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JET Plasma Equilibrium Reconstructions Using Magnetic and MSE Measurements and Including the Effects of the Iron Core

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Introduction – In the JET tokamak, the presence of the iron core distorts the configuration of the magnetic fields that holds the plasma in equilibrium, changes the shape of the plasma boundary, and substantially affects the plasma configuration. Therefore, a correct iron core model can improve the accuracy of an equilibrium code. We report here a description of the finite element CREATE-I reconstruction code, in which the iron transformer is implemented in a different way with respect to the code routinely used at JET (EFITJ [1]), both in terms of geometry and treatment of the equations. Results of comparative calculations using magnetic and MSE experimental data are presented.

Features of the reconstruction tool – In axisymmetric toroidal geometry, using a (r, φ, z) cylindrical co-ordinate system, the MHD equilibrium equation can be written as:

$$-\frac{\partial}{\partial r}\left(\frac{1}{\mu r}\frac{\partial \psi}{\partial r}\right)-\frac{\partial}{\partial z}\left(\frac{1}{\mu r}\frac{\partial \psi}{\partial z}\right)=J_{\varphi}=J_{plasma}+J_{cond} \qquad (1)$$

$$J_{plasma}=rp'(\psi)+ff'(\psi)/\mu_0 r, \qquad J_{cond}=\sum_{k=1}^{N_c} I_k J_{k\varphi}(r,z) \qquad (2)$$

where ψ is the poloidal flux per radian, p is the plasma pressure, f is the poloidal current density function[2-3], N_c is the number of external circuits, $J_{k\varphi}$ the toroidal component of the current density distribution associated with the k^{th} circuit current I_k, and μ is the magnetic permeability (with $\mu=\mu_0$ in nonmagnetic media). Often, $p'(\psi)$ and $ff'(\psi)$ are modelled as polynomials in ψ, with linear coefficients α_n and β_n. Many choices of the basis functions are possible. The optimal number of parameters α_n and β_n depends critically on the amount of the experimental data available, but a large number of coefficients could introduce aliasing problems by using the polynomials set. In order to avoid this problem we can choose sinusoidal functions. Adopting the weighted residual approach and assuming the ferromagnetic constitutive relation $\underline{B}=\mu(B^2)\underline{H}$, Eq. (1) can be solved using Picard iterations:

$$\int_{\Omega} \frac{\nabla\psi_{m+1}\cdot\nabla w}{\mu_0 r} d\Omega = \int_{\Omega} wJ_{\varphi} d\Omega + \int_{\Omega i}\left[\frac{1}{\mu_0} - \frac{1}{\mu(B_m^2)}\right]\frac{\nabla\psi_m \cdot \nabla w}{r} d\Omega_i \ \forall w \qquad (3)$$

where Ω is the poloidal cross-section of the whole region of interest, $\Omega_i \subset \Omega$ is the iron region, m denotes the iteration cycle, and w is any weighting functions.

The Grad-Shafranov equation (1) is then used as a constraint to determine the unknown coefficients α_n and β_n by minimising the functional $F(\underline{\alpha},\underline{\beta})=\frac{1}{2}\Delta\underline{s}^T\{W_s\}\Delta\underline{s}$, where $\Delta\underline{s}$ is the difference between measured signals and simulated measurements, and $\{W_s\}$ is an appropriate weighting matrix taking into account the covariance of the measurements. The circuit currents could be considered as exactly known, making use of their measured values.

However, since in a real experiment there are always errors associated with the measurements, they can also be treated as unknowns to be determined by the fit. For the solution of Eq. (3) we use a finite element computer code working in a MATLAB™ environment by evaluating ψ at mesh nodes [4]. The amount of computational time required to perform a reconstruction using the CREATE-I code depends on the number of unknowns and the relative error on ψ (or the magnetic energy) desired. A typical calculation on a PC (Pentium III 650 MHz) requires few minutes using a mesh with 16409 nodes and 32576 first order elements, with a 10^{-4} relative error on ψ.

Iron Core Model Geometry – The JET iron core is an eight-limbed magnetic circuit described in [5-6]. Its equivalent 2D axisymmetric model used in EFITJ reconstructions is described in [5]. Fig. 1 shows the equivalent model used in the present study.

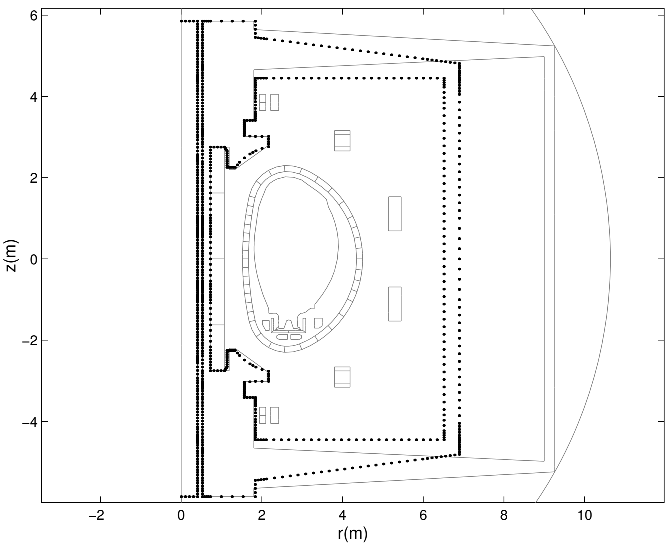

Figure 1. Equivalent 2D axisymmetric models of JET magnetic circuits: model described in [5] (solid line) vs. geometry used in the present study (dotted line)

The new model has the real shape of the polar shoes (previously modelled as straight lines) in order to reproduce the concavity of the cross section. The geometry of the plasma facing side of the limbs (previously shifted away and tilted) is straight and located as in the real 3D geometry in order to ensure a correct boundary condition (field lines perpendicular to air/iron interface) when the iron is not saturated.

Equilibrium Identification of a JET L-Mode Discharge – Firstly, we analysed standard L-mode experiments. In particular we selected shot #53154 at t=44.06s, both using and neglecting the MSE (motional Stark effect) data. The MSE measurements are included in the code as vertical field measurements inside the plasma. The magnetic diagnostics used in the calculations consist of 42 magnetic pick-up coils, 14 saddle loops and 15 virtual flux-loops (obtained by combining one reference flux loop and saddle loops signals). The externally applied poloidal field (PF) is provided by ten external circuits, their currents are fitted. A measure of the deviation between theoretical and measured data is given by the quantity χ:

$$\chi = \sqrt{\sum_{i=1}^{N_m} (M_i - C_i)^2} \Bigg/ \sqrt{\sum_{i=1}^{N_m} (M_i)^2} \tag{4}$$

where M_i, C_i and N_m denote the measured value, the computed value and the total number of all the measured data. Table I shows the comparison (χ *for each set of measurements*) of the code reconstruction results using the two different iron core models (either exploiting or neglecting the MSE data) with the experimental results for the magnetic probes, saddle loops, flux-loops, and, the PF circuits. It also reports the reconstructed values of the axial safety factor q_0, the radial co-ordinate of the magnetic axis and the poloidal beta for the different study cases. In Fig.2 we show the fitting of the magnetic field coils measured by the 42 pick-up coils. In Fig. 3 we can see the comparison of reconstruction results using different iron models with the experimental B_z measurements given by MSE data.

Table I. Reconstruction of the equilibrium configuration for JET pulse #53154 at 44.06 s.

Study cases	χ_T **magnetic probes**	χ_T **saddle loops**	χ_T **flux loops**	χ_T **circuit currents**	R_{axis} *(m)*	β_{pol}	q_0
New iron model (without MSE data)	1.92%	1.84%	0.46%	4.26%	3.02	0.15	1.39
Old iron model (without MSE data)	2.07%	2.85%	0.58%	7.59%	3.02	0.15	1.42
New iron model (with MSE data)	4.54%	4.61%	0.18%	3.23%	3.03	0.22	0.82
Old iron model (with MSE data)	5.04%	5.10%	0.30%	4.73%	3.03	0.22	0.97

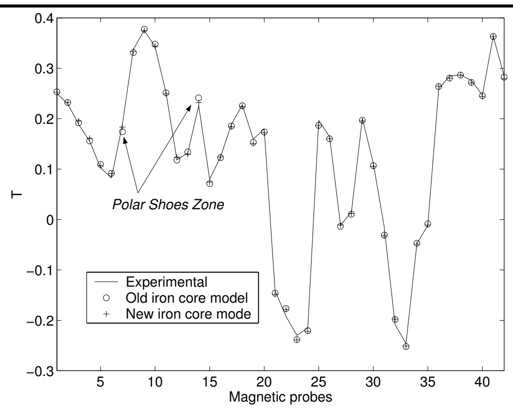

Figure 2. JET pulse #53154 at 44.06 s: measured vs. reconstructed values of magnetic field probes

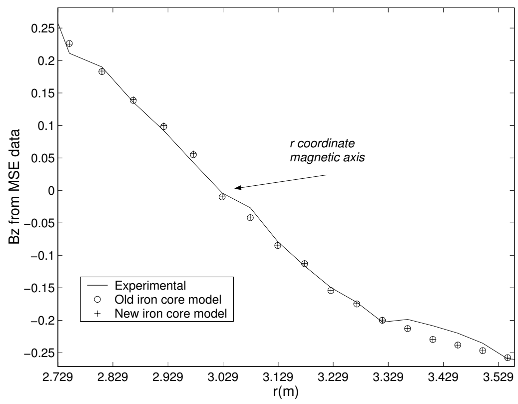

Figure 3. JET pulse #53154 at 44.06 s: measured vs. reconstructed values of MSE signals

Table I and Figs 2-3 show that the fitting of the measurements is better using the new iron core model than the old one. The probes located close to the polar shoes using the old iron geometry model give the higher χ. When using the MSE data, χ for the magnetic signals increases (because more diagnostics are used), and the difference between the results obtained with the two different core models decreases (probably because the use of additional internal profile information in the equilibrium analysis reduces the uncertainties associated with the parameterisation and the representation of the current profile used in the fitting).

A Quasi Double Null Equilibrium Experiment – Here we consider a quasi double null configuration: JET pulse #53714 at t=63.8s. In this shot the X-point, as seen by a camera, is located in the divertor region. We used the two different iron core models in two different cases: 1) all circuit currents fixed; 2) two of the PF circuit currents (providing control of radial and vertical plasma position) treated as unknowns, not fitted. Table II and Fig. 4 show that the reconstructed separatrix shape in this case can have a large uncertainty. The iron core model does affect the X-point location. The results are very sensitive to the choice of the $\{W_s\}$ matrix. Therefore, a more careful analysis is needed.

Conclusions – The CREATE-I reconstructions are satisfactory for standard JET plasmas in L-mode experiments. The new iron model appears to improve the fit. MSE measurements can successfully be included in the plasma current density profile identification tool. For more complex plasma configurations, including ITB (internal transport barrier) reversed shear discharges, further studies are needed.

Table II. Reconstruction results for JET pulse #53714 at t=63.8s

Study cases	All PF circuit currents assigned	All PF circuit currents assigned except two
New iron model	X-point up	X-point down
Old iron model	X-point up	X-point up

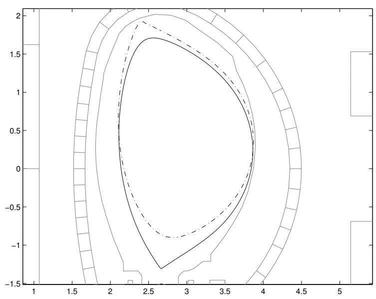

Figure 4. Separatrix reconstructions (JET pulse #53714 at t=63.8s)

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