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Intrinsic Transport Barriers in Ergodic Divertor Plasmas

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Introduction:

Normal operation of an ergodic divertor(ED) tokamak produces a transport barrier that is evident in the electron temperature profile and affects mhd stability. The barrier occurs just within the region of enhanced transport ("Chirikov parameter">1; ie. where islands overlap) produced by the ED. The barrier is apparent from a steep temperature gradient. A striking feature is that it restores the interior temperature profile to that obtained without the ED. This paper demonstrates that this behaviour results from an inward pinch, generated by the steep gradient of the ED fields towards the divertor coils. Figure 1 shows Tore-Supra[1] and Text[2] data illustrating this behaviour. Analysis of the mhd stability of Tore-Supra plasmas[3] shows that a barrier must be present whenever the ED coils are energised.

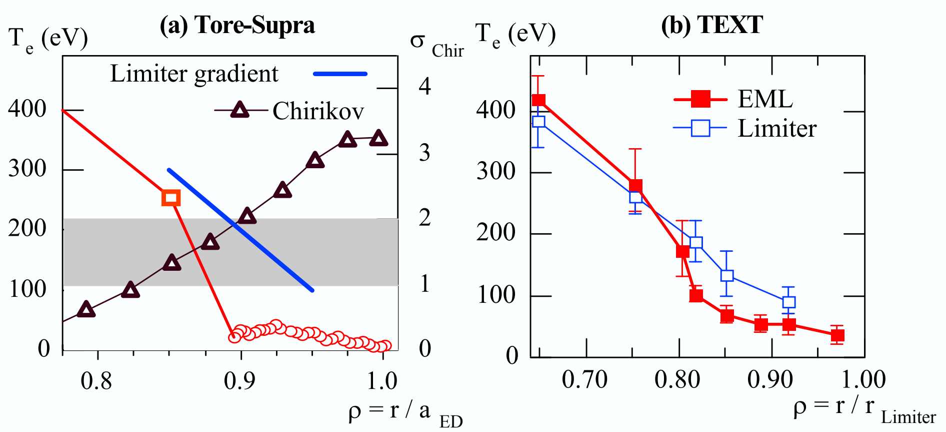

Figure1: *Electron temperature profiles from (a) Tore-Supra and (b) TEXT (EML≡ED). In both cases the blue curve corresponds to the ED off and the red to the ED on.*

If normal, anomalous transport is undisturbed by the ergodisation and is represented by conductivity κ_A, then the experimental behaviour is reproduced if the ED causes conduction, represented by κ_{ED}, and an inward pinch $v = -d\kappa_{ED}/dr$. Some trivial algebra shows that the resulting temperature, in slab geometry, is given by:

$$T(x) \quad = \quad S \,.\, (1-x) / (\kappa_A + \kappa_{ED}(x)) \qquad (1),$$

where S is the heat flux. This solution shows that if κ_{ED} is a rapidly growing function of x, as one would expect for multipole fields, then T would be unchanged at small x, would be eroded away where $\kappa_{ED} >> \kappa_A$ and would have a "barrier" in between. (1) still holds for κ_A being a function of x, as long as it also obeys the same relationship, with a corresponding pinch, as κ_{ED}. Notice that the barrier is caused by the pinch rather than a reduction in κ.

The next section will outline the functioning of the ergodic divertor coils and will introduce a simple model to describe their fields. This model is analysed by an expansion in the strength of the ergodic fields. At second order, the model satisfies v=dD/dr, as required to restore the interior profile. The following section moves to a discussion of physically more realistic models and shows that the divergence free property of magnetic fields naturally produces a barrier of the kind observed experimentally. In the final section, the main results are reiterated and the implications for future applications are highlighted.

A Simple Model for Ergodic Divertor Fields:

The Tore-Supra had six ED coils installed on the interior, low field side of the vacuum vessel. Each coil consisted of a meander of seven conductors, pitched at the angle of the tokamak magnetic fields and joined by vertical connections. The current feeds entered from the top and bottom of the coils. The magnetic flux was drawn out between the conductors, onto pumped neutralisers. Drawings of the ED coils can be found in references [1] and [3]. Each coil subtended a toroidal angle of $\sim\pi/13$ and an effective poloidal angle of $\pi/3$. The resulting magnetic spectrum peaked around m=18 and n=6, so resonating with q=3.

Since the ED fields fell off rapidly in both radial and toroidal directions, it is convenient to simplify modelling by taking cylindrical geometry and to replace the magnetic field by a radial kick from each passage in front of a coil that is given by:

$$r_{n+1} - r_n = \xi_E . r_n^{m1} . \cos(m\theta_n) . H(\theta_{max} - |\theta_n|) \quad (2),$$

where ξ is the single pass displacement at the coil face, the exponent m1 has a value of about 10, m is 18, H is the Heavyside function and θ_{max} is $\pi/6$. Connection length maps and Poincaré plots obtained with (2), together with $\delta\theta = \delta z / q(r)$ to represent the tokamak fields ($q(r) = q_0 + (q_a - q_0) . r^2$ was used) , show good accord with those obtained with the field line following code MASTOC[4].

For modelling purposes, the heat conduction equation can be substituted by particle diffusion, since the form of equations is identical. Thus the effect of the ED "field", given by (2), on the temperature profile can be studied by following an ensemble of test particles over a range in minor radius $r_{min} \leq r \leq 1.0$, with constant toroidal velocity and anomalous transport represented by a random walk of step size ξ_A. Particles leaving the domain in r are recycled to $r_{min}+\xi_A$. This model can be simulated using the Monte Carlo method and is amenable to analysis. The analysis assumes that the kicks imparted by the ED are uncorrelated. This assumption is not only incorrect but has a significant effect on some of the results, as we will see shortly. Expanding to second order in ξ_E and ξ_A and using slab geometry for simplicity, the density(temperature) is given by:

$$n(r) = \Gamma . (1-r) / \{\xi_A^2/2 + [(\xi_E^2/4) . r^{2.m1} . (\theta_{max}/\pi)]\} \quad (3),$$

where Γ is the particle (heat) flux. Equation 3 (green lines) is compared with Monte-Carlo simulations (blue points) for three values of ξ_E/ξ_A (=10, 20 & 40) in figure 2. Curves are also shown for the corresponding analytic case with $\xi_E = 0$ (red lines) so that the scale of the effect of the ED fields can be gauged.

Figure 2 shows that the analytic result (3) is reasonably good for ξ_E/ξ_A up to 20 or so. At low values, whilst there is a clear barrier, that of the Monte Carlo is not so strong as the barrier from (3) because of the strong correlation between adjacent steps where q is around 3. The diffusion induced by the ED, close to r = 1, is much stronger due to these correlations than from random steps and the relationship with the pinch velocity is broken. As a result, the interior profile is not recovered, as can be seen in the figure.

For values of above 20, the agreement between (3) and the Monte Carlo simulations breaks down completely. This comes about because of strong anti-correlations between steps for $0.65 < r < 0.75$, where q is around 2. It can be shown, by dropping terms of order 1/m, that:

$$\langle\delta r_{n+j}\delta r_n\rangle \approx \langle\delta r_n\delta r_n\rangle * \cos(j.m.\ \pi/(3.q)) . (1 - j.\ \pi/(6 . q . \theta_{max})) \quad (4)$$

as long as the last term in brackets is positive. Thus, one step is strongly anti-correlated with the next(about -50%) when q is around 2 for the Tore-Supra case. This anti-correlation has the opposite effect to a correlation; reducing the pinch velocity slightly whilst strongly reducing the diffusion. The reduced diffusion greatly increases the timescale for the Monte-Carlo simulation and makes high statistics, well converged cases difficult to obtain

The assertion that (anti-)correlations are the main cause of disagreement between (3) and the Monte-Carlo simulations has been tested by randomising the poloidal angle at each passage past a divertor coil. Good agreement is then restored with (3), as shown in figure 2d. This is also the case at small ξ_E/ξ_A.

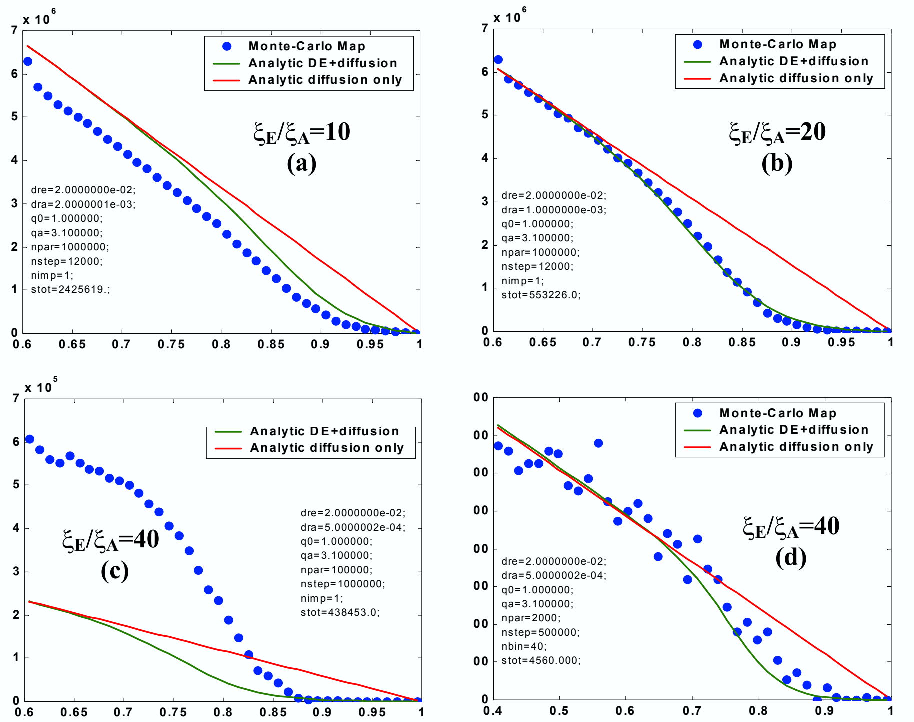

Figure 2: *Comparisons between Monte-Carlo calculation (blue dots), analytic form with ED pinch(green) and analytic form without the pinch(red) for ξ_E/ξ_A=10 (a), 20 (b) and 40 (c and d). The latter has all toroidal correlations switched off.*

It is pertinent to ask why experiment only produces profiles like that of the model for $\xi_E/\xi_A = 20$: It could be coincidence, or it could be that toroidal decorrelation occurs, due to turbulence, or that experiments avoid profiles like fig. 2(c) because the resulting current gradient at q=2 would produce a disruption.

Real Ergodic Fields:

An important defect in the model for the ED fields, given by (2), is that it is not time reversal invariant. A particle following a real magnetic field line will find itself back where it started, if time is reversed, whereas a particle in the model system above will not. This can be remedied by replacing equation 2 by:

$$r_{n+1} - r_n = \xi_E/2.\{r_n^{ml}.\cos(m\theta_n).H(\theta_{max}-|\theta_n|) + r_{n+1}^{ml}.\cos(m\theta_{n+1}).H(\theta_{max}-|\theta_{n+1}|)\} \qquad (5)$$

By itself, (5) results in v = (1/2).dD/dr, as is readily shown by analysis or modelling. This comes about because half the step length is determined by the particle's destination. Hence this contribution to the flux is symmetric around any reference surface and can only produce diffusion. In contrast, the other part depends on the coordinates of the particle's origin and so can produce a pinch through the radial dependence of the step length, as done by (2).

A poloidal magnetic field, such as that of the ED, obeys the relation $\underline{B}_p = \underline{n}_z \wedge \underline{\nabla}\phi$, where ϕ is the poloidal flux function and $\underline{n}_z$ is the unit vector in the toroidal direction. The displacement vector $\delta\underline{x}$, undergone by a particle passing an ED coil must obey the same relationship as B_p. Thus the poloidal displacement vector $\delta\underline{x}$ is given by:

$$\underline{x}_{n+1} - \underline{x}_n = \underline{n}_z \wedge(\underline{\nabla}\phi(\underline{x}_{n+1}) + \underline{\nabla}\phi(\underline{x}_n))/2 \qquad (6)$$

When this form for the single step displacement is used to calculate transport coefficients, it can be shown that these are related by v = dD/dr. The poloidally averaged contributions to the drift velocity come half from the radial variation and half from the poloidal movement, which concentrates particles towards the maximum radial movement. This sharing between radial and poloidal motion is what makes (6) an area(flux) conserving map as well. Presumably, the area conservation is connected with the transport property in some profound way that, as yet, has not revealed itself to the authors.

The treatment of an expression for a realistic representation of an ED is much more complicated than (6). The displacement becomes an integral over the trajectory produced by all the current elements and the tokamak fields:

$$\underline{x}(z_{n+1}) - \underline{x}(z_n) = \Sigma_{\text{conductors},i} \int d\underline{z}' \wedge \underline{\nabla}\phi_i(\underline{x}(z')) \qquad (7),$$

where $\phi_i(\underline{x}(z')) = S_i / |\underline{x}(z') - \underline{x}_i|$ is the flux due to a conductor element with strength S_i and located at $\underline{x}_i$. Thus far, the use of (7) in a Monte-Carlo simulation has proven to be computationally prohibitive. The integral must be solved accurately, otherwise spurious pinch velocities are obtained. An attempt to tabulate values for (7) and interpolate the displacements ran into the same problem of accuracy versus speed. Expansion of (7) in terms of the conductor strengths yields (6), in the limit that the integral is estimated using the trapezoidal rule. It has also been verified that for single passes of a realistic system of conductors that $2.\langle\delta r\rangle_\theta = d/dr \langle\delta r^2\rangle_\theta$, as would be required to give v = dD/dr.

Conclusions and Discussion:

The paper shows that the transport barrier produced in ED plasmas requires a particular relationship between pinch velocity and diffusion coefficient, v = dD/dr, and that this is obeyed by the field lines themselves in the limit that toroidal correlations between ED coils are neglected. In this sense it is proposed that the transport barrier is intrinsic to the ED. It has been shown that correlations and anti-correlations between successive passages in front of divertor coils are significant in the limit of weak anomalous transport, relative to the ED transport. If this occurred experimentally, enhanced core temperatures would be obtained. Whether this is not observed because this limit cannot be accessed experimentally or because some other agent, such as poloidal diffusion, limits the toroidal correlations is not clear and will be the subject of future investigation.

It is noteworthy that the condition for the ED-type of barrier requires that the anomalous transport also obey v=dD/dr. In so far as the region between the centres of turbulent vortices is source free, this might arise because the drift in the turbulent electric field has the same form as poloidal magnetic fields; ie. $\underline{v}_p = \underline{n}_z \wedge \underline{\nabla}\phi/B$, where ϕ is the electric flux.. In this case, all the above discussion will pertain for the anomalous transport.

The intrinsic transport barriers are a completely general property of applied ergodic fields where there is a strong gradient in the perturbing fields. Thus transport barriers should be observed, particularly with the Textor DED[5] and perhaps in ergodic region at the separatrix of stellerators[6], depending on the field gradients involved. The application of ergodic fields to control ELMs[7] might be affected because reduction of the pressure gradient in one place, due to induced transport, might lead to an increase at the intrinsic barrier. At very least, an experimental test will be made more interesting!

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