

A NEW FAST AND ACCURATE KINETIC CODE FOR THE STUDY
OF
THE LOW FREQUENCY DYNAMICS IN A MAGNETISED
PLASMA, DRIVEN BY AN EXTERNAL ELECTROSTATIC PUMP

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Abstract

This paper deals with the upgrade of a Vlasov-Maxwell kinetic code, which, although able to describe with high precision the electron and the ion dynamics in the (x, v_x, v_y) phase space, turns out to be exceedingly demanding in terms of run times. In the modified code, which is described here, the ions are treated kinetically, the electrons are attributed their $\mathbf{E} \times \mathbf{B}$ drift velocity, and the Maxwell equations for the electromagnetic components of the self-consistent fields are eliminated. The new code is faster up to a factor 100 and the results are accurately comparable to the ones of the fully kinetic program. It is therefore possible to use our model to investigate the interaction of electrostatic waves (specifically, ion Bernstein waves) with a magnetised plasma, considering actual experimental parameters.

The problem

With the purpose of describing the generation by ion Bernstein waves (iBw) of a sheared poloidal ion flow, suitable for improving the energy confinement in a tokamak plasma, we investigate the interaction of an externally pumped, low frequency, purely electrostatic wave of finite amplitude, propagating normally to the ambient uniform magnetic field, with a collisionless plasma. The geometry is a uniform slab with the magnetic field in the z direction and the wave vector and the wave electric field in the x direction; the expected ion flow results in the y direction.

In order to normalize the equations to be solved, the following dimensionless variables have been introduced: $\omega_{pi}t \rightarrow t$, $\mathbf{v}/c \rightarrow \mathbf{v}$, $\omega_{pi}x/c \rightarrow x$, $f_a c/n_{0a} \rightarrow f_a$, $eE(B)/m_i c \omega_{pi} \rightarrow E(B)$, $\Lambda_a = -\text{sign}[q_a]m_i/m_a$. $E_{\text{dr}}(x,t) = a \sin(\omega_0 t - k_0 x)$ is the externally applied es wave, $\omega_0 \approx 4\omega_{\chi i}$, $\omega_{\chi i}$ the ion cyclotron frequency. Moreover, q_a , m_a , T_a are the electric charge, the mass and the temperature of the a -species ($a = e, i$), respectively; c

is the speed of light, e is the modulus of the electron charge, $\beta_a = v_{ta}^2/c^2$, $v_{ta} = (2T_a/m_a)^{1/2}$. Thus we obtain:

$$\frac{\partial f_a}{\partial t} + v_x \frac{\partial f_a}{\partial x} - \Lambda_a \left\{ [E_x(x,t) + E_{dr}(x,t) + B_z v_y] \frac{\partial f_a}{\partial v_x} + [E_y - B_z v_x] \frac{\partial f_a}{\partial v_y} \right\} = 0 \quad (1.a,b)$$

$$\frac{\partial E_x}{\partial x} = \iint dv_x dv_y f_i(x, v_x, v_y, t) - \iint dv_x dv_y f_e(x, v_x, v_y, t) \quad \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \quad (2.a,b)$$

$$\frac{\partial B_z}{\partial x} = \frac{\partial E_y}{\partial t} - \iint dv_x dv_y v_y f_i(x, v_x, v_y, t) + \iint dv_x dv_y v_x f_e(x, v_x, v_y, t) \quad (3)$$

The theoretical study of such a problem would benefit of a kinetic code able to describe, with high resolution in (x, v_x, v_y, t) space, the evolution of the ion distribution function for times long enough to achieve a stationary state [1]. At the same time, the integration time step has to be kept small to appropriately follow the coupled fast electron dynamics. The code in use up to now (VM code [1]) is very accurate but it needs long run times, despite the use of a reduced ion-to-electron mass ratio ($\Lambda_i = -1$, $\Lambda_e = 50$) to artificially decrease the time scale separation [2] between the ion and the electron dynamics. This choice, however, requires a rescaling of the normalised quantities, in order to maintain the characteristics of the wave-plasma interaction, thus loosing the matching with the parameter values, for instance, of IBW-FTU experiment [3].

A new model allowing a consistent speed-up is needed, this paper shows the analytical and numerical justification for the new approximations and the comparison between the result from the new fast code and the original fully kinetic one.

Averaging over the electron Larmor gyration

In the limit of massless electrons ($\Lambda_e \rightarrow \infty$), and considering that the VM code shows that the electron distribution function remains very close to a Maxwellian during the interaction, we get the equation $V_{ex}(E_x + E_{dr}) + V_{ey} E_y = 0$ linking the x and y components of the electron fluid velocity. By assuming, on physical ground, that the electron motion in the y direction follows the drift motion in the presence of a slowly varying electric field E_x , that is $V_{ey} \approx v_{ed} = -E_x/B_z$, we get $V_{ex} \approx E_y/B_z$. Finally, since Eq.(2b) shows that E_y remains always at least two orders of magnitude smaller than E_x , V_{ex} is taken strictly equal to zero. In Fig.1 the electron fluid velocity V_{ey} (solid line) is displayed as a function of time in a generic position of the simulation box,

with superimposed the normalized drift velocity $V_{ey} = v_{ed}$ (dashed lines), as they result from the VM code. It can be seen that, since on the typical electron time scale, the applied field oscillates very slowly, it is a good approximation to consider the electrons as executing slow drift oscillations in the y direction, superimposed to their fast Larmor rotation. In addition, the electron plasma density (not shown) regularly oscillates at $\omega = \omega_0$ in time and $k = k_0$ in space, with oscillations of $\pm 5\%$ around its unperturbed value. As this behaviour is quite general, one can avoid the integration of the Vlasov equation for electrons, Eq.(1b), by substituting their fluid velocity in Eq.(3) with the corresponding drift velocity in the y direction, and their density in Eq.(2a) with its unperturbed value (LFD code). A comparison between the VM and the LFD codes shows an excellent agreement on the ion dynamics, and a speed-up factor of the order of 10 is achieved.

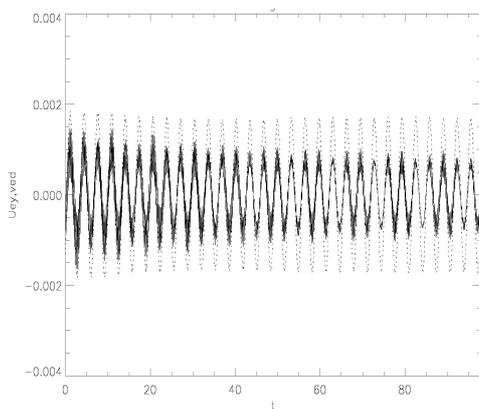


Fig1

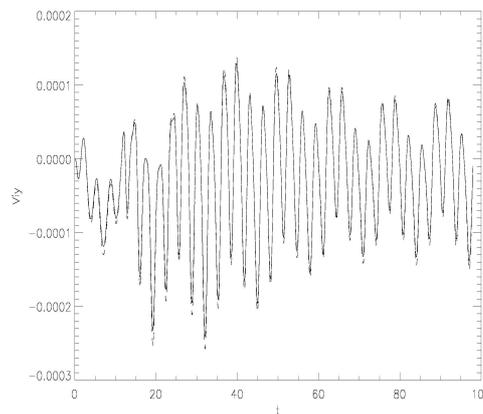


Fig2

The electrostatic approximation

A next step in simplifying the code can be done by exploiting the smallness of the electromagnetic components of the self-consistent fields if compared with the electrostatic ones. The outputs of both VM and LFD codes show that the $|E_y| \ll |E_x|$, and that the fluctuations of B_z are completely negligible, all along the temporal evolution. Then, we put $E_y = 0$ and $B_z = \text{const}$. The plasma response to the action of a purely electrostatic pump wave can be considered electrostatic to a large extent. The integration of Eqs.(2b,3) can therefore be eliminated (ES code), thus obtaining another speed-up factor of the order of 10, while the ion fluid dynamics as well as the ion distribution function evolution remain in good agreement with those obtained with the VM code. Fig.2 displays the ion fluid velocity V_{iy} as a function of time in a generic position of the simulation box. The solid line represents the output of VM

code, while the dashed one was obtained running the ES code with the same parameters. The matching is excellent, for all the time extent of the simulations.

First results with full ion-to-electron mass ratio

It is now possible, via the ES code, to study the IBW-plasma interaction with the actual ion-to-electron mass ratio ($\Lambda_i = -1$, $\Lambda_e = 1836$) and with the magnetic field and the frequency of the IBW-FTU experiment [3] (the wave amplitude corresponds to an electric field of 100 kV/cm). In Fig.3 the ion fluid velocity in y direction, averaged on the simulation box to avoid local effects, and in Fig.4 a contour plot (v_x, v_y) of the ion distribution functions are displayed for the early integration times of a preliminary run. They show that a flow, that is an ion fluid velocity oscillating with non zero (negative) average value, is likely to be produced in the continuing of the simulation and, most important, that the mechanism of wave-particle interaction is a Cherenkov resonance, despite the presence of a strong magnetic field, in the FTU case as we found in the reduced mass case [4].

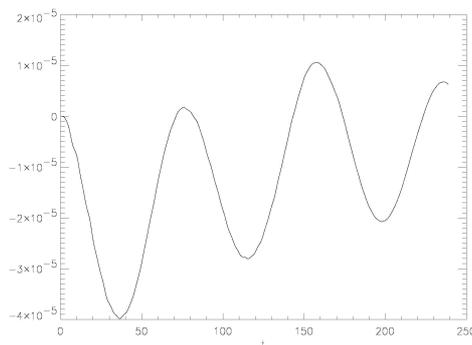


fig3

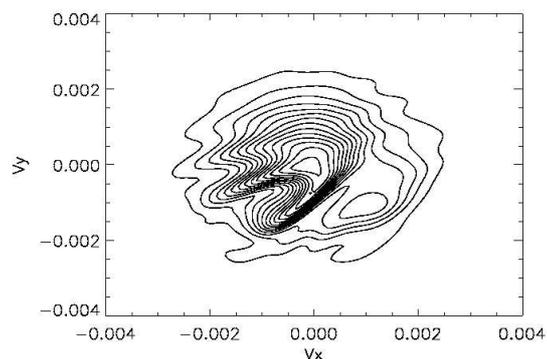


fig4

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