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A New Method for Shear Stabilization in Advanced Tokamaks via Mode-Converted Ion-Bernstein Waves

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Generation and control of internal transport barriers (ITBs) is central to the development of advanced tokamak reactors. It is generally believed ICRF waves generate plasma flows through ion absorption; but given the high electron temperatures expected in a reactor, avoidance of electron Landau damping imposes a very low upper limit on usable $k_{||}$ for waves near the fundamental cyclotron frequencies. For example, at T_e=15 keV and 40 MHz (Ω_T at $B = 7.9$ T), $\omega/k_{||} > 2\,v_{th,e}$ requires $k_{||} < 1.7$ m^{-1}. Use of higher-frequency ion-Bernstein waves (IBWs) mitigates this problem. As an IBW approaches a majority ion resonance its wavelength becomes less than a thermal ion gyroradius, its polarization is nearly electrostatic, and its radial group velocity becomes very small, leading to large amplitudes. In consequence, strong ion absorption occurs even at higher cyclotron harmonics (n=5, 6, …). Use of the IBW is also motivated by theory that indicates the IBW drives poloidal flow by a ponderomotive mechanism connected to absorption. In addition, if the absorption layer is on the high-field side (HFS), toroidicity converts HFS perpendicular heating to radially separated counter parallel flows on the low-field side (LFS). Experiments on PLT, Alcator-C, PBX-M, and FTU (waveguide antenna) in which IBWs were launched from the outboard edge have yielded confinement increases consistent with IBW-created sheared-flow transport barriers. Nevertheless, IBW launch faces many difficulties, including those arising from sensitivity to density and its gradient, sheath effects, parametric decay, ponderomotive density expulsion, and coaxial mode excitation.

Generating an IBW by mode conversion from a fast magnetosonic wave (FW) within the plasma offers a favorable alternative to LFS IBW launch. It is possible to achieve nearly complete mode conversion of a fast wave incident from the high-field side on the 2nd harmonic resonance of a small minority (5%) hydrogen component ($\omega = 2\Omega_H$) in a reactor D/T plasma. Conversion is negligible if hydrogen is absent, but the amount needed is little more than the residual level present in tokamaks.

As a concrete example, consider the following model reactor. Assume major and minor radii of $R_0 = 6.5$ m and $a = 2.5$ m, and assume $B \sim 1/R$ with $B = 6.07$ T at R_0. Choosing $f = 200$ MHz places $\omega = 5\Omega_T$ at $R = 5$ m. The conversion layer is then slightly to the HFS of the $2\Omega_H/4\Omega_D/6\Omega_T$ resonance at $R = 6$ m, which, assuming a modest Shafranov shift of 0.5 m, is 1 m to the HFS of the magnetic axis. Lower aspect ratio is desirable as it reduces the separation between conversion and absorption surfaces as a fraction of minor radius and increases Shafranov shift, both of which allow greater separation between the magnetic axis and the mode conversion zone, thereby minimizing 2-D complications. In our scheme, a fast magnetosonic wave (FW) is launched from a HFS antenna situated near the midplane into a D/T plasma with a minority H component. A comb-line antenna could provide the desired k_ϕ spectrum: sharply peaked at a low value. At this frequency the wave propagates in vacuum up to $k_\phi = 4.2\ \mathrm{m}^{-1}$, permitting a large antenna/plasma gap. The wave is unaffected by the $3\Omega_D$ layer in the low-temperature HFS plasma edge; it next encounters the $5\Omega_T$ resonance, but there too FW absorption is negligible. On reaching the mode conversion layer some absorption may occur, primarily on H, but it can be minimized by spectral control and adequate H density. Transmission is negligible. The mode-converted IBW then propagates back toward $5\Omega_T$.

The conversion dynamics are well-modelled by 2 coupled 2nd-order linear differential equations for E_x and E_y ($E_z = 0$) based on a 2nd-order gyroradius expansion. In the limit $k_{||} = 0$ these can be written as the standard tunneling equation, with a tunneling factor given by: $\eta = 8.62\,\eta_H\,F(\eta_H)\,n_{e20}^{1/2}\,\beta L$ where $F = (A+B)^{1/2}(B/A-1)^{5/2}$ and $A = \frac{8}{15}\eta_D + \frac{12}{35}\eta_T + \frac{4}{3}\eta_H$, $B = 2 + \frac{2}{15}\eta_D + \frac{2}{35}\eta_T + \frac{2}{3}\eta_H$, $\eta_j = n_j/n_e$, $n_{e20} = n_e$ in $10^{20}\ \mathrm{m}^{-3}$, $\beta = .0403\dfrac{2\,n_{e20}T_H}{B^2}$, T in keV, B in Tesla, and L is the B scale length (= R). The transmitted FW power fraction is $e^{-2\eta}$. For $\eta_D = \eta_T = (1-\eta_H)/2$, F varies almost linearly over the range of interest from $F(0) = 44.3$ to $F(0.1) = 27.2$. Under reactor conditions a small amount of H has a large effect. Even at $\eta_H = 4\%$, $n_{e20} = 1$, $T = 10$, we have $\eta = 1.4$ and transmission is only 6%. Physically $\eta \sim k_{\perp\infty}\Delta$ where $k_{\perp\infty}$ is the non-resonant FW wave number and Δ is the width of the conversion zone. Since $k_{\perp\infty} \propto n_e^{1/2}$ it is apparent the bulk plasma acts to broaden the conversion zone, i.e., it causes the mode conversion point (wherethe roots of the dispersion relation coalesce) to move well away from the resonance toward the high-field side. The effect is proportional to the plasma beta if H is at the bulk temperature. For finite $k_{||}$, a wave with a not-too-large $k_{||}$

encounters the mode conversion point before the Doppler-broadened H absorption zone. As $k_{||}$ increases, the latter widens, absorption increases and mode conversion diminishes, establishing an upper limit on usable $k_{||}$. For other parameters fixed, the minimum η_H is established by requiring the mode conversion fraction exceed some value for some maximum $k_{||}$. Note: although the Doppler width increases as $T_H^{1/2}$, the conversion zone widens as T_H. Also, only in fusion conditions does it matter that η_H be small as possible—to minimize fuel dilution.

We have written a 1-D full-wave code to calculate the fields, power absorption profile and scattering parameters for this scenario. Boundaries are chosen such that wave modes are independent there. The code, based on an invariant embedding technique, provides an alternative formulation to existing codes for purposes of comparison and further development. B_z and $k_{||}$ both vary as $1/(R_0 + x)$. Since the fast wave is well characterized by a single, real $k_\perp$ at the boundaries, the expression for kinetic flux from weak damping theory is valid for that mode (which is > 90% electromagnetic anyway). Using this, the mode conversion fraction can be computed by subtracting the transmitted and absorbed powers from the incident (reflection is negligible on the HFS). This obviates a cumbersome calculation of the IBW kinetic flux (often $k_\perp \rho_T > 1$ at the HFS boundary). Also, the code computes absorption only for H. This is reasonable because: i) D absorption is smaller by at least a factor of $(n_D/n_H)/400$ for cases considered, and T absorption is always negligible; and ii) We are primarily interested in identifying cases where absorption is low, in which case the error in the mode conversion coefficient will be very low.

The equations for E_x and E_y are stable to integration from the HFS (left) boundary, xL, to xM, a middle point chosen a bit beyond the mode conversion (MC) point. Thus a set of 3 fundamental solutions for the left side (xL to xM) are generated by integrating the original equations using 3 independent sets of initial conditions. (A fourth is unnecessary because one mode is zero at xL). These are used to form a transfer matrix relating values at xL to values at xM. Invariant embedding is used to the right of the MC point where the IBW modes become evanescent, making straightforward ODE integration numerically unstable in either direction. The original equations are first transformed into modal form by a similaritytransformation. The mode that grows exponentially to the right, call it IBW$^+$, and the left-propagating FW, call it FW$^-$, are both zero at the LFS boundary, xR. Each is represented as a linear combination–with x-dependent coefficients–of the other two modes, IBW$^-$ and FW$^+$. Substitution into the original mode equations yields 4 coupled

Riccatti equations for the 4 unknown coefficients that are stable to integration from xR to xM and have zero initial values. Solving yields values at xM, which are used with values from the left-side transfer matrix in a continuity condition at xM to obtain the unknown boundary values. The boundary values at xL then weight the fundamental left-side solutions to form the final left-side solution. Finally, the values at xM are used as initial values for a pair of coupled linear ODEs for IBW$^-$ and FW$^+$ obtained by substituting the IE representations of IBW$^+$ and FW$^-$ into the original mode equations. The equations are stable to integration from xM to xR. The Table summarizes a few representative cases from our model reactor. $k_{||ant}$ in m^{-1} is specified at $R = 4$ m.

case	n_{e20}	% H	T	$k_{\|\|ant}$	% trans	% abs	% MC
1	1.4	5	5	5	7.2	12	80.7
2	1.4	5	20	5	0.02	1.9	98
3	1.4	5	20	7	0.02	20.1	79.9
4	1	5	10	5	4	37	59
5	2	5	5	5	1.5	14	84
6	2	5	5	9	1	68.6	30.4
7	2	2.5	10	5	1	37	62
8	2	5	10	5	0.04	1.7	98
9	2	5	10	7	0.05	14.1	85.8
10	2	5	20	7	0	2	98
11	2	5	20	9	0	21.1	78.8
12	2	2.5	20	7	0.03	61.4	38.5
13	2	2.5	20	5	0.04	14.7	85
14	2	2.5	20	3	0.04	0.6	99
15	2	3.5	20	5	0	1.5	98.5

Two-dimensional IBW ray-tracing solutions incorporating advanced tokamak equilibria and poloidal magnetic fields have been carried out. For a range of initial $k_{||}$, the results for elongated, triangular equilibria with very slight reverse poloidal curvature near the absorption zone midplane illustrate low electron absorption and the mode-converted IBW reaching the 5T resonance where strong, localized ion absorption can lead to substantial shear generation and ITB formation.