

A Family of FE/FV Grid Representations to model Tokamak Edge Plasmas

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Introduction

Finite Element (FE) / Finite Volume (FV) fluid modelling of the scrape-off layer - or the entire poloidal cross section of the reaction torus if necessary [1] - is performed using a group of related mesh generation tools. The basic software is a 2D finite element generator which produces adaptive unstructured grids with special regard to the flux surface [2]. Most of the existing FV codes apply structured computing grids, but some recent developments employ adaptive unstructured grids which are most suitable for both FE and FV methods [3]. A wide range of steady-state and transient problems can be treated this way, like the resolution of e.g. steep fronts, associated with essential radiation, ionisation, and recombination processes. They provide for more enhanced operation features which allow the user and even the codes automatic control to follow more closely the evolving transient behaviour, when one considers the complex geometry of ITER like fusion devices.

Recent enhancements-overview

In order to cover the mesh handling capabilities of common FE/FV codes, some enhancements have been implemented into the modelling program. Most of the enhancements were made to keep the codes solving support functions most favourable to the user and to the applicability of triangular and quadrilateral elements already formulated for fusion process simulation applications. Some of the enhancements were made to prepare the code for automated operation an option supposed to be introduced as a further step in a row.

The first modification employs triangular elements with one side parallel to the magnetic field. After local refinement/coarsening the grid is realigned by subsequent projection of nodes. Elements containing intermediate nodes are converted into simple triangular elements by iteration. As a further enhancement a rectangular grid can be superimposed and integrated into a grid made up from arbitrary quadrilateral elements enabling the treatment of very complex geometrical shapes. Mesh adaptation is performed primarily on the rectangular portions of the grid. Another code modification creates quadrilateral grids avoiding transition elements with one intermediate node by replacing them by compositions of simple quadrilaterals as transition structures. Two of these new features will be described in more detail in the following.

Quadrilateral mesh without intermediate nodes

The basic software tool produces adaptive and aligned unstructured quadrilateral grids. Fully aligned quadrilateral elements - with each two of their edges following equipotential lines and lines of slopes - are best suited for simulation of fusion plasmas because this procedure respects the extreme anisotropy of plasma transport coefficients along and perpendicular to the magnetic field as well as the special topology of the tokamak magnetic flux surfaces.

For modelling the transition zones between regions of different refinement level special quadrilaterals are used with intermediate nodes up to one at any element edge as required. Provisions for an accurate and stable numerical treatment of these transition elements is one of the major issues of adaptive grid schemes [4]. Even if this is done in a satisfactory way, the adaptive algorithm becomes dependent on the specific numerical method used. Therefore it would be very desirable to produce meshes avoiding intermediate nodes. For this reason we have modified the code in order to replace transition elements by structures of simple quadrilateral elements (see Fig. 3). Subdividing a quadrilateral element into a simpler one is sensitive to the number of edges with mid-side nodes, as the following cases could be observed, for subsequent refinement the following problems had to be addressed properly:

- One edge includes an intermediate node: This case is treated later on in detail
- Two not adjacent edges include intermediate nodes: Division into 2 simple quadrilaterals is possible
- Two adjacent edges include intermediate nodes: Division into 3 simple quadrilaterals is possible
- Three edges include intermediate nodes: Division into 4 simple quadrilaterals and generation of one new intermediate node at the free edge reduces this situation to the first one

Therefore only the case with one intermediate node remains to be solved adequately.

Unfortunately there is no way to divide such an element into quadrilaterals without generating new intermediate nodes. In “the worst case” subdividing must proceed by subdividing all adjacent elements until the boundary of the area meshed is reached. In case there are a number of neighbouring elements of such kind, especially at longer boundaries between regions of different refinement levels, then elements can be transformed two by two into 6 simple quadrilaterals.

But with superseding one element the subdivision process can be changed:

A 3-element-structure as a replacement enables to connect two one-intermediate-node-elements or allows at least to divert the subdivision propagation to the nearest boundary.

Alignment for the major part - but not for all - of the new elements can be achieved by proper choice of the internal nodes. Some elements turn out unaligned or their angles are far from rectangular. Further adaptation of the mesh in these zones is therefore most likely to provoke serious problems. Therefore we have successfully employed a strategy not to refine *these* elements, but provided an option to return to the parent mesh that still includes the transition elements. The elements of the original grid at the same location replace the new elements flagged for subdividing. Iterative alignment/adaptation/realignment is done to the original mesh and then the algorithms mentioned produce a simple quadrilateral grid.

Adapted and aligned triangular mesh

Triangular grid elements are aligned with the flux surfaces as soon as one of the three element edges lies on the same flux level surface [5]. A well-aligned and adapted triangular mesh can be derived from the original quadrilateral mesh (with transition elements) using the same strategy mentioned in the above (Fig. 4). Subdivision of transition elements into triangles this way becomes a reasonably simple task.

This way of discretization of the flux surface is intended to serve in particular cases when the FE/FV method is applied for “orthogonal behaviour” of any dynamic process to be treated. The more general advantage is that elements of basic nature can be used exclusively without derived properties like intermediate nodes, integration points or distribution algorithms required in such an environment. It can be expected that solver application quality and the accuracy of the results can take advantage of these improved discretization options.

Restrictions

The subdividing process is done half-automatically at this time. Because of the extensive workload a fully automatic execution will be set up in a project under preparation.

References

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Figures

Figure 1 **Area selected for Demonstration**

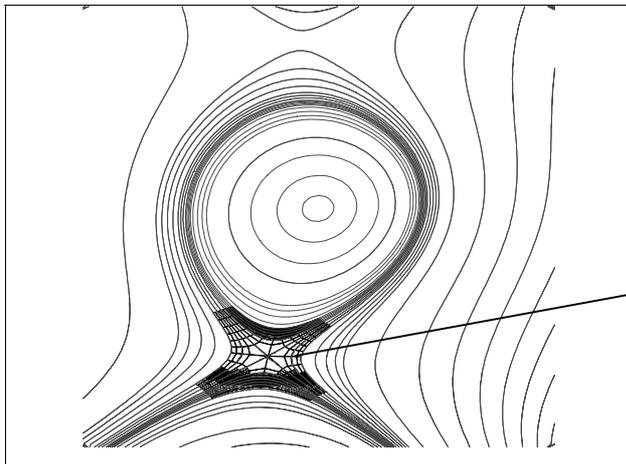


Figure 2 **Refinement close-up**

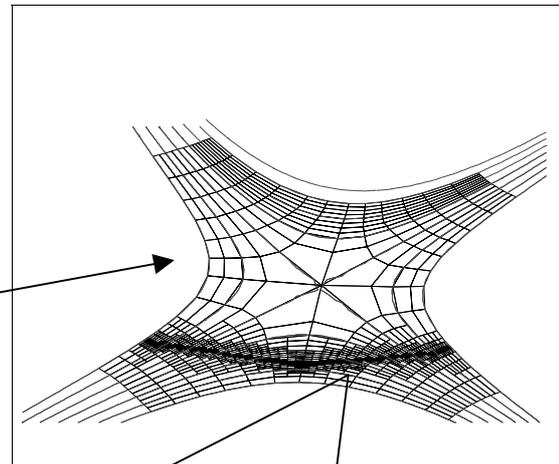


Figure 3 **Mesh entirely quadrilateral**

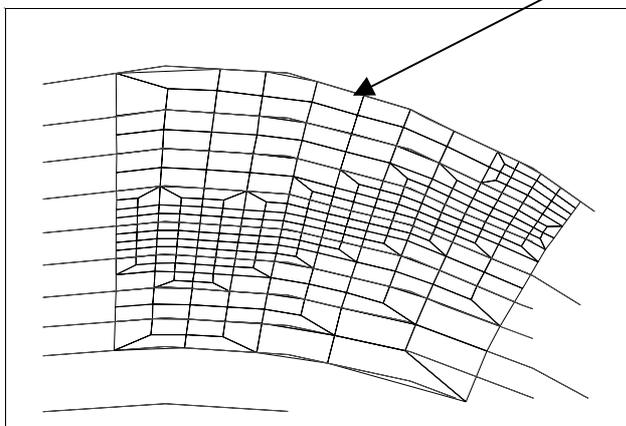


Figure 4 **Mesh entirely triangular**

