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# Confining instability in a complex plasma

S.V. Vladimirov, A.A. Samarian, and B.W. James

*School of Physics, University of Sydney, New South Wales 2006, Australia*

**Abstract.** It is shown that the stability of the vertical and horizontal confinement of colloidal "dust" particles levitating in a complex plasma appears as a non-trivial interplay of the external confining forces as well as the interparticle interactions and collective processes such as the plasma wake.

In the laboratory experiments, the micrometer sized highly charged dust grains levitate in the sheath region under the balance between the gravitational and electrostatic forces acting in the vertical direction as well as the externally imposed confining potential applied in the horizontal plane [1]. The vertical confinement involving the gravity force and the electrostatic force acting on the dust particles with variable charges is a complex process exhibiting oscillations, disruptions and instabilities [2-6]. A characteristic feature of the particle confinement is also the strong influence of plasma collective processes such as the plasma wake [7, 8].

Consider two colloidal particles of mass $M$ and charges $Q$, separated by the distance $x_d$ horizontally (i.e., aligned along the $x$-axis), see Fig. 1a or $z_d$ vertically (aligned along the $z$-axis), see Fig. 1b. In the simplest approximation, the particles interact via the

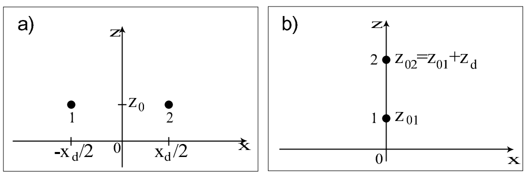


**FIGURE 1.** Sketch of the particle configurations.

screened Coulomb (Debye) potential $\phi_D = Q^2\exp(-|\mathbf{r}|/\lambda_D)/|\mathbf{r}|$ where $\lambda_D$ is the plasma Debye length. For particles levitating in the plasma sheath, the interaction potential in the vertical direction is asymmetric because of the ions flowing towards the negatively charged electrode. However, it is also instructive to consider the case with Debye only interaction even in the vertical direction.

We assume that the external confining force acting in the $x$-direction can be written as $F^{\text{ext}} = -\gamma_x(x-x_0)$, where $\gamma_x \sim QdE_x^{\text{ext}}/dx$ is a constant and obtain the balance of the external confining and Debye repulsion forces

$$\frac{2Q^2}{x_d^2}\left(1+\frac{x_d}{\lambda_D}\right)\exp\left(-\frac{x_d}{\lambda_D}\right) = \gamma_x x_d. \tag{1}$$

The balance of forces in the vertical direction includes also the gravitational force $F_g = Mg$ and the sheath electrostatic force $F_{el} = QE_z^{\text{ext}}(z)$. In equilibrium, we assume that the sheath electric field near the equlibrium position can be linearly approximated $F_{el} - Mg = -\gamma_z(z - z_0)$, where $\gamma_z \sim QdE_z^{\rm ext}/dz$ is a constant. For the vertically aligned particles (Fig. 1b) the lower and upper equilibrium positions are $z_{01}$ and $z_{02} = z_{01} + z_d$, respectively. In this case, the equilibrium balance of the forces in the vertical direction can be written as $F_{el,1(2)}(z_{01(2)}) - M_{1(2)}g + F_{1(2)}^{D,W}(z_{02} - z_{01}) = 0$, where $F_{1,2}^{D,W}$ are the forces of the interaction between the particles due to their interaction Debye and/or asymmetric (wake) potentials $\Phi_D$ and/or $\Phi_W$, respectively: $F_1^D(z_{02} - z_{01}) = Qd\Phi_D(|z|)/d|z||_{|z|=z_d}$, and $F_2^{D,W}(z_{02} - z_{01}) = -\,Qd\Phi_{D,W}/(|z|)d|z|\Big|_{|z|=z_d}$. In the case of Debye only interaction between the particles, we obtain equation similar to (1), with the obvious change of $x$ to $z$. In the case of the asymmetric wake potential, the equilibrium condition for the levitation of two identical particles gives us

$$\frac{Q^2}{z_d^2}\left(1 + \frac{z_d}{\lambda_D}\right)\exp\left(-\frac{z_d}{\lambda_D}\right) - \gamma_z^W(z_d - z_W) = \gamma_z z_d, \tag{2}$$

where $z_W$ is the distance between the minimum of the asymmetric attracting potential characterized by $\gamma_z$ and the upper particle.

Now, consider horizontal perturbations of two horizontally aligned particles, Fig. 1a. By including the phenomenological damping $\beta$ and linearly expanding the interaction forces, we obtain two oscillation modes with the frequency

$$\omega_{xx,1} = -\frac{i\beta}{2} + \left(-\frac{\beta^2}{4} + \frac{\gamma_x}{M}\right)^{1/2}, \tag{3}$$

for the two particle oscillating in phase with equal amplitudes $A_1 = A_2$, and

$$\omega_{xx,2} = -\frac{i\beta}{2} + \left[-\frac{\beta^2}{4} + \frac{\gamma_x}{M}\left(3 + \frac{x_d^2/\lambda_D^2}{1 + x_d/\lambda_D}\right)\right]^{1/2} \tag{4}$$

for the particles oscillating counter phase ($A_1 = -A_2$). Both modes are always stable.

The next case involves vertical oscillations of two horizontally aligned particles, Fig. 1a. We obtain that the two oscillation modes have the frequency similar to (3), with the change of $x$ to $z$, for the two particle oscillating in phase ($A_1 = A_2$), and

$$\omega_{xz,2} = -\frac{i\beta}{2} + \left(-\frac{\beta^2}{4} + \frac{\gamma_z}{M} - \frac{\gamma_x}{M}\right)^{1/2} \tag{5}$$

for the two particles oscillating counter phase ($A_1 = -A_2$). While the first mode is always stable, the counter phase mode *can now be unstable*, depending on the ratio $\gamma_x/\gamma_z$. This instability arises because of the *confining* potential in the direction *perpendicular* to the direction of particle oscillations.

By introducing small vertical perturbations $\delta z_i$ of the vertically aligned particles, we obtain for the case of Debye only interactions equations analogous to the first case of horizontal vibrations of horizontally aligned particles. There are two oscillations modes;

the first one has the frequency similar to (3) for the two particle oscillating in phase with equal amplitudes $A_{1,2}$, and the second mode's frequency is similar to (4), both with the obvious change of $x$ to $z$. Taking into account the asymmetry of the interaction potential, we obtain that the first oscillation mode, for the particles moving in phase with equal amplitudes $A_1 = A_2$, is unchanged while the second frequency is now given by

$$\omega_{zz,2}^W = -\frac{i\beta}{2} + \left[ -\frac{\beta^2}{4} + \left( \frac{\gamma_z}{M} + \frac{\gamma_z^W}{M} \left( 1 - \frac{z_W}{z_d} \right) \right) \left( 3 + \frac{z_d^2/\lambda_D^2}{1 + z_d/\lambda_D} \right) + \frac{\gamma_z^W}{M} \frac{z_W}{z_d} \right]^{1/2} \quad (6)$$

for the counter phase oscillations; their amplitudes are not equal in magnitude: $A_1 = -\left(2 + \frac{z_d^2/\lambda_D^2}{1+z_d/\lambda_D}\right)\left(1 - \frac{z_W}{z_d} + \frac{\gamma_z}{\gamma_z^W}\right) A_2$. Both modes are always stable.

Now, consider horizontal oscillations of two vertically aligned particles. When the particle interaction is symmetric of Debye type, we obtain two modes of oscillations, the first one corresponds to to the particles oscillate in phase (with equal amplitudes), and its frequency is equal to (3). The second one is similar to (5), with the frequency

$$\omega_{zx,2} = -\frac{i\beta}{2} + \left( -\frac{\beta^2}{4} + \frac{\gamma_x}{M} - \frac{\gamma_z}{M} \right)^{1/2}, \quad (7)$$

and $A_1 = -A_2$. We see that the counter phase mode can be unstable, the condition for this instability is somewhat opposite to the condition of the instability of the mode of vertical vibrations of two horizontally arranged particles (5). If to take into account the plasma wake, the equation of horizontal motion of the upper particle in this case is the same as for the symmetric Debye only interaction, while the lower particle is oscillating in the wake potential characterized by $\gamma_x^W$ which is its horizontal strength in the parabolic approximation. For our purposes here it is sufficient to assume that $\gamma_x^W$ is a positive constant of order (or slightly more) than $\gamma_z^W$, see, e.g., numerical simulations [8]. The frequency of the first mode coincides with (3) while the frequency of the second mode is given by

$$\omega_{zx,2} = -\frac{i\beta}{2} + \left[ -\frac{\beta^2}{4} + \frac{\gamma_x}{M} + \frac{\gamma_x^W}{M} - \frac{\gamma_z}{M} - \frac{\gamma_z^W}{M} \left( 1 - \frac{z_W}{z_d} \right) \right]^{1/2}. \quad (8)$$

Now, we see that the wake potential can *stabilize* possible horizontal instability of two vertically algned particles; note that for the supersonic wake potential this stabilization occures only within the Mach cone. The amplitudes of the second mode of oscillations are related by $A_1 = \frac{\gamma_x^W A_2}{\gamma_z + \gamma_z^W (1 - z_W/z_d)}$. Thus for the asymmetric interaction potential, the second mode of oscillations does not correspond to the counter phase motions: the vibrations of particles are *in phase* now, with unequal amplitudes.

The proposed mechanism can be related to experimentally observed phenomena, for example, for the two-particle system in planar rf-discharge, involving horizontal oscillations of two particles aligned in the vertical string and hysteretic phenomena in the disruptions of the horizontal and vertical arrangements, see Fig.2. The stability diagram for the two-particle system, Fig.3, reveals two extreme regions: one is the region (I)

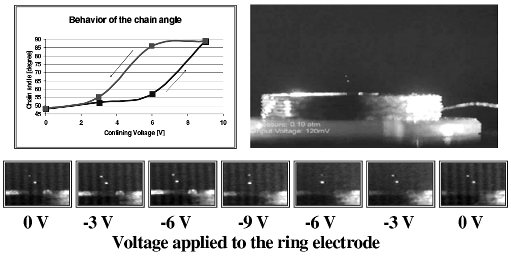


**FIGURE 2.** Experiment: levitation of two particles.

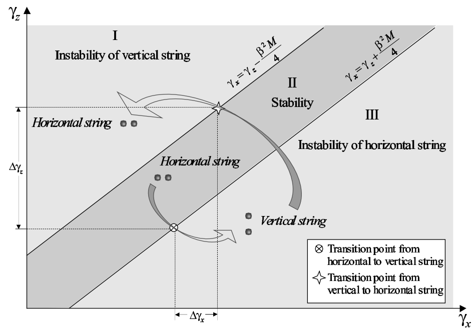


**FIGURE 3.** Stability diagram of the particle arrangements.

where $\gamma_z > \gamma_x + M\beta^2/4$, corresponding to the vertical string unstable with respect to the horizontal motions of the particles, another is the region (III) where $\gamma_x > \gamma_z + M\beta^2/4$ corresponding to the horizontal string unstable with respect to the vertical motions of the particles, as well as the central region (II) where both structures can be stable.

## ACKNOWLEDGMENTS

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