COLLECTIVE CHARACTER OF THE INTERACTION IN PLASMAS AND NEW RESULTS IN THE STOPPING POWER THEORY

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Though Coulomb scattering is a most basic process in plasma and has been studied for a century [1], doubts concerning the treatment as a sequence of binary collisions remain [2], and recent analysis has revealed [3] that this standard assumption is not justified in general and requires revision. The new point is that a test particle is scattered collectively by the $N$ particles residing in a Debye sphere rather than by successive binary collisions. Here, $N = \frac{\pi n \lambda^3}{\lambda}$ involves plasma density $n$ and the screening length $\lambda = \max(\nu_T/\omega_p, v/\omega_p)$, where $\nu_T$ is the thermal velocity, $v$ the velocity of the test particle, and $\omega_p$ the plasma frequency. This collective scattering can be represented qualitatively as interaction of the test particle with the total un-compensated charge in the Debye sphere which is determined by the fluctuations of $N$. This is equivalent to scattering on an effective charge $e\sqrt{N}$ rather than the charge $e$ of the individual plasma particles that is the main assumption of binary Coulomb collision theory. This has deep consequences for the Coulomb logarithm, because it drastically shifts the borderline between classical and quantum Coulomb scattering, extending the domain in which the quasi-classical approximation applies. This shift is visualized in Fig. 1.

The present analysis is based on the full quantum description of the plasma. This is necessary to identify the borderline between classical and quantum scattering regimes. On this basis, an expression for the momentum evolution of the test particle is derived which holds generally and is analyzed for both quasi-classical and quantum regimes. Beside the Coulomb scattering parameter $\alpha = Z_0e^2/\hbar v$, it contains the plasma parameter $N$ expressing scattering by the fluctuating plasma microfield. The momentum distribution obtained from the present theory is confirmed by molecular dynamics (MD) simulation. It is also compared with the standard model of binary Coulomb collisions which deviates significantly from the MD results.

Using the method of [4], we obtain an analytical expression of the probability of finding the test particle at time $t$ in a state which has a perpendicular component of momentum $q_\perp = (q_x, q_y)$

$$M(t, q_\perp) = \int \exp(-i\mathbf{q}_\perp \cdot \mathbf{u}_\perp) \, F_e(\mathbf{u}_\perp) F_i(\mathbf{u}_\perp) \, d\mathbf{u}_\perp,$$

while the test particle had momentum $\mathbf{p} = (0, 0, Mv)$, $v \gg \nu_T$ at time $t = 0$, where $F_e(\mathbf{u}_\perp)$ and $F_i(\mathbf{u}_\perp)$ are characteristic functions describing the scattering of a test particle by electrons and ions, $\mathbf{u} = r/\hbar$, $\mathbf{u}_\perp = (u_y, u_z)$, $\mathbf{q}_\perp = (q_y, q_z)$ and $F_e(\mathbf{u}_\perp) = \exp(n f_e(\mathbf{u}_\perp))$, $f_e = \int (\cos [eZ_0 \Delta G/2\hbar v] - 1) \, d\mathbf{r}$, $\Delta G = g(\mathbf{u}_\perp) - g(\mathbf{u}_\perp = 0)$, $g = \int_0^t \int V_0 (a + \zeta)/2, y + \hbar u_y, z + \hbar u_z) \, d\zeta$, $V_0(\mathbf{r})$ is the screened Coulomb potential.

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FIG. 1. For demonstration we show the change of the Coulomb logarithm in the Landau collisional integral due to effects under consideration. In the framework of the binary collision theory the area of the applicability of the quantum and classical logarithms depends only on energy of colliding particles. The whole high-energy area is described by the quantum (Bethe) logarithm and the classical (Bohr) logarithm occupies only a tiny part of of the of physically interesting parameters. Taking into account of collective Coulomb scattering has drastically changed this prediction: the classical logarithm is applicable for describing of high energy case, its applicability depends on plasma density and only the region of small $N$ is described by the quantum logarithm. There are areas of strongly non–ideal (the borderline $T \sim n^{1/3}$) and degenerate plasmas (the borderline $T \sim n^{2/3}$), which are beyond our consideration. In the binary collision theory the border between the classical and quantum logarithm corresponds to $\alpha \approx 1$ ($T = \text{const}$), yet taking into account of collective Coulomb scattering gives rise to the new border $\alpha^2 N \approx 1$ ($T \sim n$).

The expression for $f_e$ is valid for those $\mathbf{q}_\perp$ for which the main contribution in the integral is not related to the domain $\min((y + \hbar u_y)^2 + (z + \hbar u_z)^2, y^2 + z^2)) < Z_0^2 e^4 / \mu^2 v^4$, where $\mu = m_\alpha M/(m_e + M)$, $m_e$ is the electron mass. Aiming for logarithmic accuracy, we do not need to be more accurate. The expression giving $M$ clearly demonstrates that the scattering by electrons and ions is statistically independent. This allows us to investigate the scattering by electrons independently from the scattering by ions. It means that $M(t, \mathbf{q}_\perp)$ can be represented as a convolution of $M_e(t, \mathbf{q}_\perp)$ and $M_i(t, \mathbf{q}_\perp)$, which are Fourier–images of $F_e$ and $F_i$.

The function $f_e$ defines the scattering of a test particle by plasma electrons. The analytical structure of $F_e$ shows that this scattering depends on the dimensionless parameters $N = \pi n \lambda^3$ and $\alpha = Z_0 e^2 / \hbar v$. We find three quite different regimes of scattering which are governed by these dimensionless parameters. One can see that the dependence of the scattering on this $N$ is a clear indication of essentially collective nature of Coulomb collisions in plasma, because this parameter is absent in the two–particle collision problem.
FIG. 2. Comparison between the MD simulation results (bar graph), predictions of the present theory (solid line), and predictions of the traditional diffusion approximation (dashed line). We have simulated a test particle of an initial velocity $2.297 \times 10^8$ cm/s (15 eV) moving in a volume containing globally uniform randomly-distributed immovable Coulomb centers of charge $\pm e$ at concentration $n_+ = n_- = n = 1.054 \times 10^{17}$ cm$^{-3}$, the simulated test volume was of the size $V = 2L_V \times 2L_V \times 6L_V$ with $L_V = 7.239 \times 10^{-6}$ cm. Test particle has a charge $2e$ and a mass of an electron. Classical equations of motion were solved using a second-order scheme with an adaptive time-step. $2.06 \times 10^5$ independent trials were conducted. The bar graph shows simulated distribution of the square transverse momentum $q_T^2$ for $t = 2L_V/v = 6.30 \times 10^{-14}$ s. The scale of $q_T^2$ is given by $q_T^2 = 2\pi nZ_0^2 e^4 t/v = 3.87 \times 10^{-41} (g \times \text{cm}/s)^2$. The distribution of $q_T^2$ predicted by the present theory, namely $M(t, q_T^2)dq_T \equiv \pi M(t, q_T) dq_T^2$, is shown by a solid line. The solid line has been obtained numerically from $M$ with $F_i = F_e = \exp(nf_e)$ and $f_e$ corresponding to $\alpha > 1$, $\alpha^2N\ln N > 1$.

In the case $\alpha > 1$, $\alpha^2N\ln N > 1$, $t > \lambda/v$, we obtain $f_s(u_{\perp}) = -\nu t \zeta u_{\perp}^2/v$, where $\nu = 2\pi nZ_0^2 e^4$, $\zeta = \ln(\lambda v/Z_0 e^2 \kappa)$, $\kappa = \max(|u_{\perp}|, 1/\mu v)$. As $\hbar$ does not appear in $f_e$, the scattering can be described in the quasiclassical approximation. A non-trivial feature is a logarithmic singularity at $u = 0$ that is responsible for a power-law tail of $M_e$. Eq. (7) yields $M_e(t, q_{\perp}) = \exp(-q_{\perp}^2/2p_0^2)/(2\pi p_0^2)$, where $q_{\perp}^2 > 2p_0^2 \ln(L/4)$, $L$ is a solution of $L = \ln(2\pi nL^2 v t) \gg 1$, $p_0^2 = \nu LT/v$, and the power-law tail on the transferred transverse momentum is given by $M_0(t, q_{\perp}) = 2p_0^2/(\pi L q_{\perp}^2)$ (look at “the tail” on Fig. 2) for $p^2 > q_{\perp}^2 > 2p_0^2 \ln(L/4)$, $p^2 = 2\mu v$. One can see that for times $t > \tau^* = 2m^2 v^3/\nu L \ln(L/4)$, such that $p^2 < 2p_0^2 \ln(L/4)$, the power-law distribution function tail is absent. Comparison of the results of molecular dynamics (MD) simulations with our theory taking into account the fluctuating plasma microfield shows good agreement (see fig.2). The Coulomb logarithm corresponding to the MD situation equals $L_c = 8.34$. The traditional binary Coulomb collision theory describes the scattering in plasma as a sequence of instant binary collisions (the limit when collision duration goes to zero under the condition that collision time is a constant). This results in a diffusion approximation for test particle kinetics in an almost ideal plasma, predicting a Gaussian distribution for $q_{\perp}$, shown by the dashed line. The theory of binary Coulomb collisions demonstrates sizeable deviations from the
results of MD simulations. We emphasize that this area of parameters is described by the classical Coulomb logarithm both in the theory of binary Coulomb collisions and in the correct treatment of scattering as a scattering of fluctuating plasma microfield. However the details of the scattering reveal the essential difference in these two approaches.

For \( \alpha < 1 \), \( \alpha^2 N \ln(\alpha^2 N) > 1 \), \( t > \lambda/\nu \) we have \( f_e(u_\perp) = -\nu t \zeta_2 u_\perp^2 / \nu \), where \( \zeta_2 = \ln(\lambda/\hbar \kappa) \), \( \kappa = \max(\{u_\perp \}, \alpha/\mu \nu) \). In spite of the quantum nature of the screened binary collisions on account of \( \alpha < 1 \) \( f_e \) depends on \( \hbar \) at most logarithmically. Moreover, comparing \( \zeta_1 \) with \( \zeta_2 \) one finds that \( M_e \) depends on \( \hbar \) only due to the expression \( \ln(\alpha^2 N) \). Furthermore, the conditions \( \alpha < 1 \), \( \alpha^2 N \ln(\alpha^2 N) > 1 \) imply that \( \ln N \gg \ln(1/\alpha^2) \). Therefore, we can write \( \ln(\alpha^2 N) \approx \ln N \) with logarithmic accuracy. This brings us back to the quasiclassical description of scattering in an almost ideal plasma, as it was firstly realized in [4]. Notice that this is a new quasiclassical regime occurs due to \( \alpha^2 N \ln(\alpha^2 N) > 1 \). In the case of \( \alpha < 1 \), \( \alpha^2 N < 1 \) making use of \( M \) comes to the quantum logarithm coinciding with the prediction of screened binary collision theory. This result is evident from a physical point of view, since the number of plasma particles \( N \) simultaneously involved in the interaction is not large enough. Moreover, this case covers only a narrow parameter area.

\( M_e \) contains all information about scattering and gives an opportunity to calculate the Coulomb logarithm. We give only the final results for a particular case of scattering of a test ion \( M \gg m_e \): 1) the area of classical binary collisions \( \alpha > 1 \) is described by the classical (Bohr) logarithm \( L_{cl} = \ln(m_e v^2 \lambda / Z_0 e^2) \); 2) in the area of quantum binary collisions \( \alpha < 1 \) a new scattering regime occurs provided \( \alpha^2 N \gg 1 \) and this parameter area is also described by the classical logarithm \( L_{cl} = \ln(m_e v^2 \lambda / Z_0 e^2) \); 3) in the quantum binary collisions area the quantum (Bethe) logarithm \( L_q = \ln(\alpha m_e v^2 \lambda / e^2) \) occurs provided \( \alpha^2 N \to 0 \).

Our results prove that a new physical phenomenon — suppression of quantum effects by collective effects — occurs in the weakly-coupled plasma, and this demands the revision of the standard theory of many-particle Coulomb systems.


