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Ion Drag in Complex (Dusty) Plasmas

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Abstract

The problem of estimating the ion drag force in complex plasmas is considered. It is shown that the standard calculation of the ion-dust elastic scattering (orbital) cross-section often fails in complex plasmas. This is because the range of ion-grain interaction typically exceeds the Debye length whilst the standard approach uses the cut-off at the impact parameter equal to the Debye length. A new simple analytical approach to estimate the orbital cross-section is proposed. Our analytical results agree well with the available numerical results. The ion drag turns out to be significantly larger than previously estimated. It exceeds the electrostatic force in the limit of weak electric field for micron-sized grains. We suggest that this is the cause of the central "void" observed in microgravity complex plasma experiments.

The ion drag force, $F_{\rm I}$, consists of two parts often referred to as *collection* and *orbital* forces. They are associated with ion collection by the grain, and ion elastic scattering in the electric field of the grain, respectively. The calculation of $F_{\rm I}$ has been addressed recently in several works [1, 2]. Barnes *et al.* [1] modified the standard theory of pair collisions of charged particles in plasmas by taking into account the finite grain size and ion collection by the grain. A numerical calculation of the momentum-transfer cross section for elastic ion scattering was reported by Kilgore *et al.* [2] for a point-like grain, with the potential distribution derived from a self-consistent numerical solution of the Poisson-Vlasov equation. We show below that the analytical expression derived for the orbital part of the ion drag in [1] underestimates significantly the numerical results of [2] in a case of a bulk plasma (subthermal ion drift) for typical dust and plasma parameters.

In this work we propose a simple approach to improve the estimation of the ion drag force. We make the following assumptions:

- Small, $a \ll \lambda_{\rm D}$, isolated, $\Delta \gg \lambda_{\rm D}$, negatively charged spherical grain (a, Δ, and $\lambda_{\rm D}$ are the grain radius, intergrain separation, and plasma Debye length, respectively).
- Singly charged collisionless ions, $l_i \gg \lambda_{\rm D}$ (l_i is the ion mean free path).
- Weak electric field $E \ll T_i/el_i$ and subthermal ion drift, $u \ll v_{T_i}$ (u and $v_{T_i} = \sqrt{T_i/m}$ are the ion drift and thermal velocities, respectively).
- Shifted Maxvellian distribution function for ions $f(\mathbf{v}) = f_0(v)(1 + \mathbf{uv}/v_{T_i}^2)$.
- Screened Coulomb potential of interaction between ion and grain during collision.
- Ormital motion limited (OML) theory for grain charging is applicable.

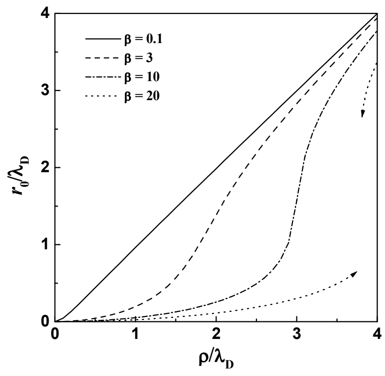

Figure 1: Normalized distance of the ion closest approach to the grain during collision, r_0, vs. the normalized impact parameter, ρ, for different values of β. The curves are calculated for a screened Coulomb potential. When β exceeds the critical value ($\beta \geq 13.2$) a discontinuity appears due to a potential barrier for the ions moving towards the grain. We do not consider this regime in the present work.

The general expression for the ion drag force is

$$\mathbf{F}_{\rm I} = m \int \mathbf{v} v f(\mathbf{v})[\sigma_{\rm c}(v) + \sigma_{\rm s}(v)] d\mathbf{v}, \tag{1}$$

where $\mathbf{v}$ is the ion velocity, m is the ion mass, and $\sigma_{\rm c}(v)$ and $\sigma_{\rm s}(v)$ are the (velocity dependent) momentum-transfer cross sections for the ion collection and scattering, respectively. They are given by

$$\sigma_{\rm c}(v) = \int_0^{\rho_{\rm c}(v)} 2\pi \rho d\rho, \qquad \sigma_{\rm s}(v) = \int_{\rho_{\rm c}(v)}^{\rho_{\rm max}} (1 - \cos\chi) 2\pi\rho d\rho, \tag{2}$$

where ρ is an impact parameter and $\chi(\rho)$ is the scattering angle. The impact parameter corresponding to ion collection is given by the OML theory: $\rho_{\rm c}(v) = a(1+2e|\phi_{\rm s}|/mv^2)^{1/2}$, where $|\phi_{\rm s}|$ is the grain surface potential. The collection cross-section is

$$\sigma_{\rm c}(v) = \pi a^2(1 + 2e|\phi_{\rm s}|/mv^2). \tag{3}$$

The estimation of the orbital cross-section is a main problem addressed in this work. For the screened Coulomb potential and small grain, the scattering of ions is characterized by one dimensionless parameter

$$\beta(v) \simeq Ze^2/mv^2\lambda_{\rm D}, \tag{4}$$

where Z is the absolute magnitude of grain charge. Physically β is the ratio of Coulomb radius to the Debye length. For subthermal ion drift the characteristic quantity is $\beta(v_{T_i})$. The standard approach – "Coulomb potential + cut-off at $\rho_{\rm max} = \lambda_{\rm D}$" – is valid only if $\beta(v) \ll 1$. This is because the ratio of momentum transfer due to ions with $\rho < \lambda_{\rm D}$ to the momentum transfer due to ions with $\rho > \lambda_{\rm D}$ is proportional to $\ln[1/\beta(v)]$ in this case. Therefore, to within logarithmic accuracy, it is sufficient to consider ions with impact parameters below $\lambda_{\rm D}$. For these ions the use of a bar Coulomb potential (instead of screened one) is a good approximation. For $\beta(v)$ of the order of unity or greater this approach fails: The range of ion-dust interaction can exceed the Debye length, the ions are scattered with large angles even if $\rho > \lambda_{\rm D}$. This is illustrated in Fig. 1.

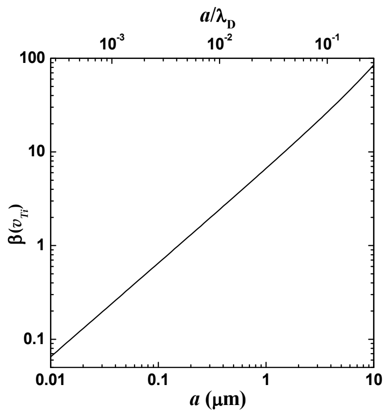

Figure 2: The parameter $\beta(v_{T_i})$ vs. grain radius, a. The curve is calculated for typical bulk plasma parameters: Ar gas, electron temperature $T_e = 2.5$ eV, electron to ion temperature ratio $\tau = 100$, electron concentration $n_e \simeq n_i = 10^9$ cm^{-3}

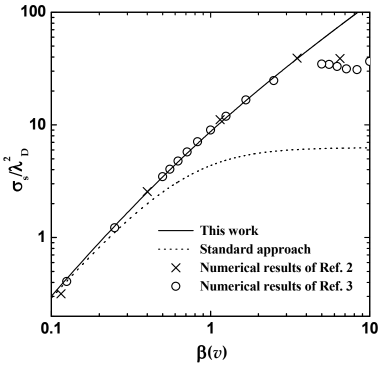

Figure 3: Orbital cross section of ion-grain collisions, $\sigma_{\rm s}$, normalized to the squared Debye length, $\lambda_{\rm D}^2$, vs. the parameter β for a point-like grain. Shown are results of the proposed approach (solid line), standard approach (dotted line), and numerical results of Refs. 2 (crosses), and Ref. 3 (circles).

The standard approach works well for electron-ion collisions in a usual plasma, where $\beta(v_{T_i}) \sim N_{\rm D}^{-1} \ll 1$ ($N_{\rm D}$ is a number of ions inside the Debye sphere). For ion-grain collisions in complex plasmas, $\beta(v_{T_i}) \simeq z\tau a/\lambda_{\rm D}$, where $z = Ze^2/aT_e$, $\tau = T_e/T_i$. For typical bulk plasma parameters and micron-sized grains $\beta(v_{T_i})$ is comparable or larger than unity as shown in Fig. 2. In this case the standard approach neglects a significant fraction of the ion momentum transfer (from ions with impact parameters above $\lambda_{\rm D}$).

To improve the evaluation of the orbital cross-section it is necessary to take into account ions with impact parameters above $\lambda_{\rm D}$. We propose the following simple procedure which is then justified by comparison with the available results of numerical calculations. To obtain analytical results we keep the approximation of the bar Coulomb potential, but the determination of $\rho_{\rm max}$ is revised. We take into account all the ions that *approach* the grain closer than $\lambda_{\rm D}$. The the determination of $\rho_{\rm max}$ is $r_0(\rho_{\rm max}) = \lambda_{\rm D}$. The orbital momentum transfer cross-section is then

$$\sigma_{\rm s}(v) = 4\pi\lambda_{\rm D}^2\beta^2(v)\Gamma, \quad \Gamma(v) = \ln\left[\frac{1+\beta(v)}{a/\lambda_{\rm D}+\beta(v)}\right], \tag{5}$$

where Γ is a *modified Coulomb logarithm.* For a point-like grain $\Gamma(v) = \ln[1 + 1/\beta(v)]$, whilst within the standard approach $\Gamma(v) = \frac{1}{2}\ln[1 + 1/\beta^2(v)]$ (both forms are equivalent for $\beta(v) \ll 1$). The proposed modification of the Coulomb logarithm is in good agreement with self-consistent simulations [2] and with numerical results for an attractive Coulomb screened potential [3] up to $\beta(v) \sim 5$, as shown in Fig. 3.

Substitution of (3) and (5) into (1) gives for the ion drag force

$$F_I = \frac{8\sqrt{2\pi}}{3} a^2 n_i m v_{T_i} u \left[1 + \frac{1}{2} z\tau + \frac{1}{4} z^2 \tau^2 \Lambda \right], \tag{6}$$

where Λ is the modified Coulomb logarithm integrated over $f(\mathbf{v})$. $\Lambda \simeq 2F[\beta(v_{T_i})/2]$, where $F(x) = \int_x^\infty \exp(x-t)/t dt$. In a special case $\rho_c(v_{T_i}) \simeq \lambda_D$ or $a/\lambda_D \sim 1/\sqrt{2z\tau}$ the present approach gives for F_I the result $\sim \sqrt{8z\tau} \sim 40$ times higher than that of Ref. 1 for plasma parameters of Fig. 2. This large difference is due to that the orbital part was neglected in [1] for $\rho_c > \lambda_D$, whilst it still dominate over the collection part, according to our results.

Next we compare the magnitudes of the electrostatic, F_E and the ion drag force, F_I. Assuming $u = \mu_i E$, with $\mu_i = e l_i v_{T_i}/T_i$ we obtain for their ratio $F_I/F_E \simeq \delta l_i/\lambda_D$, where $\delta = \frac{1}{3\sqrt{2\pi}}\beta(v_{T_i})\Lambda$. δ is a slowly increasing function of $\beta(v_{T_i})$, ranging from ~ 0.3 to ~ 0.5 for $1 < \beta(v_{T_i}) < 10$. Our results were derived for the "collisionless" limit, so that $l_i \gg \rho_{\max} \geq \lambda_D$. Hence, in the limit of weak electric fields the ion drag is stronger than the electrostatic force (as long as the proposed approach is applicable). This conclusion leads to a more physical insight into the mechanism of a "void" (dust-free region in the central part of a rf discharge) formation in complex plasma experiments under microgravity conditions. The electric field is weak in the center, and the ion drag (which is pointed outward) exceeds the electrostatic force (which is pointed to the center). The individual grains are pushed out of the center, leaving a void – as observed.

In conclusion, a simple procedure is proposed to improve the evaluation of the orbital part of the ion drag, which is justified by comparison with earlier (numerical) results. The ion drag force is significantly larger than previously expected. This might be important for understanding of the void formation, wave propagation, long-range interactions and other processes in complex plasmas.

References

[1] M.S. Barnes, *et al.*, Phys. Rev. Lett. **68**, 313 (1992).

[2] M.D. Kilgore *et al.*, J. Appl. Phys. **73**, 7195 (1993).

[3] H.-S. Hahn, E. A. Mason, and F. J. Smith, Phys. Fluids **14**, 278 (1971).