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Plasma Mach-probe with unmagnetized ions

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Abstract

The spatial distribution of ion flux collected by a spherical object of radius much larger than the Debye length, in a collisionless flowing plasma, is calculated using a particle-in-cell code. The results provide the first rigorous theoretical calibration of a “Mach probe” in a plasma with negligible magnetic field.

1 Introduction

A sphere in a collisionless plasma is arguably the simplest possible problem involving a bounded object in a plasma. The classic assumption, adopted here, is that the surface simply removes ions (and electrons) incident on it, by neutralization.

The stationary plasma problem was not definitively solved for non-zero ion temperature until the 1960s [1, 2]. Adding a background plasma flow, or equivalently a motion of the sphere through a stationary plasma, breaks the symmetry of the problem and undermines the previous analysis approach, because angular momentum is no longer conserved. This asymmetric problem gained early attention mostly for applications to spacecraft in the ionosphere (see e.g. [1]), which generally move at substantially supersonic speeds, but no studies have provided the quantitative flux to the sphere.

For fusion and industrial processing plasmas, the various forms of Mach probe, to measure the plasma flow by measuring separately the upstream and downstream ion collection currents, are a very important application. The calibration of Mach probes in *magnetized* plasmas (that is, when the ion Larmor radius is smaller than the probe dimension) has been established by theory [3, 4] and verified in experiments [5]. But, remarkably, the simpler-seeming problem of unmagnetized Mach probes, which is the subject of the present study, has not previously been solved [6].

This paper reports an essentially fully consistent solution of the flowing collisionless plasma and sphere problem, with the only approximations being that the electrons are governed by a Boltzmann factor and that the Debye length, λ_D is infinitesimally small. The calculations are performed using a particle-in-cell computer code written specifically for this problem in spherical coordinates, dubbed SCEPTIC (Specialized-Coordinate Electrostatic Particle and Thermals In Cell). The parallel-processing code has been run on a 36-node Beowulf cluster. It advances typically 7.2 million particles to steady state in the self-consistent potential calculated on a spherical mesh with special boundary conditions designed to accommodate the sheath singularity. A fuller description of this work is submitted for publication[7].

2 Flowing Plasma Results

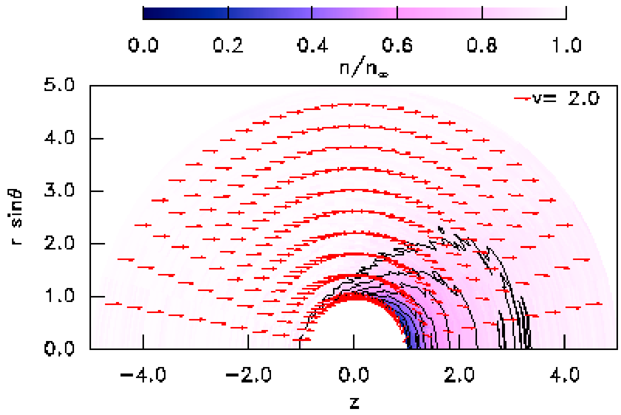

Figure 1: Contour plot of the density near a probe with $T_i = T_e$ and $v_f = 2$. Mean (fluid) velocity at a subset of the cells is shown by vector arrows.

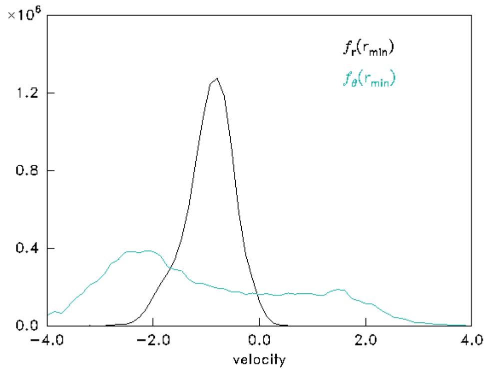

Figure 2: Ion distribution functions in radial and azimuthal velocity, for the case of Fig 1, in the first downstream angular cell, at the sheath-edge.

In Fig 1 are shown examples of density and velocity for a flow velocity (normalized to $\sqrt{ZT_e/m_i}$) $v_f = 2$. Figure 2 shows the distribution functions in the radial and the azimuthal components of velocity, showing the non-Maxwellian nature of the ion distributions, and how important a particle kinetic treatment therefore is.

Fig 3 shows the normalized ion flux as a function of $\cos\theta$ for a range of flow velocities (marked on the curves) and T_i (measured in units of T_e).

It transpires that a reasonable compact universal analytic fit to all the curves is obtained with the 3-parameter form

$$\Gamma(v_f, \cos\theta) = \Gamma_0 \exp\{v_f[(1-\cos\theta)K_u - (1+\cos\theta)K_d]/2\}, \quad (1)$$

with $\Gamma_0 = 0.62$, $K_u = 0.64$, and $K_d = 0.70$. It agrees with the numerical results within their uncertainty for $v_f \lesssim 1.2$, when $T_i = T_e$.

To summarize the results, the ratio of upstream to downstream ion current on the symmetry axis is shown in Fig 4. This is the quantity determining the calibration factor of a Mach probe. The results are close to the (solid) line obtained from eq (1), namely

$$\frac{\Gamma_{\rm up}}{\Gamma_{\rm down}} = \exp(K v_f), \quad (2)$$

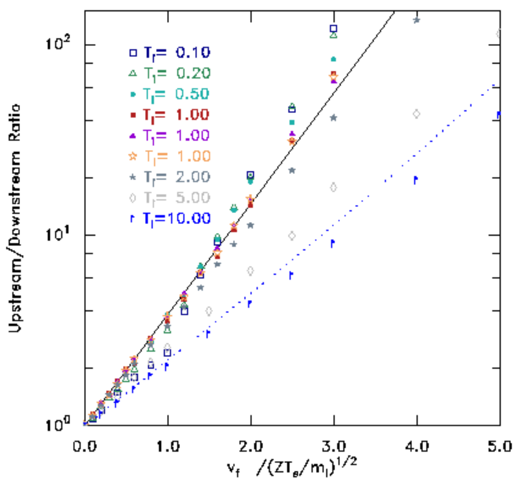

Figure 4: Ratio of current density upstream to downstream for the same data as Fig 3.

with $K = K_u + K_d = 1.34$, for $T_i \lesssim 3T_e$. When $T_i = 10T_e$ or higher the analytic free-flight expression (dotted line) is quite a good fit, and the flow velocity for a specific ratio becomes proportional to $\sqrt{T_i}$.

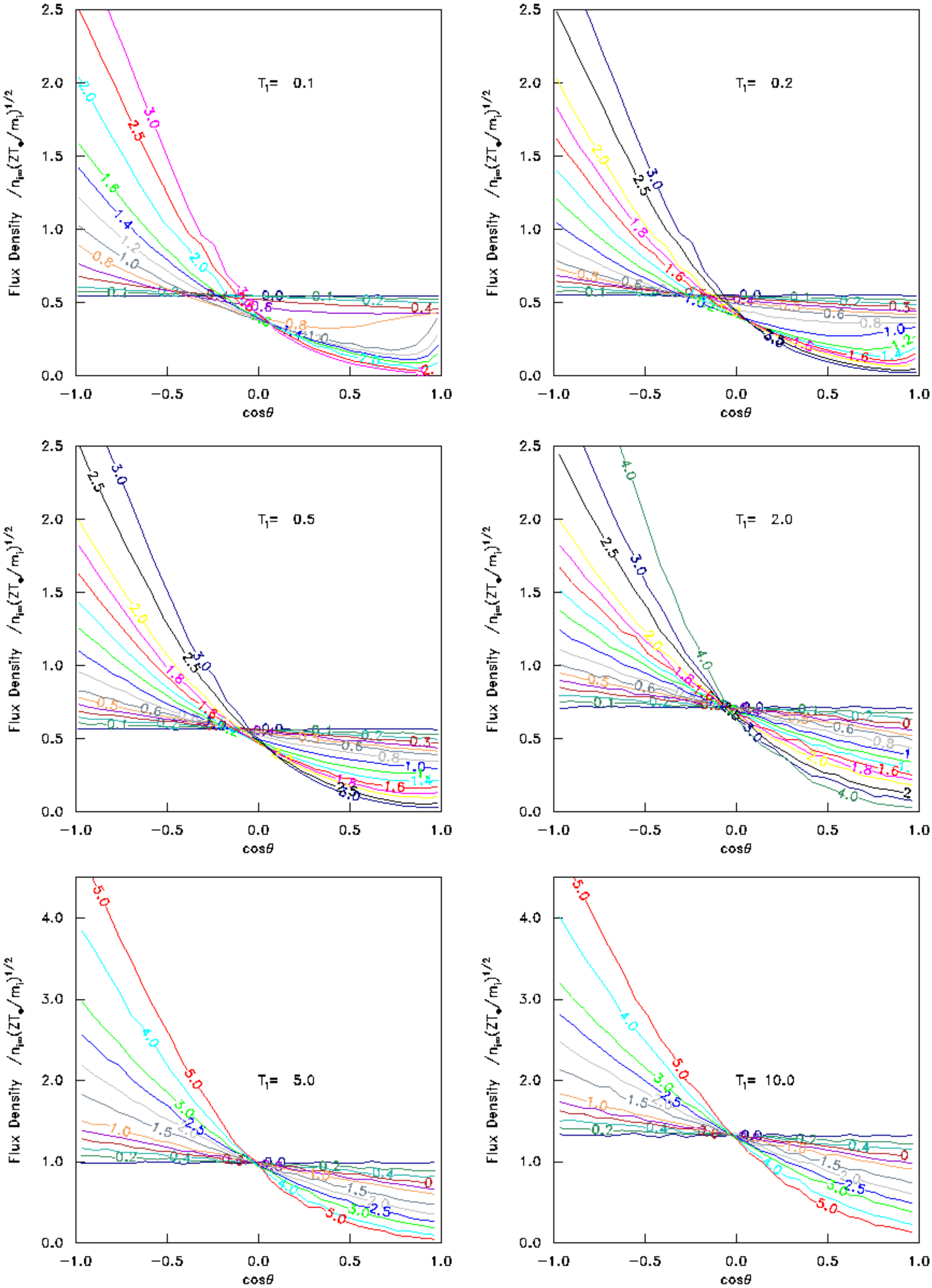

Figure 3: Ion flux density normalized to $n_{i\infty}(ZT_e/m)^{1/2}$, as a function of $\cos\theta$ for different flow velocities, as indicated on the curves, and a range of ion temperatures.

In Table 1 are compared different published estimates of the calibration factor. The heuristic Hudis and Lidsky formula [8], popular for its analytic simplicity, but unjustified by physics, substantially overestimates the flux asymmetry, or underestimates the flow velocity when $T_i = T_e$. The experimental results with independent flow measurements seem to be consistent with the calibration from SCEPTIC. As anticipated, the asymmetry is less than when the ions are magnetized.

Table 1: Comparison of Mach Probe calibration factors

Source	K formula	$K(T_i = 1)$	Description	Ref
Present work	1.34 for $T_i \lesssim 3$	1.34	fit to SCEPTIC	
Analytic Integration	$\sqrt{2\pi T_e/T_i}$	2.5	Free-flight	[1]
Hudis & Lidsky	$4(T_iT_e)^{1/2}/(T_e+T_i)$	2	Heuristic	[8]
Chung et al	1.26 $@T_i = 0.1$		Experiment	[9]
Oksuz et al	1.3 $@T_i = 0.1$		Experiment	[10]
Hutchinson	$(0.43\sqrt{1+T_i/T_e})^{-1}$	1.64	Magnetized, fluid	[3]

The present data provide a rigorous theoretical calibration of unmagnetized Mach probes, at least for this particular spherical geometry. Further studies are necessary to establish how sensitive the calibration is to different probe geometries and to situations where the sheath thickness is not negligible.

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