Modeling of the Force Transferred from the Magnetic Perturbation Field of the DED to the TEXTOR-94 Edge Plasma

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1. Introduction

The TEXTOR-94 tokamak in Jülich is now under construction for the dynamic ergodic divertor (DED) experiment, which has been suggested as an alternative scheme for transport and exhaust control [1]. In the DED experiments, the resonant perturbation is applied to the tokamak plasma in order to create an ergodic magnetic field structure at the edge; due to the rotating perturbation field (up to 10kHz), a rotation of the edge plasma can be induced. For the understanding of the dynamic aspect of the DED experiments, it is an important issue to analyze the force transferred from the perturbation field to the plasma.

In this paper we suggest a model to describe the plasma response to the external resonant field, which was derived from a single fluid MHD equations. Using this model the induced current inside the plasma and the behaviour of the perturbation field are analyzed, with which we can estimated the force acting on the plasma through the \( j_z \times B_r \) force, where the \( j_z \) and \( B_r \) are the toroidal component of the perturbation current and the radial component of the perturbation field, respectively. The analysis for the several different conditions of the plasma current profile is presented, followed by the discussion of the validity of the model.

2. Description of TEXTOR-DED

The TEXTOR-94 has a major radius of 1.75 m and the plasma column is restricted by the ALT limiter at the low field side or by the DED target plate at the high field side. In this analysis the radius of plasma column is set to 0.477 m. The DED coils will be installed on high field side with the poloidal coverage of +/- 60 degrees and a fundamental mode number of \((m, n) = (12, 4)\) [1]. The radial location of the coils is 0.535 m. The coils are powered by a four-phase AC current up to 15kA peak value and produce the (mainly) poloidally traveling perturbation. They can be operated at 50 Hz, 1 kHz, a band from 1 to 10 kHz, as well as DC. The perturbation field resonates with the \( q = 3 \) flux surface. But, since the pitch of the tokamak field lines is shallower at the high field side than at the low field side due to the toroidal effect, the spacing of the DED coils is set to be narrower than \( m = 12 \) value. The effective mode number in this case is 20. Additionally, \( m_{\text{eff}} = 5 \) and 10 will be available by changing the connections of the coil array. These modes are used by mixture with the main mode in order to control the degree of ergodization and to decouple the perturbation strength.
at the edge and its penetration depth. But we concentrate on the $m = 12$ as a typical one in this paper.

3. Model

The basic equations are the single fluid MHD equation, Ohm’s law;

$$\rho \frac{\partial \mathbf{u}}{\partial t} = -\nabla p + \mathbf{j} \times \mathbf{B},$$  \hspace{1cm} (1)

$$\eta \mathbf{j} = E + \mathbf{u} \times \mathbf{B},$$  \hspace{1cm} (2)

and Maxwell’s equations. The analysis is made in a cylindrical coordinate system $(r, \theta, z)$ and the perturbation is assumed to have the form of \( \exp[i(m\theta - nz/R_0 - \omega t)] \), where $r$, \( \theta \), $z$, $m$, $n$, $R_0$ and $\omega$ are the radial, poloidal and toroidal coordinates, poloidal and toroidal mode number, major radius and perturbation frequency, respectively. It is convenient to introduce the vector potential for the perturbed magnetic field, $\mathbf{B}_1 = \nabla \times \mathbf{A}$, where the subscript “1” represents the perturbation field and $\mathbf{A}$ is defined as complex value, $\mathbf{A}(r) = A_r(r) + iA_{\theta}(r)$. If $R_0 m / \eta n >> 1$ the derivative in the $z$ direction is neglected. After the linearization and the projection of the coordinate to the field line at the resonance surface, the relevant component of the Ohm’s law is

$$\eta j_{z1} = E_{z1} + u_{r1} B_{\theta0}(1 - qn / m),$$  \hspace{1cm} (3)

where the subscript “0” represents the equilibrium field. From the Eqs. (3) and the Maxwell’s equations, we obtain

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} A_z \right) - \frac{m^2}{r^2} A_z + i \omega \mu_0 \sigma A_z + \mu_0 \sigma u_{r1} B_{\theta0}(1 - qn / m) = 0.$$  \hspace{1cm} (4)

In the momentum equation we neglect the pressure gradient term (this is a good approximation in the ergodic layer) i.e. $\nabla p_0 = j_0 \times B_0$ in the zeroth order. This means either that $j_0 = 0$ or that $j_0$ is in parallel to the magnetic fields. The first option was analyzed in ref. 2, and here we treat the later option to see the effect of the equilibrium plasma current. Then the radial component of the plasma velocity is

$$u_{r1} = -\frac{i}{\mu_0 \rho \omega} \left[ \frac{1}{r} \frac{\partial}{\partial r} (r \frac{\partial A_z}{\partial r}) - \frac{m^2}{r^2} A_z \right] B_{\theta0}(1 - qn / m) + \frac{i}{\rho \omega} j_{z0} \frac{\partial A_z}{\partial r}.$$  \hspace{1cm} (5)

Substituting Eq. (5) to (4), we obtain the equation governing the behaviour of perturbation in the ergodic layer,

$$\frac{1}{r} \left( \frac{\partial}{\partial r} r \frac{\partial A_z}{\partial r} \right) - \frac{m^2}{r^2} A_z + \frac{1}{(v_{Apol} / \omega)^2} A_z + \frac{1}{(\omega / v_{Apol})^2 + 1} B_{\theta0}(1 - qn / m) \frac{\partial A_z}{\partial r} = 0;$$  \hspace{1cm} (6)

here,

$$\delta^2 = \frac{1}{i \mu_0 \sigma \omega}, \quad v_{Apol}^2 = \frac{\left( B_{\theta0}(1 - qn / m) / \mu_0 \rho \right)^2}{\mu_0 \rho}.$$  \hspace{1cm} (7)
where $|\delta|$ and $v_{Apol}$ are the skin depth and poloidal Alfvén speed, respectively. Hereafter we call third and forth terms on lhs of eq.(6) $j_{z1}^{(1)}$, $j_{z1}^{(2)}$, respectively.

4. Results

As seen from eq. (6), the amplitude of $j_{z1}^{(2)}$ strongly depends on the current profile at the edge, for which we assumed $j_{z0}(r) = (\nu + 1) f_p (1 - (r/a)^2)^\nu / (\pi a^2)$ with the peaking factor of $\nu$. Figure 1 shows the radial profile of the perturbation field in the case of $\nu = 1$ (peaked current profile), for the several frequencies in the case of 50 eV, Ip = 400kA, $l_{mode} = 15$kA, toroidal field = 2.25 T and $a = 0.477$ m. The DED coils are at 0.535 m and the density is assumed constant at $10^{19}$ m$^{-3}$. The resonance surface for this mode is located at $r = 0.42$ m. Due to the induced current at the resonance surface, the magnetic field is modified around $r = r_s$. The most attenuation takes places around resonance surface and becomes large as the frequency increases.

The profile of $j_{z1}^{(1)}$, $j_{z1}^{(2)}$ for the different frequencies are shown in Fig. 2 (a) and (b) in the expanded view around the resonance surface. It is found that $j_{z1}^{(1)}$ is highly concentrated at the resonance surface, while $j_{z1}^{(2)}$ is rather widely distributed over the radial direction. For $\nu = 5$, $j_{z1}^{(2)}$ becomes two order smaller than $j_{z1}^{(1)}$, where the modification of the magnetic field is found to be mainly coming from $j_{z1}^{(1)}$.

The force acting on the plasma can be calculated from the data obtained in Fig. 1 and 2, which is plotted in Fig. 3 for the case of 7 kHz showing the each contribution from $j_{z1}^{(1)}$ and $j_{z1}^{(2)}$ with $f^{(1)}$ and $f^{(2)}$, respectively. Because of the wide distribution of $j_{z1}^{(2)}$, the $f^{(2)}$ acts the plasma in wide radial extent while $f^{(1)}$ is restricted around $r = r_s$. It is clear that the radial...
integration of Fig. 3 results in the large contribution from \( f^{(2)} \) to the total force transferred to the plasma. As mentioned above, the contribution to the total force changes depending on the plasma current profile and we found that for the case of \( \nu = 5 \), \( j_{z1}^{(1)} \) (i.e., \( f^{(1)} \)) dominates in determining the total force.

5. Discussion

In the preceding sections we presented the results as an one of the models to describe the plasma response to the resonant external field. The plasma response to the resonant perturbation is well described by the tearing mode theory, which was first formulated by H. P. Furth et al. in 1963 [3]. Taking the limit of ideal plasma, \( \delta \to 0 \) and also \( \omega \to 0 \), one can easily see the difference between the present model and the tearing mode model, i.e., our model reduces to,

\[
\frac{1}{r} \left( \frac{\partial}{\partial r} \left[ r \frac{\partial A_z}{\partial r} \right] \right) - \frac{m^2}{r^2} A_z + \frac{\mu_0 j_{z0}}{B_{\phi0} (1 - nq / m)} \frac{\partial A_z}{\partial r} = 0
\]

(8), which does not come along with the standard tearing mode theory. In the tearing mode theory the third term is replaced with the current gradient term that drives the instability at the resonance surface, while in our model the term represents the interaction between the equilibrium plasma current and the poloidal component of the perturbation field that would create the radial momentum through the Lorentz force. Although the both terms indicate the resonant behaviour (i.e., singularity) at the resonance surface, the physical interpretation is quite different each other. At the moment it is considered that this discrepancy might come from the several assumptions that we made considering the ergodized plasma edge in deriving the model. In order to make this point clear, it is now under consideration to make another approach along the standard theory. One of the possible models would be the one derived by T. H. Jensen et al. [4], which can provide the information on the behaviour of island excited by the external perturbation.

References