Active feedback stabilization of high-beta modes in tokamaks

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1. Introduction

Active control of MHD modes is receiving increasing attention as a way to increase the beta limit of advanced tokamaks. Currently, experiments on active control of \( n = 1 \) external kink modes are in progress on DIII-D [1, 2]. Here, we present a combined MHD and Control analysis for the stabilization of resistive wall modes (RWM), when the no-wall beta limit is exceeded. The MHD modeling uses fully toroidal, ideal computations, that produce low-order rational transfer functions. These express the fluxes in sensor coils, produced by the plasma-wall system, in response to currents in the feedback coils. We use the transfer functions for controller design, subject to constraints of good performance, robustness and limited gain. The control analysis determines what controller structures and design parameters give acceptable control. Results will be presented for two basic types of controller.

The number of coils in the toroidal direction is assumed large enough, so that one can consider modes with a single toroidal mode number \( n \). Our previous analyses [3, 4] have shown that a single set of coils in the poloidal direction is sufficient. The performance of the feedback is rather insensitive to the poloidal width of the active coils, but for standard advanced tokamak \( q \)-values, and active coils rather close to the plasma, a coil width of about 25\% of the poloidal circumference is optimal. Concerning sensor design, we have found that small sensors are best [4].

\[
P_1(s) = \frac{\psi_s}{M_{sf}^{(0)} I_f}
\]

(1)

Here, \( M_{sf}^{(0)} \) is the mutual inductance between the feedback and sensor coil, in the absence of the plasma and wall. The other response function expresses the flux induced in the feedback coil itself, taking into account the response of the plasma and the resistive wall

\[
P_2(s) = \frac{\psi_f}{L_f^{(0)} I_f}
\]

(2)
For active coils in the form of broad strips or fat conductors, \( P_2(s) \) varies considerably with frequency. However, for thin-wire coils, where most of the inductance comes from the near field of the wires, \( P_2(s) = 1 \) is a good approximation. In order to keep the maximum coil voltage \( V_f \) low, a low-inductance active coil is preferable. However, in the present study we assume \( P_2 \equiv 1 \) for simplicity.

### 3. Control systems

#### 3.1 High-resistance coil

In [4, 5, 6], we considered a control model where the flux perturbation \( \psi \) is fed back via a PD controller to the voltage on the active coil \( V_f \). Such a "\( \psi \)-sensor" control system is illustrated in Fig. 1. Here, the process to be controlled is that from the feedback voltage \( V_f \) to the magnetic perturbation

\[
G(s) = \frac{P_1(s)}{1 + \tau_f s P_2(s)}
\]

(ignoring normalization constants). The gain is normalized such that \( K \) is the ratio, at the sensor position, of the radial field perturbation directly from the active coil to the measured total \( b \) (which can be either radial or poloidal), assuming low frequency response \( \omega \ll R_f/L_f \). The properties of the active coil and controller can then be characterized by the response time \( \tau_f = L_f^{(0)}/(R_f + R_g) \), where \( R_f \) and \( R_g \) are the resistances of the coil and the controller respectively. \( R_g \) need not be a physical resistance, but can be produced by using an approximate current controller for \( I_f \).)

We consider a controller acceptable if it meets certain performance criteria, that involve the open loop transfer function \( L(s) = G(s) K(s) \), the sensitivity function \( S(s) = 1/[1 + L(s)] \), the complementary sensitivity \( T(s) = L(s)/[1 + L(s)] \) and the control activity \( KS \). The following criteria were applied

\[
J_S = ||S||_\infty = \max_{\omega \in \mathbb{R}} |S(j\omega)| \leq c_S = 2, \\
J_T = ||T||_\infty = \max_{\omega \in \mathbb{R}} |T(j\omega)| \leq c_T = 2.5 \quad (3)
\]

which ensure acceptable stability margins and damping, and

\[
J_u = ||KS||_\infty = \max_{\omega \in \mathbb{R}} |KS(j\omega)| \leq c_u = 10, \quad (4)
\]

which limits the control activity. A trade-off is possible, such that good control (low \( J_S \) and \( J_T \)) can be achieved by increasing the control activity \( J_u \). The system is considered controllable if all the limitations (3), (4) can be met. In [5], these requirements were applied to a set of advanced tokamak equilibria listed in Table 1.

It was found possible to stabilize the \( n = 1 \) RWM by a PD controller for all the equilibria with an upper limit for the normalized response time \( \tau \equiv \tau_f/\tau_w \) of about 1 [5]. The three first equilibria could be controlled for \( \tau \lesssim 3 \). Moreover, the controllers were robust in the sense that the controller optimized for equilibrium 4, with the highest pressure, works well for the three cases with lower pressure.
3.2 Low-resistance coil

The control system in Fig. 1 is not well adapted for superconducting active coils, where \( \tau \) is large (unless the internal resistance of the generator is significant). We therefore consider another controller design for the case of large \( \tau \). It is then necessary to introduce more phase-lead into the system. We do so by considering as output from the plant the voltage \( V_s = d\psi_s/dt \), rather than the flux. We now choose normalization for the controller gain such that \( K \) is the ratio, at the sensor position, of the direct field from the feedback coil, to the measured field in the high frequency limit \( \omega \gg R_f/L_f \). The control system is illustrated in Fig. 2. Now, the process transfer function is \( M_0(s) = s(L_f + R_f) \) (which resembles current control \( \tau_f \) in Fig. 1). For the system in Fig. 2, the controller must have some integral action, so that any initial condition will decay to zero. In the absence of integral action, the system in Fig. 2 can be stable, but will in general settle down with a nonzero flux perturbation.

Better control is possible with the \( \bar{V} \) sensors, mainly because the extra differentiation decreases the phase lag. For convenience, we have split the controller \( K(s) = U(s)W(s) \) into a PI part

\[
U(s) = K_p + K_i/s
\]

in series with a PD controller

\[
W(s) = \frac{1+sT_d}{1+sT_d/\xi}
\]

(With the process defined as in Fig. 1, the controller in Fig. 2 would be termed a double PD controller. However, we prefer the definition of gain implied in Fig. 2, in which no time constant is involved, because a voltage rather than a flux is used to control the voltage over the active coil \( V_f \). Therefore we view the \( \psi \) sensor as PI, and the \( \bar{V} \) sensor as PID, control.)

4. Results

We have carried out optimizations for the two types of control systems. For the \( \psi \) sensor system, we have assumed that \( K(s) \) is a standard PD controller \( K(s) = K_p(1+sT_d)/(1+sT_d/\xi) \) and for the \( \bar{V} \) sensor we used the PID structure described in Sec. 3.2. With the criteria (3) and (4), the optimum for the \( \bar{V} \) sensor system occurs for \( K_i = 0 \), which means that disturbances do not decay to zero. To guarantee sufficient low frequency suppression, we therefore fix \( K_i \) at certain values (for small \( K_i \), the slowest decay rate in the system is proportional to \( K_i \)).

We have also considered two different configurations of walls and sensors, that represent very different levels of difficulty for the control design. The first configuration corresponds to conditions in present tokamaks; the wall is at \( r_w = 1.3a \) and poloidal sensors are placed just inside it. The second configuration addresses conditions for a reactor. There are two walls \( (r_{w1} = 1.3a, r_{w2} = 1.55a) \), and poloidal sensors are just outside the inner wall. (This adds a large phase lag and makes the system much harder to control.) The two configurations will be referred to as “in” and “out”, respectively. Figure 3 shows Bode diagrams: amplitude and phase of \( P_1(j\omega) \), for the third equilibrium, both geometries, and radial as well as poloidal sensors. Evidently, the poloidal sensors give a simpler frequency dependence, and we have therefore concentrated on achieving robust control for poloidal sensors.

Figure 4 shows the trade-off between per-
Figure 4: Trade-off between good performance (low \(||S||_\infty\)) and low control activity (\(||KS||_\infty\)) for voltage sensor outside the first wall.

performance and control activity that is possible for different integral action \(K_i\) (proportional to the slowest decay rate). The diagram refers to the third equilibrium with two walls and the sensors outside the inner wall. The figure shows that reasonable compromises can be found with good performance, moderate control activity, and acceptable low frequency response.

To achieve robustness, we have required the same controller to satisfy the criteria (3), (4) for all the three first equilibria. The fourth one is excluded as unrealistic (e.g., \(n = 2\) is strongly unstable). For the system with a \(\Psi\) sensor, we have computed the maximum response time \(\tau_f\), for which the specifications (3), (4) can be met. For \(\tilde{V}\) sensors, we have determined the maximum integral action \(K_i\) for which the systems meets the specifications.

The results are shown in Table 2, which contains our main conclusions. First of all, both systems can produce acceptable results when the sensors are inside the wall. For such configurations, the \(\Psi\)-sensor system works for controller time constants up to 3.2\(\tau_w\). The \(\tilde{V}\)-sensor system works well, even with a very large integral gain (much larger than necessary and desirable).

For the two-wall configuration with sensors outside the first wall, the conditions are much more demanding. The \(\Psi\)-sensor system requires a very short response time. However, the \(\tilde{V}\)-sensor system is able to control the first three equilibria and achieve an acceptable low frequency response. Consequently, active control of RWM should be possible also for a reactor. The key to good control is to introduce sufficient derivative action to counteract the phase lags from the resistive walls.

**References**