SELF-SUSTAINED NON-LINEAR OSCILLATIONS IN RADIATIVE PLASMAS

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I. Introduction. Radiative losses from radiating plasmas add new parameters to the collection of plasma parameters. One may expect the self-organization in multi-parametric systems. Self-organized structures like spiral waves have been recently observed in magnetized or rotating laboratory plasmas [1-4] and investigated theoretically [5,6]. The self-organization in chemical systems like Belousov-Zhabotinsky oscillations and spiral structures has been observed without initial equilibrium anisotropy (see, for instance, [7]). It has been shown the self-sustained uniform oscillations and the non-linear completely harmonic plane and spiral waves like chemical oscillations may occur in radiative plasmas.

IIa. Uniform oscillations. Basic equations. Uniform plasmas, consisting of fully ionised hydrogen and light impurity ions are investigated. The impurity may be described by its two most representative ions with charges z and $z+1$ [8]. For uniform perturbations, one can write, for the values, $y_z = n_z / n_I$:

$$\frac{dy_z}{dt} = n_e\left(R - (J+R)y_z\right). \tag{1}$$

Here $n_e = n_i + zn_z + (z+1)n_{z+1}$ is the electron density, n_i is the hydrogen ion density, n_z is the density of impurity ions with the charge z, $n_I = \sum_z n_z$, J and R are the ionisation rate of the ion with the charge z and the recombination rate of the ion with the charge $z+1$ respectively. The equality $y_z + y_{z+1} = 1$ is used. If the energy balance is determined by the space energy source S and impurity radiative losses, $Q = n_e\left(n_z L_z(T) + n_{z+1}L_{z+1}(T)\right)$, the temperature equation takes the form, after summing over species:

$$\frac{3}{2}\frac{d}{dt}\left(NT + \frac{2}{3}I_z n_{z+1}\right) = S - Q. \tag{2}$$

Here $N = n_e + n_i + n_I$, equality of temperatures is assumed; $T_e = T_i = T_z = T_{z+1}$, and I_z is the ionization energy of the ions with the charge z. Equations (1) and (2) may be rewritten for the perturbations $y = y_z - y_o$, and $\tau = (T - T_0)/T_*$:

$$\frac{dy}{d\zeta} = X(y,\tau), \;\; \frac{d\tau}{d\zeta} = Y(y,\tau). \tag{3}$$

Here T_0 and y_0 are the equilibrium values of T, and $y_z, \zeta = tn_I$, and T_* is an arbitrary temperature. Linear analysis yields: $\omega^2 + i\omega(X_y + Y_\tau) + X_\tau Y_y - X_y Y_\tau = 0$. If $X_\tau Y_y - X_y Y > 0$, and $X_y + Y_\tau << X_\tau Y_y - X_y Y$, the new oscillating unstable mode exists with frequency ω and growth rate $\gamma << \omega$, $\omega = \pm\sqrt{X_\tau Y_y - X_y Y}$, $\gamma = X_y + Y_\tau$. The purpose of this subsection is to find the conditions for nonlinear saturation. Introducing variables, $r = \sqrt{y^2 + \tau^2}$, and ϑ, $\sin\vartheta = y / r$, $\cos\vartheta = \tau / r$, one can get

$$\frac{dr}{d\vartheta} = r\frac{X\sin\vartheta + Y\cos\vartheta}{X\cos\vartheta - Y\cos\vartheta}. \tag{4}$$

Expanding X and Y near the equilibrium, and keeping quadratic terms one can find

$$\frac{dr}{d\vartheta} = r\Phi_1(\vartheta) + r^2\Phi_2(\vartheta). \tag{5}$$

If the magnitude of Φ_1 is sufficiently small, equation (5) may be solved with the initial conditions $r_0 = r(\vartheta_0), \vartheta_0 = 0$ for large values of r: $r^{-1} = r_0^{-1} - \int_0^\vartheta \Phi_2(\vartheta)d\vartheta$.

The mode is saturated if there are no infinite trajectories for any r_0, $\int_0^\vartheta \Phi_2(\vartheta)d\vartheta < 0$.

IIb. Physical example. The non-linear self-sustained oscillations may occur in pure carbon optically thin plasmas near temperature $T_0 \approx 3.65\ eV$. In this temperature range, only two species with charge states 1 and 2 exist [9], hence, $z = 1$. The concentration of ions with the charge $z = 1$ is significantly larger than the concentration of ions with $z = 2$. Hence, one can put $N \approx 2n_e$. In many cases the energy absorption S is proportional to the electron density. This assumption is not a defining one, but allows the presentation of a simple example. The function S is chosen as $S = 2.05 \cdot 10^{-21} n_e (T/T_*)^3 \quad erg/(cm^3 s),\ T_* = 1.6 \cdot 10^{-12}\ erg$. Using the database for carbon from [8], and keeping quadratic terms in the expansions, one can find:

$$\frac{d^2\tau}{d\xi^2} - (0.00425 + 40\tau)\frac{d\tau}{d\xi} - 0.883\left(\frac{d\tau}{d\xi}\right)^2 = -11\tau - 15\tau^2. \qquad (6)$$

Here $\tau = T/T_*$, $\xi = tn_I/(t_0 n_0)$, $n_0 = 10^{12}\ cm^{-3}$, $t_0 = 1s$. The solutions for small ($d\tau/d\xi = 0.05$) and large ($d\tau/d\xi = 0.23$) initial perturbations are shown in Fig. 1 and 2 respectively.

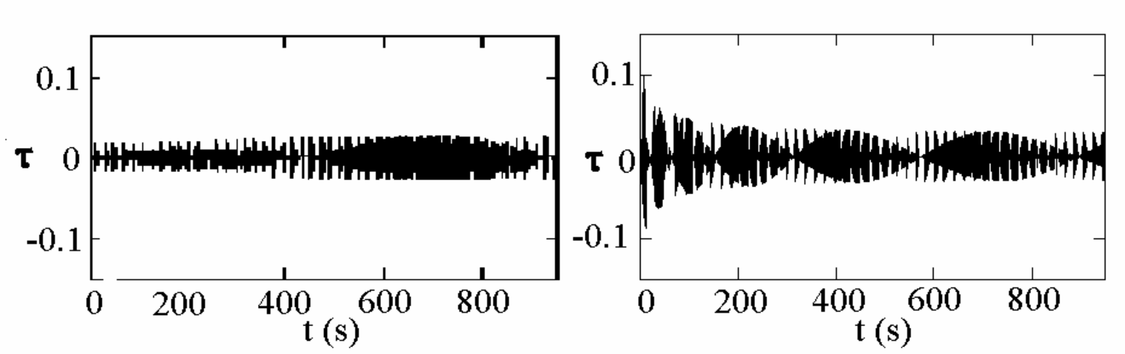

Fig. 1. Fig. 2

The solutions consist of many harmonics in both cases. However, one can find completely harmonic non-linear oscillations and travelling waves.

III. Non-linear harmonic oscillations and waves (plane and spiral). If the right hand of equations (3) takes the form

$$X = X_s \sin\varphi + X_c \cos\varphi,\ Y = Y_s \sin\varphi + Y_c \cos\varphi \quad , \qquad (7)$$

after substituting the functions y and τ in the form

$$y = p_0 + p_1 \sin\varphi + p_2 \cos\varphi,\ \tau = q_0 + q_1 \sin\varphi + q_2 \cos\varphi \quad , \qquad (8)$$

one can find the coefficients p_j, q_j, at least for small oscillations. Here $\varphi = \omega\zeta$ for uniform oscillations, $\varphi = \omega\zeta - kx$ for plane waves, and $\varphi = \omega\zeta + l\theta - \eta(\rho)$ for spiral waves, x, ρ and θ are space coordinates for plain and cylindrical geometry. Expanding the right hand of (3) and substituting (8), one can get a form for X :

$$X \approx a_0 + a_1 \sin\varphi + a_2 \cos\varphi + a_3 \sin^2\varphi + a_4 \cos^2\varphi + a_5 \sin\varphi\cos\varphi +$$
$$a_6 \sin^3\varphi + a_7 \sin^2\varphi c \cos\varphi + a_8 \sin\varphi\cos^2 + a_9 \cos^3\varphi \quad , \tag{9}$$

where up to cubic terms of the expansion are kept. The same form may be obtained for Y . Expression (9) takes the form (7) if:

$$a_3 = a_4 = -a_0\,,\; a_5 = 0\,,\; a_6 = a_8, \text{ and } a_7 = a_9\,. \tag{10}$$

Finally, one gets 4 equations from (7) equating terms with $\sin\varphi$, and $\cos\varphi$. Also 5 equations (10) and 5 analogue equations for Y must be satisfied. In total, 14 equations must be satisfied in order to find 8 coefficients p_j, q_j, ω, and k (or η for spiral waves). Hence, one needs in 6 fitting parameters. First, the impurity concentration n_I is a controlled parameter. Second, the energy source S is a controlled function. Up to cubic approximation, this gives four fitting parameters, S_0, S_τ, $S_{\tau\tau}$, and $S_{\tau\tau\tau}$. Third, in many cases the heating leads to the existence of electronic tails producing additional ionisation. The ionisation rate may be represented in the form $J = J_{th} + J_{ext}$. Here J_{th} is the thermal ionisation rate, J_{ext} is defined by the non-thermal effects and may be controlled. Fourth, in many cases non-equilibrium hydrogen neutral atoms penetrate the plasma and may influence the level of radiation losses significantly [9,10]. Thus, at least seven fitting parameters may be found.

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References

[1] M.V. Nezlin and E.N. Snezhkin, Rossby Vortices, Spiral Structures, Solitons. Springer-Verlag, Berlin, 1993.
[2] K.S. Fine, A.C. Case, et al., Phys. Rev. Lett., **75**, 3277 (1995).
[3] T. Ikebata, H, Tanaka et al., Phys. Rev. Lett, **81**, 1853 (1998).
[4] W.E. Amatui, D.N. Walker et al., Phys. Rev. Lett, **77**, 1978(1996).
[5] J.R. Penano, G. Ganguli, et al., Phys. Plasmas, **5**, 4377 (1998).
[6] M. Kono, and M.Y. Tanaka, Phys. Rev. Lett, **84**, 4369 (2000).
[7] Oscillations and travelling waves in chemical systems, ed. by R.J. Field, and M. Burger, J. Wiley\&sons, NY, Chichester, Brisbane, Singapore, 1985.
[8] Gervids, V.I., Kogan, V.I., Morozov, D.Kh., Plasma Phys. Rep, **27**, 994 (2001).
[9] Gervids, V.I., Morozov, D.Kh, Plasma Phys. Rep, **26**, 439 (2000).
[10] Gervids, V.I., Zhidkov, et al., In Reviews of Plasma Phys, v. 12, p. 207, part 1.3, ed. by Leontovich A.M. and Kadomtsev B.B., Consultant Bureau, NY-Lnd, 1987.