

## ION DRAG AND BROWNIAN MOTION IN DUSTY PLASMAS

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### Abstract

The ion drag force is calculated for dusty plasmas with ion absorption. Influence of strong particle interactions in dusty plasmas on drag force is found.

### 1. Introduction

Recent experiments on dust dynamics in RF discharge [1,2] demonstrated the existence of voids in the dust cloud for some discharge parameters. In particular in [2] parameters of the voids were found (internal and external radii) and the dynamics (including rotations) of the 2D dust cloud with and without voids, as functions of gas pressure and RF power. In void creation the role of ion flow seems very important [3,4]. There are recent considerations [5-8] of the ion drag force (the scattering and collecting parts of it) based on orbit motion limited theory [9]. At the same time the existent theories for ion drag force are not sufficiently justified and cannot provide agreement between modeling and the experiments [4]. It means that the consideration of the ion drag force has to be done on a more rigorous basis to clarify the limitations of the existing models and to derive more exact expressions for the drag force. As a result it can be clarified, in particular, that the ion drag force is really dominant for the process of void creation or that it is necessary to find some other mechanisms, at least for some regions of the discharge parameters.

In the present work we focus on the evaluation of the ion drag force in dusty plasmas. In general there are three components, which characterize the ion drag force:  $F_1$ ,  $F_2$  and  $F_3$ , where  $F_1$  appears due to the ion scattering by charged grains,  $F_2$  is connected with ion collection by grains and  $F_3$  describes the momentum transfer to the grains due to reflection of the collected particles or, more often (as for argon discharges), evaporation of other particles (atoms) created by recombination of bombarding ions with electrons on the grain surface. The forces  $F_1$  and  $F_2$  can be calculated on basis of rigorous kinetic considerations [10-11], for the force  $F_3$  it is possible to use a model similar to [12]. If atoms leave the surface of the grain diffusively (with the temperature of the surface after ion hits and electron-ion surface recombination), the approximations similar to [12] are applicable.

The more complicated processes require the development of the appropriate kinetic theory. In this work, we consider, the influence of rather strong interaction of ions with grains and between grains on the processes of ion scattering by grains, effects, which as far as we know, have not been reported before for the drag force in dusty plasmas. In general this interaction can provide energy transfer to the grains not only by heating of grains, due to absorption of the ions, but also due to excitation of some collective modes in the dust subsystem. Here, we consider only the influence of strong interaction between the grains on the ion drag for the case of elastic ion-grain scattering. We discuss also the influence of strong ion-grain interaction and relatively high ion drift velocity on the ion drag force. We calculate the collection drag force for an arbitrary relation between the ion drift and thermal velocities. Important particular problems, as a completely positive Coulomb logarithm for ion scattering and some other new results of kinetic theory for dusty plasmas are discussed.

## 2. Kinetic description of ion drag

For the understanding of dusty systems the extension of kinetic theory and inclusion of new kinetic variables for the dust particles play a crucial role [10-11,13-16]. If the mass of the grains is supposed to be constant we can use the results of [10-11]. In these papers a consistent kinetic theory for dusty plasmas has been developed in an approximation taking into account charging collisions, as dominant, but also elastic collisions between grains and ions and neutral atoms or molecules. This approximation permits to justify results of semi-phenomenological considerations as the average charge of the grains in OML or the charging frequency. At the same time essential new results as the charge and velocity distributions of grains, the anomalous effective kinetic temperatures for dust and the Brownian dynamics for grains have been found. The grain's dynamics can be described on basis of a generalized Fokker-Planck equation in velocity-charge ( $\mathbf{v}, q$ ) space. The concrete form of the friction coefficient  $\beta(\mathbf{v}, q)$ , diffusion coefficient  $D(\mathbf{v}, q)$  and the mixed coefficients can be determined. It is easy to show that the effective force on the grains can be written as:

$$F = -m_g \int d\mathbf{v} \mathbf{v} \beta(\mathbf{v}, q) f_g(\mathbf{v}, q), \quad (1)$$

where  $m_g$ ,  $q$  and  $f_g(\mathbf{v}, q)$  are the mass, charge and one-particle distribution function of grains respectively, and  $\beta(\mathbf{v}, q)$  can be calculated according to Eq. (59) of [10]:

$$\beta(\mathbf{v}, q) = -\sum_{\alpha} (3m_{\alpha} / m_g) \int d\mathbf{v}' (\mathbf{v} \cdot \mathbf{v}') / v'^2 \cdot |\mathbf{v} - \mathbf{v}'| \cdot \sigma_{g\alpha}(\mathbf{v} - \mathbf{v}', q) \cdot f_{\alpha}(\mathbf{r}, \mathbf{v}', t) \quad (2)$$

Here  $m_{\alpha}$ ,  $\sigma_{g\alpha}$  and  $f_{\alpha}(\mathbf{r}, \mathbf{v}', t)$  are the mass, cross-section for absorption of electrons and ions ( $\alpha=e,i$ ) and one-particle distribution function. Let us consider homogeneous plasmas and assume, for simplicity, that the charge distribution is very sharp, therefore all grains have the same charge  $q$ . For the case of isotropic (for example Maxwellian) distribution  $f_{\alpha}$  the Eqs. (1), (2) describe the effective friction force for moving grains. The function  $\beta(\mathbf{v}, q)$  possesses a remarkable peculiarity: due to the processes of absorption  $\beta$  can change sign for some velocity ("negative friction"), if parameter  $\Gamma = (q e^2) / a T_i \geq 1$ , where  $a$  is the radius of grains. For the limit  $\mathbf{v}=0$  it was found in [10,11].

In this work we consider the other case, namely anisotropic distributions  $f_{\alpha}$ , which is typical, in particular, for the region of a strong electric field in the discharge near the sheath, where the ion drift velocity can be high. For the case of slow motion of the grains ( $v_g \ll v_{\alpha}$ ,  $u$ , where  $v_g$ ,  $v_{\alpha}$ , and  $u$  are the typical grain, thermal and drift ion velocities, respectively) Eq. (1) can be simplified by use of Eq. (2) and expansion in powers of the ratio  $v / v'$  of the velocities:

$$F = n_g \sum_{\alpha} m_{\alpha} \int d\mathbf{v} \mathbf{v} |\mathbf{v}| \sigma_{g\alpha}(\mathbf{v}, q) f_{\alpha}(\mathbf{v}, \mathbf{u}). \quad (3)$$

Here  $n_g$  is the average density of scatterers and we added the drift velocity  $\mathbf{u}$  as an additional argument of  $f_{\alpha}(\mathbf{v})$  to stress the existence of regular flow of the light plasma components. It is necessary to mention, that in case of dusty plasmas with dominant charging collisions in Eqs. (2) and (3)  $\sigma_{g\alpha}(\mathbf{v}, q)$  is the cross-section for absorption only. To apply the results to real plasmas we have to generalize (3) to include scattering collisions with rather strong interactions between grains and scatterers. As was proposed in [10] on basis of physical reasons we summarize the differential cross-sections for absorption  $d\sigma^a_{g\alpha}$  and transport cross-section for scattering  $(1-\cos\chi) d\sigma^s_{g\alpha}$  in the formulae under consideration.

### 3. Scattering part of the ion drag: grain - grain and ion - grain correlations

Even in the liquid-like state the interaction between the highly charged grains is strong. It means that the effective cross-sections for scattering are different from that for isolated grains. A similar effect is important for liquid metals [17,18]. To estimate the influence of the grain-grain correlation we use for classical systems the method [18] and find for  $F_1$

$$F_1 = n_g \sum_{\alpha} m_{\alpha} \int d\mathbf{v} \mathbf{v} |\mathbf{v}| f_{\alpha}(\mathbf{v}, \mathbf{u}) \int (1 - \cos\chi) d\sigma^s_{g\alpha}(|\mathbf{v}|, \chi, q) S_{gg}(|\mathbf{v}|, \chi), \quad (4)$$

The value  $2m_{\alpha} |\mathbf{v}| \sin(\chi/2)$  is the momentum transfer. Usually integration by  $\chi$  is restricted due to screening of the  $\alpha$  - g interaction and due to the ion collecting process. If we take into account corrections for large values of scattering angles [19] and the DLVO structure [20] of the Debye potential, the limitations take the form:  $\lambda_{Li}/(\lambda_{Li} + a) > \sin(\chi/2) > \lambda_{Li}/(\lambda_{Li} + \lambda_D + a)$ , where  $\lambda_{Li}(v) = qe^2 / mv^2$  and  $\lambda_D$  is the Debye radius. As is easy to see the structure factor  $S_{gg}$  for these values of  $\chi$  can be very important for calculation of the integral, because the position of the first maximum  $Q_0$  in  $S_{gg}$  is usually situated in the region of integration of transferring wave vectors or even at the right side of this region. Here we consider analytically Eq. (4) for the parameters  $(\lambda_{Li} + a) \gg 1/Q_0$ , when  $S_{gg}(Q) \equiv S(0)$ . In general only numerical integration is possible. For the case under consideration introducing the dimensionless parameter  $s = 2(u/v_{Ti})^2$ , where  $v_{Ti} = (T_i/m_i)^{1/2}$  is the ion thermal velocity, we can rewrite  $f_1 = F_1 / [n_g n_i a^2 T_i S(0)]$  as:

$$f_1 = (4\pi^{1/2}/3) \Gamma^2 s^{1/2} \int_0^{\infty} d\xi \exp(-\xi) \ln \left\{ \frac{[\Gamma + 2\xi(1 + \lambda_D/a)]}{[\Gamma + 2\xi]} \right\}, \quad s \ll 1 \quad (5)$$

$$f_1 = (8\pi \Gamma^2 / s) \ln \left\{ \frac{(\lambda_D + \lambda_{Li}(u) + a)}{[\lambda_{Li}(u) + a]} \right\}, \quad s \gg 1 \quad (6)$$

The expressions for  $F_1$  includes the structure factor  $S(0)$ , which is small and the always positive Coulomb logarithm, in contrast with [10,19]. After integration Eq. (5) results in:

$$f_1 = (4\pi^{1/2}/3) \Gamma^2 s^{1/2} \left\{ \exp(\Gamma/2) \text{Ei}[-\Gamma/2] - \exp[a\Gamma/2(\lambda_D + a)] \text{Ei}[-a\Gamma/2(\lambda_D + a)] \right\}$$

For  $\Gamma/2 \gg 1$   $f_1 \cong 2\lambda_D t / a\Gamma$ , if  $a\Gamma/2(\lambda_D + a) \gg 1$  and  $f_1 \cong -t \{ \ln[a\Gamma/2(\lambda_D + a)] + 0,577 \}$ , if  $a\Gamma/2(\lambda_D + a) \ll 1$ , where  $t \equiv (4\pi^{1/2}/3) \Gamma^2 s^{1/2}$ .

The logarithmic approximation is not valid in cases, where the logarithm is small. It is possible for large grain charge, if  $\lambda_{Li}$  is one order of magnitude larger than  $\lambda_D$ , which is typical, for example, for the case of highly charged grains in dusty plasmas. It seems that in that case the exact cross-section for scattering in a Debye-like potential (DLVO) has to be used, instead of the Coulomb potential with Landau-type cutoff. Then larger impact parameters with weak scattering can play a role, because the region of strong scattering is very narrow. However this statement is not quite correct. The condition  $\lambda_{Li}(v_{Ti}) \approx \lambda_D$  as, is easy to see, really means, that for large  $q$  interaction between the ions and grains can be strong and linearization of the Poisson equation and DLVO approximation for the potential is not applicable. In that case the concept of the effective charge can be used [21,22]. Instead of the bare charge  $q$ , the considerably smaller effective charge determines in that case the effective potential, which can be again similar to the DLVO one. This problem is still not sufficiently clarified for dusty plasmas. In the case  $s \gg 1$  the approximation used above is good enough, because  $\lambda_{Li}(u) \ll \lambda_{Li}(v_{Ti})$  and the inequality  $\lambda_{Li}(u) \ll \lambda_D$  can be satisfied.

In the case considered analytically there is an essential decrease of the ion drag due to strong interaction between the grains. In general, as follows from Eq. (4), decrease or increase of the scattering ion drag depends on the actual parameters. If the region of integration of momentum transfer is close to the maximum of  $S(Q)$  an increase of  $F_1$  will take place.

#### 4. Collecting part of the ion drag force

The general expression (3) for the part  $f_2 = F_2 / n_g n_i a^2 T_i$  for the usual form of  $\sigma_{g\alpha}$  and arbitrary  $s$  can be calculated and gives:

$$f_2 = (\pi^{1/2} / 2 s^{3/2}) \{ [s^2 + 4s(1+\Gamma) - (8\Gamma+4)] (\pi s)^{1/2} \Phi(s) + [2s^2 + 4s(2\Gamma+1)] \exp(-s/4) \}, \quad (8)$$

where  $\Phi(s)$  is the error function. The asymptotic forms of  $f_2$  for small and large  $s$  are:

$$f_2 = (4\pi^{1/2}/3) s^{1/2} (2 + \Gamma), \quad s \ll 1; \quad f_2 = (\pi/2) [s + 4(1+\Gamma)], \quad s \gg 1, \quad (9)$$

For  $4(1+\Gamma) \gg s \gg 1$  the function  $f_2(s)$  has a plateau, but increases again for  $s \geq 4(1+\Gamma)$ .

The drag reactive force due to surface recombination and evaporation of the created atoms to the plasma is also calculated and can play an essential role, especially for surface temperatures higher than the ion temperature  $T_i$ .

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