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New Algorithm for Time-Domain Simulation of Neo-classical Transport

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In this paper, we present a new efficient algorithm for time-domain simulation of neoclassical transport in small tokamaks. The algorithm is based on a two-dimensional axisymmetric variational finite-elements equilibrium solver and finite-difference solution of one-dimensional transport equations. We discuss a new scheme for easily solving circuit equations without referring to the mutual inductance matrix of the poloidal coils. As well, we solve the Ampere law instead of Grad-Shafranov equation. This scheme results in great simplification of the problem and elimination of the need for knowledge of plasma transport. Therefore, the equilibrium and transport equations become decoupled, and the inside and outside plasma regions can be treated at once. In this case, the plasma boundary conditions are automatically satisfied. The variational finite-element code has been optimized so that each time step takes a few seconds on a simple Windows based PC, with a code developed on MATLAB. We have considered our small tokamak, Damavand, and tested the applicability of our algorithm.

1. Introduction

The self-consistent simulation of plasma equilibrium and transport is usually done, by solution of the nonlinear Grad-Shafranov equation

$$\Delta^*\Psi = -r^2\mu_0 \frac{dp}{d\Psi} - I\frac{dI}{d\Psi}, \tag{1}$$

inside the plasma in which $\Delta^* = r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2}$ is the Grad-Shafranov operator, p is the plasma pressure, $I=rB_t$ is toroidal field function, and Ψ is poloidal flux. The poloidal flux outside plasma can be found by solution of linear axisymmetric Ampere law

$$\frac{1}{r}\Delta^*\Psi = -\mu_0 J_t, \tag{2}$$

using any magnetostatic code, where J_t is the toroidal current density. Please notice that (2) is mostly referred to as the Grad-Shafranov equation, however, in order to distinguish between (1) and (2), we shall use different nomenclatures. After simultaneous solution of (1) and (2) and matching them across the plasma boundary, one has to solve the set of transport equations for plasma density n, electron and ion temperatures T_e and T_i. In this case, the poloidal field B_p is found by time-domain integration of radial derivative of toroidal electric field E_t. As well, J_t is found by taking divergence of B_p. These two latter derivations are approximate and destroy conservation of magnetic flux. Also, there remains problem of determination of E_t inside plasma.

Various methods have been developed for matching (1) and (2) over plasma boundaries. However, none of them is as simple and efficient as the method we present here.

In this paper, we use (2) instead of (1) inside plasma with the toroidal current density function J_t including the plasma toroidal current density. The dependences of p and I on Ψ incorporate through proper building of J_t distribution as described. Therefore, one can elegantly compute the toroidal electrical field E_t, induced by poloidal coils and hence the solution of circuit equations would be greatly simplified, as well as preservation of magnetic flux.

2. Description of Algorithm

The flowchart of our algorithm is illustrated in Fig. 1. The simulation starts by initialization of variables and prescribing start profile values for plasma density, electron and ion temperatures.

The next step is to solve the circuit equations. In general, the toroidal electric field over the entire two-dimensional solution region should be already known from previous time step, however, initially it may take on a start value as well. The jth non-feedback poloidal coil discharges its capacitor bank with capacitance of C_j, which is assumed to be initially charged at a voltage of V_j, through a series resistance R_j and an inductance L_j, after a tunable delay of τ_i. The voltage U_j induced in L_j by other poloidal coils and itself, depends on the inductance matrix and knowledge of current variations in other coils. However, we can avoid complication of the mutual couplings by simply using the equation $U_j=2\pi r E_t$ at the position of the coil. The current flowing in the coil I_j, then can be calculated simply by integration of toroidal current density σE_t over the coil cross section area, where σ is the copper conductance.

The next step is to determine the two-dimensional J_t function over the inside and outside plasma region. J_t for outside plasma is determined by the poloidal coils, and therefore known. However, inside the plasma a special care should be made. Since the plasma profiles of n and T_e are already known in one-dimension, the plasma conductance is also known. If we are at the first step, there is a known initial plasma configuration and therefore the plasma current distribution is readily known. But for the successive time steps the situation is rather different. We here suppose that the entire plasma is allowed to exist in a two-dimensional window determined by the limiters and the vacuum vessel. Now one can notice that J_t is indeed a function of poloidal flux Ψ, known from the last equilibrium solution, and thus having the plasma profiles known on the one-dimensional axis z=0, the entire plasma window can be filled out with plasma by comparing the values of Ψ at each point to the values of Ψ at the z=0 axis. Subsequently, the associated toroidal current density J_t within plasma would be known.

In the next step, we store the last known poloidal flux Ψ somewhere, being required later for calculation of toroidal electric field, and proceed to solution of equilibrium. Since we are treating all the solution region at once, the inside and outside plasma regions are solved simultaneously and proper boundary conditions are automatically satisfied. In this case, we no more need artifical mixed plasma boundary conditions. Although there are many numerical methods for solution of the two-dimensional equation (2), we propose to employ our recently developed variational finite elements approach [1] for solution of magnetostatics problems. In our approach, (2) is solved by extremizing the functional

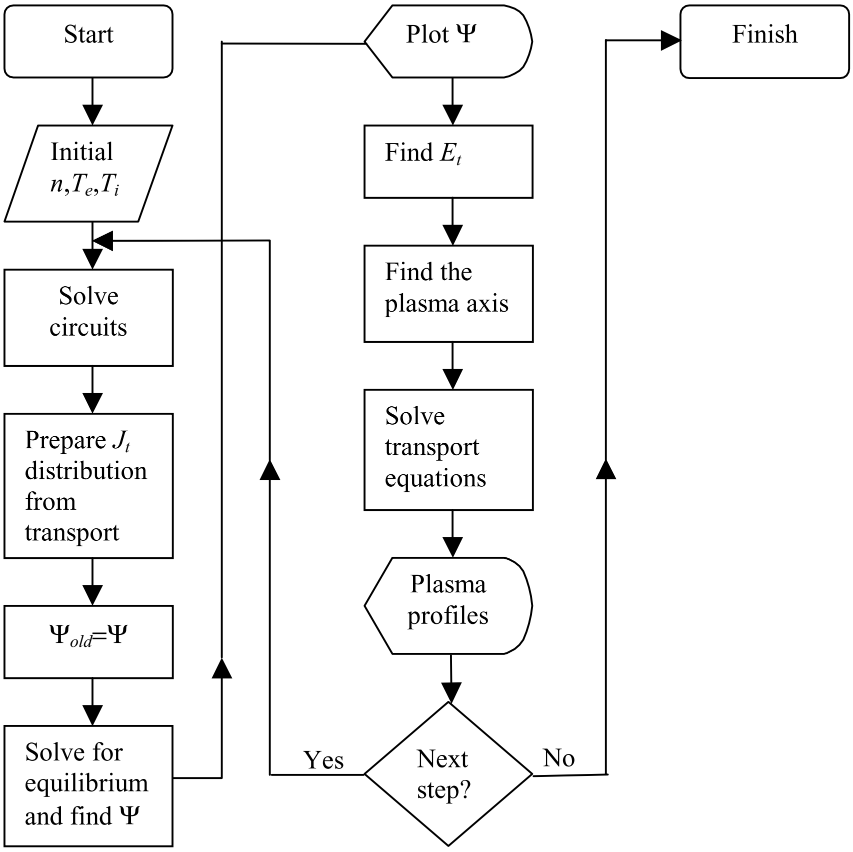

Fig. 1. Flowchart of simulation of the self-consistent plasma equilibrium and transport.

$$\Pi(\Psi) = \iint \left(\frac{1}{2r} |\nabla\Psi|^2 - \mu_0 J_t \Psi \right) drdz \, , \tag{3}$$

where $\nabla = \partial/\partial r \, r + \partial/\partial z \, z$. After descritizing (3) onto a grid of N nodes, a linear system of equations is found, which reads

$$\mathbf{K}\boldsymbol{\Psi}=\mathbf{F}. \tag{4}$$

Here, **K** and **F** are stiffness and force matrices having the dimensions NxN and Nx1 (N being the total number of nodes), respectively. **Ψ** is an Nx1 vector of known poloidal flux nodal values obtained by solution of (4). In variational finite elements **K** depends only on the meshing scheme and thus is time-independent, while **F** is a function of nodal values of the toroidal current density J_t. Therefore, once **K** is built, it can be stored and be used as many times as required. Moreover, the formulation of variational finite elements always leads to symmetric sparse stiffness matrix, so that little storage would be needed to keep **K** known. These properties greatly influence the efficiency of the equilibrium solver so that each time step requires considerably short time for computations. If the tokamak poloidal configuration is symmetric with respect to its meditorial plane z=0 (being the case for our small tokamak, Damavand), only half of the solution region can be solved for, and computation time is therefore halved. Even in such tokamaks, a small transverse vertical stability poloidal magnetic field exsits parallel to the axis, usually obtained by two small poloidal coils having opposite currents, placed symmetrically with respect to the z=0 plane. In these cases the tokamak is not exactly symmetric, but one can still first neglect the vertical stability coils and solve the symmetric poloidal system for Ψ. Then the effect of these coils can be added simply by means of the Green function technique. We have shown [2] that the required Green function of (2) is given by

$$G(\mathbf{r},\mathbf{r}') = \mu_0 \frac{\sqrt{rr'}}{2\pi} \mathrm{Q}_{1/2}\left[\frac{r^2 + r'^2 + (z-z')^2}{2rr'}\right], \tag{5}$$

where $\mathrm{Q}_{1/2}(\cdot)$ is the Legendre function of the second kind. Having the new poloidal flux Ψ obtained, the new plasma configuration is also known, without explicit referring to the functions describing plasma geometry, including minor and major radii, elongation, triangularity, and so on. In the next step, the two-dimensional electric field over the entire solution region is computed as $E_t = -1/2\pi\, \partial\Psi/\partial t$, which is a direct consequence of $\mathbf{B}_p = -1/r\, \phi \times \nabla\Psi = -1/r\, \partial\Psi/\partial z\, r + 1/r\, \partial\Psi/\partial r\, z$ with ϕ being the unit vector in the toroidal direction and the first Maxwell equation in integral form. Before we can proceed to solution of transport, one needs to determine the location of plasma. This can be done through a simple search within plasma window, for local maximum of Ψ. If the poloidal configuration system is supposed to be symmetric, the search must be limited to over the *z*-axis, that is, the plasma is allowed to move over the meditorial plane.

Finally, the transport equations are solved in one-dimension. Here, the center is the plasma axis found from the previous step, and the dimension is extended from the plasma axis towards the outer radius along *z*=0 plane. Here, we use the standard neo-classical transport equations for circular plasma [3], with the omission of estimations for B_p and J_t. These two functions are found from according to above. After finding new values for the one-dimensional parameters corresponding to the plasma density, electron and ion temperatures, and plasma toroidal current density, one can fill-out the plasma window by comparing the nodal values of Ψ at each point to the values of the one-dimensional Ψ over z=0, and employ an interpolation, if necessary, to find the values of the above-mentioned parameters in two-dimensions. In this scheme, we avoid change of variables into poloidal flux function and simply solve the transport equation with the plasma minor radius, being the free coordinate. The loop is then closed after generation of profiles of desirable parameters, and calculations are repeated from beginning until simulation is finished and loop voltage drops to zero. The plasma current in each step can be easily found by integrating the toroidal current density J_t over plasma window. Our developed code on MATLAB runs on an ordinary PC and analyzes each step in few seconds for 5,000 nodes.

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