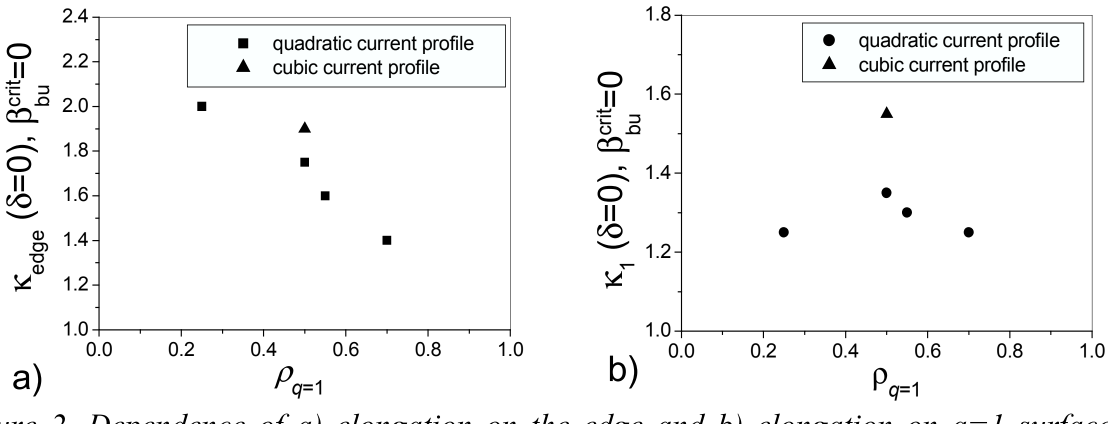

Figure 2. Dependence of a) elongation on the edge and b) elongation on q=1 surface at which the mode is destabilized at zero pressure on radius of q=1 surface

The triangularity has a stabilizing effect, which manifest itself as an increase of β_{bu}^{crit} at the same elongation with an increase of the triangularity, both positive and negative. The destabilization of the mode at low elongation and high positive triangularity is a peculiarity, which, although interesting for further studies, does not represent an interest from the point of view of experiment. The general dependence of β_{bu}^{crit} on triangularity at constant elongation and at constant $r_{q=1}$ corresponds approximately to the behavior of the volume inside q=1 surface. The stabilization by negative triangularity is less efficient than by positive triangularity.

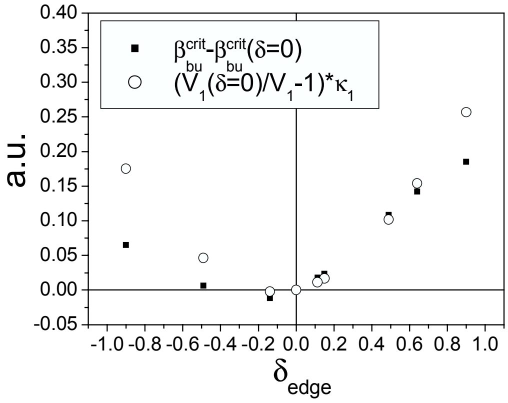

Figure 3. Increase of β_{bu}^{crit} on triangularity compared with behavior of volume inside q=1 surface (quadratic current profile, $\rho_{q=1}$=0.5, κ_{edge}=1.6)

The aspect ratio has a strong impact on the growth rate of the internal ideal kink [1] ($\gamma \propto \varepsilon_1^2$), but according to equation (1) it should not affect substantially the value of β_{bu}^{crit}. Our calculations confirm this:

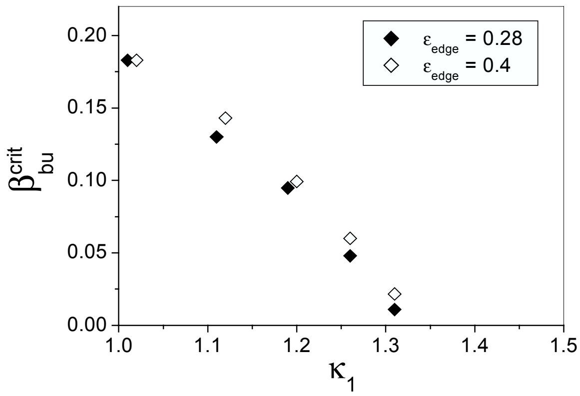

Figure 4. Dependence of β_{bu}^{crit} on elongation on q=1 surface at different aspect ratios. Quadratic current profile, $\rho_{q=1}$=0.5

Scalings of calculated dependences

The stabilization of the ideal internal kink mode by positive triangularity is known and it was shown experimentally on TCV [1]. The stabilization by the negative triangularity which appears in numerical calculations [8] has not been demonstrated in TCV experiments and it is an interesting direction for the new experiments. Therefore we have tried to find the description of this effect on basic of the plasma parameters and to simplify the prediction of the mode stability in TCV plasmas. Let us first consider the effect of elongation at zero triangularity. The equation (1) does not describe the results obtained (see Fig.5), so another scaling for quadratic type profiles is proposed :

$$\beta_{bu}^{crit}(\delta=0) = 0.33\,(1.3 - \kappa_1)\,/\,(r_1\,k_1^{1/2}\,/\,(ab)^{1/2}\,)^{2/3} \qquad (2)$$

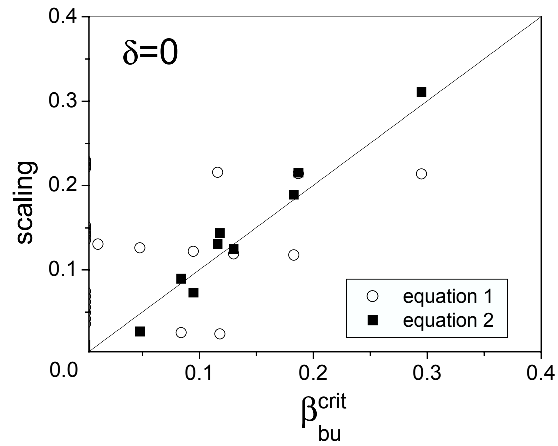

Figure 5. Scaling for β_{bu}^{crit} at zero triangularity for quadratic current profile.

By adding the term describing the stabilization by triangularity (Fig.3) and depending only on the edge triangularity (Figure 6a, compare with Figure 3), we can obtain a general scaling for cases with different elongations and triangularities for quadratic current profile

$$\beta_{bu}^{crit} = 0.33\,(1.3 - \kappa_1) / (r_1\, k_1^{1/2} / (ab)^{1/2})^{2/3} + 0.0111\,(\delta_{edge} + 4)\,(\delta_{edge} + 0.4)^2 \qquad (3)$$

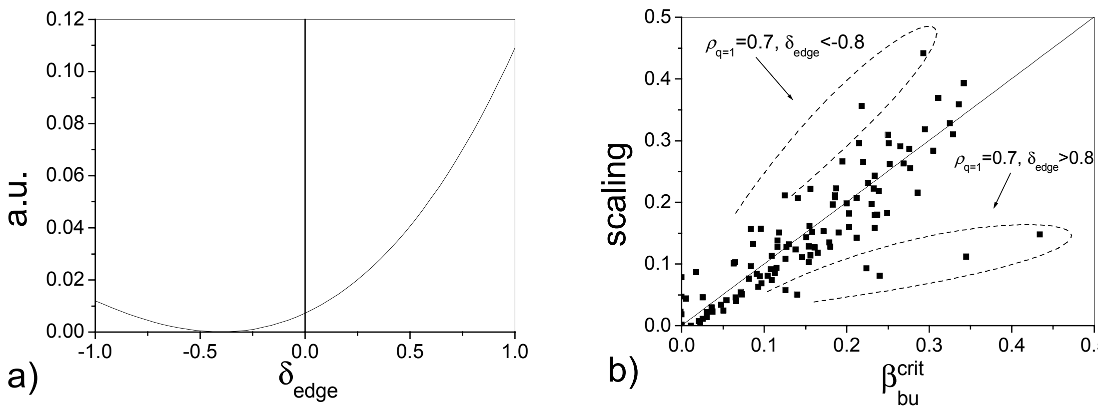

Figure 6. a) Additional term, describing the stabilization by triangularity. b) Scaling (Eq. 3) for parabolic current profile, $1.0 \leq \kappa_{edge} \leq 2.8$, $-1.0 \leq \delta_{edge} \leq 1.0$, A=3.6

The scaling (Eq. 3) describes satisfactorily the effects of elongation and triangularity for the quadratic current profile, with exception of cases with high $\rho_{q=1}$ and high triangularity, which can hardly be obtained experimentally. This scaling will be used for prediction of sawteeth behavior in TCV experiments.

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