TEMPORAL AND SPATIAL CORRELATION PROPERTIES OF NON-DIFFUSIVE TRANSPORT DATA

J.P. Graves\textsuperscript{1}, K.I. Hopcraft\textsuperscript{1}, E. Jakeman\textsuperscript{1}, R.O. Dendy\textsuperscript{2}

\textsuperscript{1}Theoretical Mechanics Division, School of Mathematical Sciences, University of Nottingham, Nottingham, NG7 2RD, UK.
\textsuperscript{2}EURATOM/UKAEA Fusion Association, Culham Science Centre, Abingdon, OX14 3DB, UK

The characterisation of non-diffusive transport and non-Gaussian fluctuations in tokamak plasmas is a central topic in magnetic fusion research. Open questions include the choice of appropriate nonlinear time series analysis techniques, and the nature of the inferences about system dynamics that can reliably be drawn from such analyses [e.g. 1,2]. Because of the inherent variability of much tokamak data, there is value in addressing these questions in the context of simpler but nevertheless complex systems such as sandpile cellular automata, which are known to display the key features of tokamak phenomenology [3]. Specifically we use the extensively diagnosed ‘ricepile’ model [4,5], which retains the identity of individual grains thereby enabling simulations akin to test-particle transport to be performed. There are further advantages for using this model as an \textit{ab initio} source of data. First, it is a paradigm for a self-regulating, non-equilibrium confinement system with nonlocal transport governed by critical gradients. Second, the data is reproducible, controlled and can be obtained in sufficient quantity to make reliable statistical inferences. Third, global and local multi-channel diagnostics can be implemented to form the spatio-temporal transport correlations in a way that can be replicated by real experiments. Fourth, the model delineates behaviours occurring on disparate temporal and spatial scales. Finally, quantities can be studied that have a direct correspondence with measurables in tokamak experiments.

Figure 1(a) shows the height fluctuation $\zeta(t) = h(t) - \langle h \rangle$ on the fuelling timescale of a specific cell in the ricepile and Fig. 1(b) shows the integrated behaviour of $\zeta$ over a changing number of fuelling times $t$. Figure 1(b) is similar in derivation to the integrated local edge density fluctuations in the DIII-D tokamak, shown in Fig. 2(a) of [1]. The data in Fig. 1(a) can be sorted into time intervals where the sign of $\zeta$ is constant, and the distribution of the durations of these intervals, which is equivalent to the distribution of the time taken for $h$ to return to its mean value $\langle h \rangle$, is shown in Fig. 2(a). Recall that diffusive stochastic processes yield return time distributions with power-law tails $\sim t^{-v}$, where for ‘fractional Brownian motion’ (the ‘strange kinetics’ generalisation of classical diffusion that accounts for correlation of step lengths), $v = 2 - H$ with $H$ the Hurst exponent [e.g. 6] which has range $0 < H \leq 1$, implying that $1 \leq v < 2$. The return time distribution for the rice pile cellular automata in Fig. 2(a) has a power law tail with $v = 1.75$, implying an ‘effective’ $H = 0.25$ and hence anti-persistence. These results are insensitive to box-
count averaging, as in the tokamak data [1]. In contrast, the analysis of the return time distribution for the density fluctuations in [1] yields $\nu = 2.4 \pm 0.2$: hence processes in addition to fractional Brownian motion must operate in this instance. The regime $\nu > 2$ is also accessible to behaviour described by rice-pile experiment [7] and automata. Indeed the distribution of times for which individual rice grains remain motionless has a power law tail $\nu \sim 2.4 \pm 0.2$ in the experiment [7], and $\nu = 2.16$ in the simulation [5]. The simulation result is explicable [8] once additional fluctuations occurring on short, intra-fuelling timescales are incorporated into the dynamics. Larger values of $\nu$ correspond to broader distributions for the additional fluctuations. This suggests that the tokamak measurements of [1] may still be compatible with an avalanching paradigm provided multi-scale dynamics are taken into account.

![Graphs](image.png)

Figure 1: (a) Time trace of the height fluctuation and (b) time integrated height fluctuation at a site in the ricepile.

![Graphs](image.png)

Figure 2: The probability distribution for the return time and (b) spatial structure function of $\zeta$. Both yield $H = 0.25$.

Both tokamak density and rice pile height profiles are accessible to multi chan-
nel diagnostics, which yield information on spatio-temporal behaviour and correlation properties. These can be quantified through $S_\zeta(\Delta x) = \langle (\zeta(x) - \zeta(x + \Delta x))^2 \rangle$, the structure function for height fluctuations separated by distance $\Delta x$ down the pile: angled brackets denote an ensemble over the fuelling time and space, and units of $x$ and $\Delta x$ represent the length of one grain. $S_\zeta(\Delta x)$ measures the ‘roughness’ of the height profile and so reflects its propensity to instability manifested through avalanches. Figure 2(b) shows $S_\zeta(\Delta x) \sim \Delta x^{2H}$ with $H = 0.25$. The fact that $H$ is the same for the temporal variations as the spatial variations is at first surprising but plausible by the following argument. If the front of an avalanche moves with velocity $\sim \Delta x/\Delta t$ such that $\zeta(x - \Delta x, t) \sim \zeta(x, t + \Delta t)$, then $S_\zeta(\Delta t) \equiv \langle (\zeta(x, t) - \zeta(x, t + \Delta t))^2 \rangle = S_\zeta(\Delta x)$. Since by definition $S_\zeta(\Delta t) \sim \Delta t^{2H}$, the equivalence of $H$ for the two processes follows.

Figure 3: (a) Probability distribution for the number of grains in an avalanche passing through a given site and (b) Spatial structure function for the number of grains.

Multi channel local flux measurements quantify the evolution of coherent structures moving across a profile. Figure 3(a) shows the probability distribution for the number of grains $n$ passing through randomly located cells through which the avalanches pass, but which are remote from the influence of the fuelling point. This behaviour occurs on the intra-fuelling timescale, and provides a means for quantifying avalanche size in a way that is directly related to the flux of grains. The dashed line in Fig 3(a) shows that the data is well fitted by the two parameter negative-binomial distribution:

$$P(n) = \binom{n + \alpha - 1}{n} \left( \frac{\bar{n}/\alpha}{1 + \bar{n}/\alpha} \right)^n \left( 1 + \frac{\bar{n}}{\alpha} \right)^{n+\alpha},$$  \hfill (1)

with mean $\bar{n}$ and cluster parameter $\alpha$. This distribution arises in diverse physical fluctuation phenomena, ranging from remote sensing of coherent radiation to quantum statistical mechanics, and can be derived from a variety of classical population processes [9]. The cluster parameter measures the degree of bunching and the smaller $\alpha$, the greater the clustering. When $\alpha \to \infty$, (1) tends to the Poisson distribution, which describes un-bunched events. The fitted value $\alpha \approx 0.7$ indicates strong clustering
and is constant across the pile, whereas $\bar{n} \sim x^{1.2}$ describes the local increase in the intermittent flow of grains with distance $x$ down the pile. The spatial variation of $\bar{n}$ is obtained simply by time averaging $n$ at each position in the pile. Figure 3(b) shows the structure function $S_n(\Delta x) = \langle (n(x) - n(x + \Delta x))^2 \rangle$ for the number of grains passing through sites separated by distance $\Delta x$. $S_n(\Delta x)$ measures the accumulation/depletion of grains in an interval, thus gauging the compressibility of an avalanche; it exhibits fractal behaviour with $S_n(\Delta x) \sim \Delta x^{1.8}$. The corresponding effective Hurst exponent is $H = 0.9$, quantifying the expected strong spatial correlation of avalanching flow.

The present statistical analysis of multichannel measurements of avalanching transport in a ricepile expands on techniques applied recently to tokamak edge density fluctuation measurements[1], and yields several intriguing results. First, it is clear that there exist nontrivial physical analogies between measured edge density fluctuations in DIII-D [1] and height fluctuations in a ricepile. We also find that, whereas the spatial and temporal properties of the ricepile profile can be characterised by subdiffusive strange kinetics with the same exponent, the flux quantities are superdiffusive. Application of these techniques to tokamak data will also reveal information on event clustering and the range of timescales (for example, intra-fuelling) involved.

This work was jointly funded by the UK Engineering and Physical Science Research Council, Euratom, the UK Department of Trade and Industry and the Leverhulme Trust.

References