CHARACTERISATION OF
ION-TEMPERATURE-GRADIENT-DRIVEN (ITG) MODES IN THE
W VII-X STELLARATOR CONFIGURATION

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1 Introduction

Micro-instabilities, ion acoustic (drift) waves, destabilized by spatial inhomogeneities,
are commonly held responsible for anomalous transport in tokamaks. Optimized stellarator
configurations such as W VII-X [1] should reduce neo-classical transport to a level close to
that found in tokamaks. At this point anomalous transport properties may become dominant.
Thus it is now necessary to develop tools for the study of drift wave turbulence in three
dimensional geometries. In this article we present results from both linear and non-linear
Particle-In-Cell, global, gyrokinetic simulations, the former in the full 3-D geometry, the latter
in an appropriate system of reduced dimensionality.

2 Linear simulation

The 3D global PIC code, EUTERPE [2][3] uses a low beta approximation within the
linear gyrokinetic model with electrons assumed adiabatic. Thus the perturbed ion
distribution function obeys,

$$
\frac{d}{dt} f(\vec{R}, v_\parallel, \mu, t) = - \frac{(\vec{E} \times \vec{B}) \cdot \vec{B}}{B^2} \frac{\partial f_0}{\partial \vec{R}} - \frac{q_i \vec{E}}{m_i} \cdot \vec{E} - \frac{1}{2} \frac{\partial f_0}{\partial v_\parallel} \left( v_\parallel \frac{\partial f_0}{\partial v_\parallel} + \frac{1}{2} v_\perp \frac{\partial f_0}{\partial v_\perp} \right) \frac{\vec{E} \cdot \vec{B}}{B^2} - \left( \vec{E} \cdot \vec{B} \right) \frac{\vec{B}}{B^2} \frac{\vec{B}}{B^2}.
$$

(1)

The equations for the particle guiding centers are,

$$
\frac{d\vec{R}}{dt} = v_\parallel \vec{h} + \frac{v_\perp^2 + v_\perp^2}{\Omega}\vec{h} \times \frac{\vec{E} \times \vec{B}}{B}, \quad \frac{dv_\perp}{dt} = \frac{1}{2} v_\perp \frac{\vec{E} \times \vec{B}}{B^2} \cdot \vec{h}, \quad \frac{d\mu}{dt} = 0.
$$

(2)

And the system of equation is closed by invoking quasi-neutrality,

$$
\frac{en_0}{Te} \phi - \nabla_\perp \cdot \left[ \frac{n_0}{B^2} \nabla_\perp \phi \right] = \int f(\vec{R}, v_\parallel, v_\perp, t) \delta^3(\vec{R} - \vec{x} - \vec{\rho}) B d\vec{R} \delta \phi d\vec{v} d\mu.
$$

(3)

This Poisson equation is solved in the PEST-1 (s, θ*, ϕ) system of coordinates using a
finite element method. As a matter of noise reduction, the right hand side of equation
3 is Fourier filtered in (θ*, ϕ). EUTERPE has been successfully benchmarked against
the helical version of the GYGLES code [5].

The goal of the linear simulations was an understanding of the coupling behavior of
the ITG mode in W VII-X. Typically the magnetic geometry of a confinement
system may lead to couplings between Fourier harmonics in the periodic coordinates.
In a tokamak the predominantly m = 1 poloidal variation in |B| leads neighboring
poloidal harmonics to couple allowing the ballooning structure to the Toroidal-ITG

1933
mode. Such behavior extends to 3D-geometries [4]. The existence of a given coupling may be determined by choosing our Fourier filter to allow only for this coupling and looking for the growth of a normal mode.

Figure 1: Toroidal harmonics of the perturbed electrostatic potential $\phi_n$. The deformed tokamak shows clear mode coupling which is entirely absent for W VII-X.

As an example, shown on the left of figure 1, are results from a tokamak configuration, helically deformed such that the spectrum of $|B|$ is dominated by $B_{m,n=0} = B_{0,0}$, $B_{2,2}$ and $B_{1,2}$, here $m$ is the poloidal mode number, $n = n_{per} \times N_{per}$ is the toroidal mode number and $N_{per}$ is the toroidal periodicity of the system. The linear growth of all modes present in the chosen filter shows that we have couplings $(m, n) \rightarrow (m + 1, n)$, $(m, n) \rightarrow (m + 2, n - 2 \times N_{per})$ and $(m, n) \rightarrow (m + 1, n - 2 \times N_{per})$ as could be expected from the $|B|$ spectrum by analogy to the case of the tokamak.

Figure 2: Comparison of growth rates and poloidal mode structures for W VII-X and the ‘equivalent’ screw pinch. W VII-X: $\gamma = 0.1529[\nu_{thi}/\alpha]$, $\omega = 0.6804$ screw pinch: $\gamma = 0.1473, \omega = 0.5283$.

In contrast, application of a number of different filter shapes, based on the dominant components of $|B|$, to the W VII-X configuration could find no evidence of coupled
modes. This led to the conclusion that here ITG modes would be ‘slab’ like, consisting of a single \((m, n)\) harmonic. This observation motivated comparison with a straight screw-pinch configuration. This ‘equivalent’ straight system was constructed by keeping the connection length \((\frac{1}{R_{eq}})\) and profile shapes unchanged. This comparison, shown in figure 2, found agreement in growth-rates to within 5% and similar mode structures, which in both cases simply conformed to the geometry of the magnetic surfaces.

3 Non-Linear simulation

We have shown that ITG modes in W VII-X have a slab like character which indeed may be well approximated by an equivalent screw-pinch equilibrium. As a first approach to it’s non-linear simulation we further approximate the geometry to that of a straight \(\Theta\)-pinch, still allowing for such slab modes. We make this approximation with the goal of achieving energy conservation in non-linear drift wave turbulence simulation.

The TORB [6] code has been adapted from the toroidal non-linear, global, gyrokinetic, \(\delta f\) PIC code ORB [7]. As with the linear code, electrons are treated as adiabatic. The markers are advanced using a 4\(^{th}\) order Runge Kutta scheme. Cubic splines are used for the finite element resolution of the Poisson equation. The charge density is Fourier filtered in the periodic coordinates. This filtering process has been shown to be energy conserving [8].

![Graphs showing field energy, kinetic energy, averaged heatflux.](image)

Figure 3: Conservation of energy and heat flux. Left 67 million tracers, filter \(m = -24:24\) \(n = 0:6\), optimized loading. Right 16 million tracers, filter \(m = -96:96\) \(n = 0:6\), solid: optimized loading, dashed: no loading optimization.

The most unstable linear modes for the equilibrium we consider are \((m, n) = \{24, 1\}; \{45, 2\}; \{48, 3\}\). Results using a filter retaining harmonics up to the fastest growing \(n = 1\) mode (left of figure 3) show very good energy conservation well past the initial saturation phase.
TORB makes use of an intelligent initial loading of markers in velocity and configuration space. A well chosen initial loading has been found to be of great assistance in improving energy conservation. This is shown on the right of figure 3, for two simulations using a filter containing all the dominant linear modes, with and without intelligent loading. Conservation comparable to that shown for the restricted filter sill requires $>120$ million tracers [8].

4 Conclusions and further work

Linear simulation has shown W VII-X to be well approximated by a straight screw pinch with respect to pure ITG instabilities. The EUTERPE code should be extended to include electron physics and the effects investigated.

Non-linear simulations of a $\Theta$-pinch approximation to W VII-X have shown good energy conserving properties. The code is now being adapted for the screw-pinich configuration suggested by linear results. Recently, correct treatment of the electron response to the field line average electrostatic potential has been implemented in TORB. This will allow consistent inclusion of the effects of zonal flows, corresponding to the $(m,n) = (0,0)$ component of the perturbed electrostatic potential.

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References