Nonlinear kinetic evolution of high frequency electron tearing modes

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Abstract

We present the results of Vlasov-Maxwell numerical simulations in the 2D-3V phase space of the nonlinear evolution of a tearing-like mode in the high frequency regime corresponding to the frequency range of whistler waves. Starting from a kinetic Vlasov equilibrium of a plasma embedded in a sheared magnetic field, we show that magnetic islands are generated and discuss the nonlinear phase of the reconnection mode and the resulting deformation of the electron distribution function.

Introduction: Magnetic Reconnection

Magnetic reconnection is a fundamental process that changes the magnetic topology of a high temperature plasma and transforms magnetic energy into plasma kinetic energy, into thermal and ordered kinetic electron energy and into accelerated particle energy. Magnetic reconnection has been recently considered also in the context of the laser-plasma interaction [1-2] where intense magnetic fields are generated by the relativistic interaction of an ultrastrong, ultra-fast laser pulse propagating in an underdense plasma [3]. As is characteristic of high temperature (high energy) plasma regimes, collisions play a minor role in the process of reconnection of the magnetic field generated by the laser plasma interaction: reconnection is then driven by effects that preserve the Hamiltonian nature of the plasma dynamics, such as electron inertia [4] and phase space resonances [5].

In the context of laser plasma interaction magnetic reconnection develops on the electron magnetohydrodynamics (EMHD) frequency range. In this range the characteristic frequency is the whistler mode frequency and ions can be considered as an immobile neutralizing background. Charge separation effects are neglected since the reconnection process occurs on time scales that are generally longer than the Langmuir period. The EMHD model is based on a fluid description of the electron dynamics and kinetic (phase space) effects are not considered. This model has often been used in the study of the evolution on time scales below the electron plasma and above the ion cyclotron frequency (and length scales shorter than the ion skin depth) of the magnetic field and of the magnetic vortices [6] generated in the interaction of an ultraintense laser pulses with a plasma. However, the EMHD model cannot account fully for the kinetic effects that characterize this interaction in particular when microscopic scales are formed. As is well known, the formation of such scales is a general phenomenon of Hamiltonian (dissipationless) nonlinear systems, such as a plasma in the EMHD description. In order to investigate the role of kinetic effects on the evolution of high frequency magnetic reconnection, we have started a numerical project based on Vlasov-Maxwell simulations choosing parameters that are relevant to laser generated plasmas.

In the simulations, as initial conditions, we adopt an electron distribution function (hereafter \text{e.d.f}) and a one dimensional sheared magnetic field that are constructed so as to mimic those obtained from our previous investigations of the development of the Weibel instability. This latter instability arises because of the anisotropy caused by the electrons accelerated by the laser pulse in the direction of the pulse propagation and is usually considered as the most important mechanism of magnetic field generation during the relativistic interaction of an ultra intense laser pulse with an underdense plasma (see [1] and references therein). The resulting
e.d.f is characterized by a spatially inhomogeneous mean velocity that corresponds to the electric current that sustains the sheared magnetic field and by a residual electron momentum anisotropy which has been shown in [2] to enhance the development of the reconnection process driven by the current inhomogeneity. In the numerical simulations presented here we focus our attention on the current inhomogeneity and adopt an initial kinetic equilibrium which is a stationary solution [7] of the Vlasov-Maxwell equation analogous to the standard Harris pinch equilibrium, modified in order to accommodate for spatially periodic conditions.

The Vlasov code

The numerical code [8] integrates the Vlasov equation in the \((x, y, v_x, v_y, v_z)\) phase space for the electrons self-consistently coupled to the Maxwell equations. Ions are considered as a fixed neutralizing background. The dimensionless equations read:

\[
\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f - (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \frac{\partial f}{\partial \mathbf{v}} = 0, \\
\frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{B} - \mathbf{J}, \quad \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E},
\]

where all quantities are normalized with a characteristic density \(n_0\), the electron mass \(m_e\), the speed of light \(c\), the electron plasma frequency \(\omega_p\) and a characteristic electric and magnetic field \(\mathbf{E} = \mathbf{B} = mc_\omega_p/e\). Periodic boundary conditions are used both in the \(x\) and \(y\) direction.

In order to give at the initial time a kinetic equilibrium (i.e., a stationary solution of the Vlasov-Maxwell equations) with a spatially periodic sheared magnetic field we adopt the solution found in [7] which reads as

\[
f(x, v) = \frac{e^{-v^2/v^2_{th}}}{\pi^{3/2} v^3_{th}} \left[ 1 + \epsilon \left( \frac{(v_z - A_z(x))^2}{v^2_{th}} - \frac{1}{2} \right) \right]; \quad A_z(x) = A_0 \cos(\sqrt{\epsilon}x) \quad (3)
\]

\[
n(x) = 1 + \epsilon A_z(x)^2/v^2_{th}; \quad B_{0y} = A_0 \sqrt{\epsilon} \sin(\sqrt{\epsilon}x); \quad J_z(x) = \epsilon A_z(x)/v_{th}; \quad B_{0z} = B_0 \quad (4)
\]

where \(f\) is the e.d.f., \(A_z\) is the potential vector perpendicular to the simulation plane, \(v_{th}\) the thermal velocity, \(B_{0z}\) the uniform magnetic field perpendicular to the simulation plane and \(\epsilon\) a free parameter. The form of the e.d.f. in the \((x, v_z)\) phase space is shown in Fig. 1.

Figure 1: The e.d.f. at \(t = 0\) in the \((x, v_z)\) phase space.

Simulation results

Here we present the results of our numerical simulations in terms of the vector potential \(A_z\) perpendicular to the the \(x-y\) plane, which corresponds to the flux function of the magnetic
field in the plane, of the magnetic field $B_z$, which in this frequency range plays a role related to
the stream function for the electron motion in the $x$-$y$ plane and is proportional to the stream
function in the limit of uniform plasma density, and of the e.d.f.

Figure 2: The vector potential $A_z$ (first frame) and the $z$-component of the magnetic field $B_z$
(second frame) in the $(x,y)$ plane.

We adopt a simulation box with $L_x = 2\pi$, $L_y = 4\pi$, and use the following parameters:
$\epsilon = 1$, $A_0 = 0.05$, $v_{th} = 0.1$, $B_{0z} = 0.1$. With the adopted normalization the electron skin depth
$d_e = c/\omega_p = 1$. This implies that the equilibrium magnetic field is strongly inhomogeneous as
it varies on the $d_e$ scale length. This choice is dictated by the necessity of finding a compromise
between the resolution in phase space and the ability to resolve the region where reconnection
takes place which scales as $d_e^2$ for small $d_e$. As a consequence, the case presented here cannot be
treated analytically within the standard asymptotic theory used e.g., for the linear (EMHD)
tearing mode in Ref. [9]. On the other hand, it is possible to solve the linearized EMHD
equations numerically. For the considered magnetic configuration only the longest wavelength
perturbation is unstable with $\gamma_{lin} = 0.19$ (normalized on the whistler time scale).

Our numerical simulations show that for the chosen parameters the reconnection instability
starts with a growth rate (normalized on the whistler time scale) that is practically equal to
the linear EMHD growth rate, and that this process leads to the formation of a growing
magnetic island, as shown by the plot of $A_z(x,y)$ at $t = 550$ in Fig 2 (first frame). The
spatial dependence of the magnetic field along $z$ shown in Fig. 2 (second frame) exhibits the
convection cell structure that is characteristic of reconnection modes. The e.d.f. is strongly
modified at around the $X$-point and develops two well separated maxima in velocity space, as
shown at $t = 600$ in Fig. 3 (see the figure caption for explanations). The distortion of the
e.d.f. is less and less pronounced moving from the $X$-point towards the $O$-point where the e.d.f.
is practically unchanged with respect to the initial distribution. This distortion of the e.d.f.
indicates that secondary kinetic instability, presently under investigation, can occur inside the
reconnection region.

Conclusions

The preliminary results presented here indicate that during the reconnection process the
e.d.f. evolves towards a multi-peaked form (in the perpendicular direction) that can excite
secondary instabilities which in turn can affect the evolution of the reconnection process. In
the case where the initial magnetic field along $z$ vanishes, $B_{0z} = 0$, an analogous simulation
(not shown here) indicates that the nonlinear evolution of the convection cells is different and
leads to the formation of sub-cells corresponding to shorter wavelength along $y$ while the e.d.f.
appears to be less distorted.
Figure 3: The $v_z$ averaged e.d.f. at $(x,y) = (\pi, 2\pi)$, $(\pi, \pi)$ and $(\pi, 0.8)$, first, second and third rows, respectively. In the first column we plot the e.d.f vs. $v_z$ (at $v_y \approx 0$) at $t = 0$ (dash-dotted line) and at $t = 600$ (continuous line). In the second and third columns we plot the e.d.f in the $(v_y, v_z)$ plane at $t = 0$ and $t = 600$, respectively.

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