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Nonlocal Transport in Fully Ionized Magnetized Plasmas

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I. Introduction

Fluid equations provide a convenient reduced description of many physical problems and are more amenable to analytic insight or numerical solution than the full kinetic equations. The most important fluid equations are the conservative equations of the density, the momentum and the energy of the particles. They are defined by the closure relations which correspond to the transport coefficients that one has to compute from the kinetic theory. An external dc magnetic field B with arbitrary strength is considered in the plasma. The transport coefficients depend in this case on the relevant dimensionless parameter $\Omega_e = \omega_c \tau_{ei}$ where $\omega_c = -eB/m$ is the electron cyclotron frequency, $\tau_{ei} = \lambda_{ei}/\mathrm{v}_t$, λ_{ei} is the electron-ion mean free path, $\mathrm{v}_t = \sqrt{T/m}$ is the electron thermal velocity. This work is devoted to the computation of the transport coefficients in magnetized plasmas.

II. Basic equations

The electron Fokker-Planck Equation (FPE) in the frame of the electrons reads

$$\frac{\partial f}{\partial t} + \left(\vec{\mathrm{v}} + \vec{V}\right) \cdot \frac{\partial f}{\partial \vec{r}} - \frac{e}{m}\left(\vec{E} + \vec{\mathrm{v}} \times \vec{B}\right) \cdot \frac{\partial f}{\partial \vec{\mathrm{v}}} - \left(\mathrm{v}_i + V_i\right) \frac{\partial V_j}{\partial x_i} \cdot \frac{\partial f}{\partial \mathrm{v}_j} = C_{ei}(f) + C_{ee}(f) \tag{1}$$

where $f(\vec{r}, \vec{\mathrm{v}}, t)$ is the electron distribution function (DF), $\vec{E}$ is the electric field, $\vec{V}$ is the fluid velocity, and in the right-hand side of (1), the two terms correspond respectively to the Landau electron-ion and to the electron-electron collision operators. In this work, we deal with small perturbations of the DF δf, the density δn, the temperature δT, the electric field $\delta \vec{E}$ and the fluid velocity $\delta \vec{V}$, around a global equilibrium function F_M. We assume that the magnetic field is directed along the z-axis, the plasma inhomogeneity depends on the spatial coordinate x, and the other vectorial fluid variables are parallel to the x-axis. In the Fourier space, ($x \Leftrightarrow k$) and the steady-state approximation, Eq. (1) becomes

$$\mathrm{ikv_x}\, \delta f - \frac{e}{m}\delta E_x \frac{\partial F_M}{\partial \mathrm{v_x}} - \frac{e}{m}\delta E_y \frac{\partial F_M}{\partial \mathrm{v_y}} - \frac{e}{m} B\left(\mathrm{v}_y \frac{\partial f}{\partial \mathrm{v}_x} - \mathrm{v}_x \frac{\partial f}{\partial \mathrm{v}_y}\right) - ik\mathrm{v}_x \delta V \frac{\partial F_M}{\partial \mathrm{v}_x} = C_{ei}(\delta f) + C_{ee}(\delta f) \tag{2}$$

It is convenient to expand δf and Eq. (2) on the spherical harmonic basis:

$$\delta f = \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} f_l^m(y) Y_l^m(\theta,\varphi) \quad \text{where } y = \mathrm{v}/\sqrt{2}\mathrm{v}_t \ .$$

Keeping only the first order nonlocal terms, proportional to $(k\lambda_{ei})^2$ in Eq. (2), we obtain:

$$C_{ee}(f_0^0) - (ik\mathrm{v}_t\sqrt{2}/\sqrt{3})yF_1^- = \frac{4\sqrt{\pi}}{3} y^2 F_M(y)\, ik\delta V \tag{3}$$

$$C_{ee}\left(F_1^+\right) + C_{ei}\left(F_1^+\right) + \omega_c F_1^- - \sqrt{2}(k\mathrm{v}_t/\sqrt{5})yF_2^+ = \frac{4\sqrt{\pi}}{\sqrt{3}} yF_M(y)\frac{e\delta E_y}{m\mathrm{v}_t} \tag{4}$$

$$C_{ee}\left(F_1^-\right) + C_{ei}\left(F_1^-\right) - \omega_c F_1^+ + 2(ik\mathrm{v}_t/\sqrt{15})yf_2^0 - \sqrt{2}(k\mathrm{v}_t/\sqrt{5})yF_2^- = \frac{4\sqrt{\pi}}{\sqrt{3}} yF_M(y)\frac{e\delta E_x}{m\mathrm{v}_t} + 2(ik\mathrm{v}_t/\sqrt{3})yf_0^0 \tag{5}$$

$$C_{ee}\left(F_2^+\right) + C_{ei}\left(F_2^+\right) + 2\omega_c F_2^- - \sqrt{2}(ik\mathrm{v}_t/\sqrt{5})yF_1^- = \frac{8\sqrt{\pi}}{\sqrt{30}} y^2 F_M(y)\, ik\delta V \tag{6}$$

$$C_{ee}\left(F_2^-\right) + C_{ei}\left(F_2^-\right) - 2\omega_c F_2^+ - \sqrt{2}(ik\mathrm{v}_t/\sqrt{5})yF_1^+ = 0 \tag{7}$$

$$C_{ee}\left(f_2^0\right) + C_{ei}\left(f_2^0\right) + (ik\mathrm{v}_t/\sqrt{15})yF_1^- = -\frac{4\sqrt{\pi}}{\sqrt{45}} y^2 F_M(y)\, ik\delta V\ , \tag{8}$$

where we have used, instead of the complex functions f_l^m, the real functions :

$$F_1^+ = -i\left(f_1^{+1} + f_1^{-1}\right), F_1^- = f_1^{-1} - f_1^{+1},\ F_2^+ = \left(f_2^{-2} + f_2^{+2}\right),\ F_2^- = i\left(f_2^{-2} - f_2^{+2}\right).$$

III. Transport coefficients

To solve the coupled integrodifferential equation, a new numerical approach is used. For this purpose, the FPE is reduced to a simple set of ordinary differential equations, which can be easily solved iteratively, with the use of standard numerical methods [1]. We have computed the components f_0^0, F_1^+, F_1^-, F_2^+ and F_2^- of the distribution function and deduced the local and the nonlocal transport coefficients.

A. Local transport coefficients

The definitions used are the ones used by Braginskii [3]:

$$en\vec{E} = -\vec{\nabla}p + \vec{j}\times\vec{B}/c + \vec{\alpha}.\vec{j}/ne - n\vec{\beta}.\vec{\nabla}T\ ,$$

$$\vec{q} = -\vec{\kappa}.\vec{\nabla}T - \vec{\beta}.\vec{j}T/e,$$

$$\pi_{\alpha\beta} = -\eta_0 W_{0\alpha\beta} - \eta_1 W_{1\alpha\beta} - \eta_2 W_{2\alpha\beta} + \eta_3 W_{3\alpha\beta} + \eta_4 W_{4\alpha\beta}$$

where $\alpha_{\perp,\wedge}$ are the electrical resistivities, $\beta_{\perp,\wedge}$ are the thermoelectric conductivities, $\kappa_{\perp,\wedge}$ are the thermal conductivities, W_{ijk} is the stress tensor and $\eta_{0,1,3}$ are the viscosity coefficients which satisfy the symmetry properties: $\eta_2(2\Omega_e)=\eta_1(\Omega_e)$ and $\eta_4(2\Omega_e)=\eta_3(\Omega_e)$. The notation $\perp, \wedge$ indicate Ox and Oy respectively. In the local approximation, terms proportional to $F_2^{\pm}$ and f_0^2 in Eqs. (4) and (5) and to $ik\lambda_{ei}F_1^{\pm} \sim (\lambda_{ei}/L)F_1^{\pm}$ in Eqs. (6)-(8) have been dropped . In this case, $f_0^0 = \sqrt{4\pi}\delta f_M(y)$ where $\delta f_M(y)$ is the perturbed Maxwellian. The transport coefficients induced by the first anisotropic distribution function computed by Braginskii [3] and improved by Epperlein and Haines [4], have been recovered. In this work, the new result is the accurate computation of the viscosity coefficients derived with only approximate method in the literature [3] and [5]. These coefficients η_1 and η_3 are given in Fig. 1.

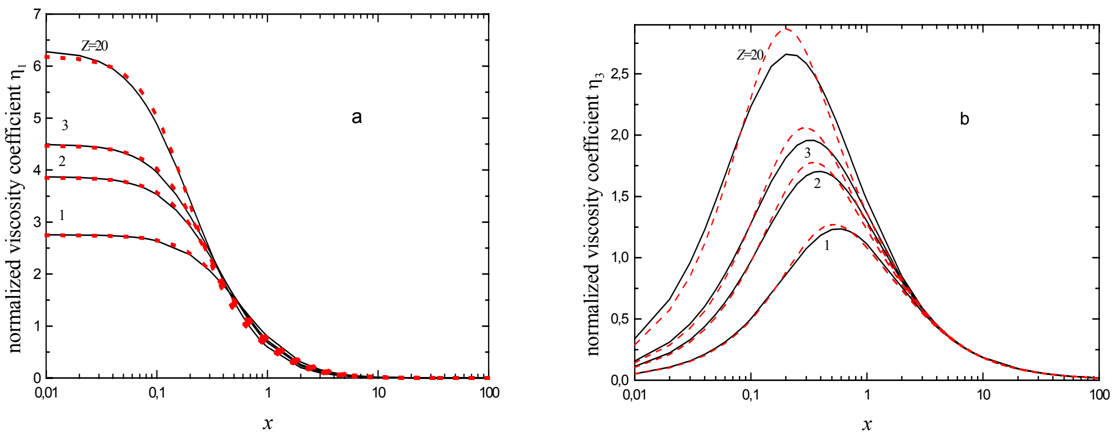

Fig. 1 Viscosity coefficients η_1 (a) and η_3 (b) normalized to the parameter $nT\lambda_{ei}/\mathrm{v}_t$ as function of $x = \sqrt{2/9\pi}\Omega_e$, for different Z. The dotted curves correspond to the analytical result of [5].

We can see in particular, that for $Z > 10$, the standard viscosity coefficients exhibit inaccuracy of about 9% in the moderate B-field range.

B. Nonlocal transport coefficients

In this case, all the coefficients mentioned above can be written as a sum of a local coefficient calculated in the collisional limit and a nonlocal coefficient proportional to $(k\lambda_{ei})^2$.

We have reduced the study to the heat fluxes and in this nonlocal approximation they read

$$q_x = \frac{q_{xlocal}}{\left(1 + k^2 \lambda_{Dx}^2\right)} \quad \text{and} \quad q_y = \frac{q_{ylocal}}{\left(1 + k^2 \lambda_{Dy}^2\right)}$$

where λ_{Dx} and λ_{Dy} are the delocalization lengths.

In Fig. (2), we present the computation of λ_{Dx} and λ_{Dy} as a function of the magnetic field.

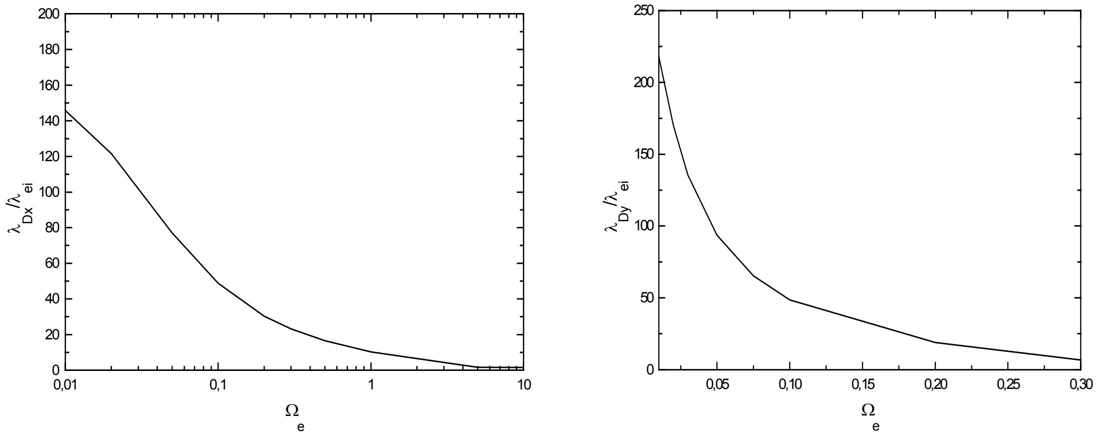

Fig. 2 Ratio of the delocalization length λ_{Dx} and λ_{Dy} to the mean free path λ_{ei} for Z=10 as a function of Ω_e.

As we see the magnetic field tends to reduce the nonlocal effects. This reduction is more pronounced for q_y than for q_x. Typically for $\Omega_e \approx 1$, the nonlocal effects disappear. In future work, we will give more quantitative results for all the transport coefficients as functions of the atomic number Z.

IV. References

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