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Electromagnetically Induced Transparency of Magnetized Plasma

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Abstract

Magnetized plasma can be made completely transparent to electromagnetic radiation at the cyclotron frequency by adding an electromagnetic pump at the appropriate frequency. The pump can be in the form of a second radiation beam, or a magnetostatic field of a helical undulator. This *classical* effect is reminiscent of the well-known *quantum-mechanical* effect of electromagnetically-induced transparency (EIT) of the three-level atomic systems. Group and phase velocities of the resulting EM wave is controlled by the pump. Possible applications include electron/ion acceleration and energy compression in the plasma.

Electron cyclotron heating (ECR) is a well-known process by which the magnetized plasma is heated by an electromagnetic wave of the appropriate polarization and frequency $\omega_1 = \Omega_0$, where $\Omega_0 = eB_0/mc$ is the cyclotron frequency. The purpose of this work is to demonstrate that ECR can be prevented, and plasma made transparent at the cyclotron frequency by exploiting the effect of electromagnetically induced transparency (EIT). This new state of the plasma, which we call plasma with EIT (PEIT), enables several interesting applications: electromagnetic energy compression and particle acceleration. In the classical plasma EIT can be caused by either a strong EM wave [1], or a magnetostatic helical undulator [2]. The latter option is very attractive since it does not require a strong electromagnetic pump, and because the entire energy of the incident wave can be converted into a plasma wave and used for particle acceleration.

The mechanism of the classical EIT is the destructive interference between the electric field of the probe ($\vec{E}_{1\perp}$) and the sidebands of the electric ($\vec{E}_{0\perp}$) and magnetic ($\vec{B}_{0\perp}$) fields of the pump. The sidebands are detuned by the plasma frequency from the pump, and are produced by the collective electron plasma oscillation along the magnetic field. The total force at the cyclotron frequency experienced by a plasma electron is given by $\vec{F}_{\rm tot} \approx -e(\vec{E}_{1\perp} + \zeta_z \partial_z \vec{E}_{0\perp} + \dot{\zeta}_z \vec{e}_z \times \vec{B}_{0\perp})$, where ζ_z is the electron displacement in the plasma wave. With only the probe wave present, the resonant $\vec{E}_{1\perp}$ generates an induced plasma current that dominates the displacement current and prevents wave propagation. With both the pump and probe present, the net force can vanish ($\vec{F}_{\rm tot} \sim 0$). Consequently, the plasma current at the cyclotron frequency is small (or even vanishing), and the probe propagates almost as if in vacuum. Numerical simulation below demonstrates that this situation is naturally achieved in a collisionless plasma.

Consider an externally magnetized plasma with $\vec{B} = B_0\vec{e}_z$ and density n_0. We assume two right-hand polarized EM waves propagating along $z-$ direction, with their electric and magnetic fields given by $2e\vec{E}_{0\perp}/mc\omega_0 = a_{\rm pump}\vec{e}_+ \exp{(i\bar{\theta}_0)} + c.\ c.$, $2e\vec{E}_{1\perp}/mc\omega_1 = a_{\rm probe}\vec{e}_+ \exp{(i\bar{\theta}_1)} + c.\ c.$, and $\vec{B}_{0,1\perp} = (c\vec{k}_{0,1}/\omega_{0,1}) \times \vec{E}_{0,1\perp}$, where $\vec{e}_\pm = \vec{e}_x \pm i\vec{e}_y$, and $\bar{\theta}_{0,1} = k_{0,1}z - \omega_{0,1}t$. The non-relativistic equation of motion of a plasma electron in the combined fields is given by

$$\frac{d^2\vec{x}}{dt^2} + \Omega_0\vec{v} \times \vec{e}_z + \omega_p^2\zeta_z\vec{e}_z = -\frac{e}{m}\sum_{m=0,1}\vec{E}_{m\perp} + \frac{\vec{v} \times \vec{B}_{\perp m}}{c}, \tag{1}$$

where $\vec{x} \equiv (z_0 + \zeta_z)\vec{e}_z + \vec{x}_\perp$ and $\vec{v} = d\vec{x}/dt \equiv c\vec{\beta}$ are the particle position and velocity. The initial conditions are $\vec{v} = 0$ and $\vec{x} = z_0\vec{e}_z$. The third term on the LHS of Eq. (1) is the restoring force of the ions.

Equation (1) was integrated for the case when only a probe field is present and the case with both the pump and the probe. The pump and the probe amplitudes were taken as increasing adiabatically in time, up to their respective peak amplitudes of a_0 and a_1, according to

$$a_{\rm pump} = \frac{a_0}{2}\left(1 + \tanh\left[(\Omega_0 t - 160)/40\right]\right), \quad a_{\rm probe} = \frac{a_1}{2}\left(1 + \tanh\left[(\Omega_0 t - 320)/40\right]\right). \tag{2}$$

Simulation results for $\omega_p/\Omega_0 = 0.3$ ($\omega_0 = 0.7\Omega_0$) are shown in Fig. 1. Without the pump, an electron is resonantly driven by the probe as shown in Fig. 1(a). Adding a strong pump with $a_0 = 0.1$ and $k_0 \approx 0.83\Omega_0/c$ dramatically changes electron motion, as seen in Fig. 1(b). After the pump is turned on but before the turning on of the probe, an electron oscillates in the field of the pump according to $\beta_{x0} = \omega_0 a_{\rm pump}/(\omega_0 - \Omega_0)\sin{(k_0z_0 - \omega_0 t)}$. Switching on the probe does not significantly alter electron motion: $\beta_x - \beta_{x0}$ appears as a barely visible dashed line. Therefore, electron response at the cyclotron frequency is suppressed, resulting in the plasma transparency to the probe. This suppression of the response to the probe is caused by the excitation of a strong plasma oscillation [shown in Fig. 1(c)], which produces a sideband of the pump at the cyclotron frequency.

EIT can also be induced by a helical undulator field. In this case one requires $\omega_p = \Omega_0$. We simulated electron motion in the combined field of an undulator, with $a_0 = 0.1$ and $k_0 = 2\Omega_0/c$, and a probe, switched on according to $a_{\rm probe} = 0.5a_1\left(1 + \tanh\left[(\Omega_0 t - 270)/60\right]\right)$, where $a_1 = 0.01$. Suppression of the electron response at the cyclotron frequency is apparent from Fig. 2(a). The force due to the electric field of the probe is canceled by the $(\dot{\zeta}_z/c)\vec{e}_z \times \vec{B}_{0\perp}$ force which is exerted on a longitudinal plasma wave by the helical magnetic field of the undulator. The plasma wave in this example can be used for acceleration of relativistic electrons because its phase velocity is $v_{ph} = (\omega_1 - \omega_0)/(k_1 - k_0) = -c$.

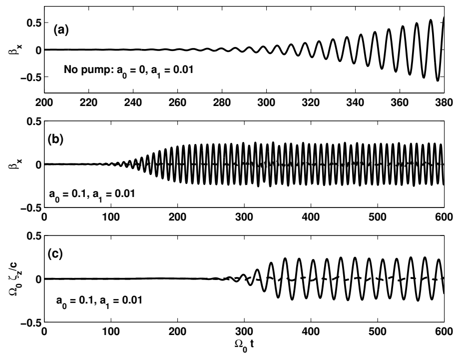

Figure 1: Numerical simulation of single particle motion in the combined field of two EM waves with ($\omega_1 = \Omega_0$,$k_1 = \omega_1/c$) and ($\omega_0 = \Omega_0 - \omega_p$, $k_0 \approx 0.83\Omega_0/c$). Both the pump and the probe are slowly turned on according to Eq. (2). (a) Without the pump an electron is resonantly driven by probe: β_x growth indefinitely; (b) With the pump, electron motion is almost unaffected by the probe. Solid line – total β_x; dashed line – $(\beta_x - \beta_{x0})$, where $\beta_{x0} = \omega_0 a_{\rm pump}/(\omega_0 - \Omega_0)\sin(k_0 z_0 - \omega_0 t)$ is the analytic result when only the pump is present. (c) Solid line: the longitudinal displacement $\Omega_0\zeta_z/c$; dashed line: $\Omega_0(\zeta_z - \zeta_0)/c$, where $\zeta_0 = 2a_{\rm pump}/a_{\rm probe}\sin\omega_p t$

The dispersion relation for the probe in the PEIT can be shown to be [2]:

$$\omega_1^2 = c^2k_1^2 - \omega_p^2\omega_1 \frac{\delta\Omega + \delta\Omega_0(k_1)}{\Omega_R^2 - (\delta\Omega)^2}, \quad (3)$$

where $\delta\Omega = \omega_1 - \Omega_0$, $\Omega_R = ck_0a_0(\Omega_0/4\omega_p)^{1/2}$ is the effective Rabi frequency, and $\delta\Omega_0(k_1) = (2\Omega_R^2\omega_0/\omega_p\Omega_0)(k_1/k_0 - 1)$. The pump field shifts plasma resonances from $\omega_1 = \Omega_0$ to $\omega_1 = \Omega_0 \pm \Omega_R$. Also, the refraction index is equal to one for $\omega_1 = \Omega_0$ when the pump is an undulator.

The PEIT dispersion relation is plotted in Fig. (3) for the same plasma parameters as in Fig. 1 and a co-propagating pump with $\Omega_R = 0.5\omega_p$. The flat band between the $\Omega_0 \pm \Omega_R$ resonant frequencies is the classical analog of the "slow light" in atomic systems. The corresponding group velocity $v_g = \partial\omega_1/\partial k_1 \approx 2c\Omega_R^2/\omega_p^2$ can also be made very small.

Slowing down of electromagnetic waves in the PEIT can be understood by considering the entrance of a probe beam of duration L_0 into the plasma. In the plasma, the "slow light" of length L_f consists of the transversely polarized field of the probe $|\vec{E}_1| = |\vec{B}_1| = a_1mc\omega_1/e$ and the longitudinal electric field of the plasma wave $E_z = 4\pi en_0(2a_1/k_0a_0)$. As the pulse enters the plasma, it loses photons to the pump at the same rate as new plasmons are created (according to the Manley-Rowe relations). The classical photon density of a field with frequency ω is proportional to the action density $\propto U/\omega$, where U is the energy density. From this one can calculate that

$$\frac{U_{\rm plas}/\omega_p}{U_{\rm phot}/\omega_1} = \frac{\Omega_0}{\omega_p}\frac{E_z^2}{2E_1^2} = \frac{\omega_p^2}{2\Omega_R^2} \gg 1 \quad \text{for} \quad \Omega_R \ll \omega_p. \quad (4)$$

Thus, most of the original pulse photons are transferred to the plasma wave. Since $ck_1 = \omega_1$, the photon energy does not change. Therefore, the loss of photons is due

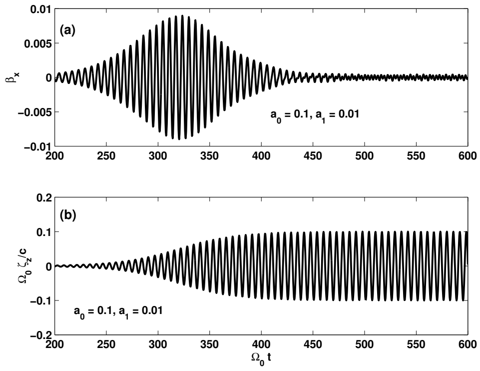

Figure 2: The same parameters as Fig. 2, except $\omega_p = \Omega_0$, $\omega_0 = 0$, $k_0 = 2\Omega_0/c$ (the static helical undulator is on continuously). (a) The transverse velocity β_x and (b) the longitudinal displacement $\Omega_0\zeta_z/c$ during and after the turn-on of the probe.

to the spatial shortening of the pulse from L_0 to $L_f = L_0 \times (2\Omega_R^2/\omega_p^2)$. Because temporal pulse duration does not change, we recover the previously calculated $v_g/c = 2\Omega_R^2/\omega_p^2$. In the case of a static undulator all the energy is transfered to the plasma and compressed by a factor v_g/c, resulting in a dramatic increase of the energy density.

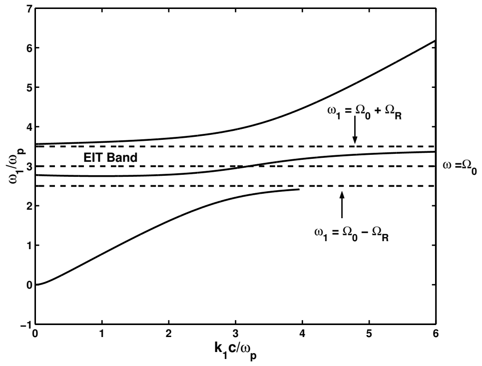

Figure 3: EIT dispersion curve, $\Omega_0/\omega_p = 3$ and $\Omega_R/\omega_p = 1/2$. Flat band above $\Omega_0 - \Omega_R$ up to $\Omega_0 + \Omega_R$ labelled "EIT Band" corresponds to "slow light" in the presence of a pump.

One interesting application of EIT in magnetized plasma is ion acceleration. The PEIT enables a short-pulse ion accelerator consisting of a "slow light" pulse in the plasma with approximately equal group and phase velocities. Acceleration is accomplished by the longitudinal electric field of the plasma wave. Counter-propagating geometry is chosen to match the phase and group velocities because $v_{\rm ph} = \omega_p/(|k_0| + k_1) \approx 0.5c\omega_p/\Omega_0$. Matching $v_{\rm ph} = v_g$ yields $a_0 \approx \omega_p^2/\Omega_0^2 \ll 1$.

1. A. G. Litvak and M. D. Tokman, Phys. Rev. Lett. **88,** 095003 (2002).
2. G. Shvets and J. S. Wurtele, Phys. Rev. Lett., in press (2002).