NONLINEAR DYNAMICS OF PARTICLES INTERACTING WITH ELECTRON CYCLOTRON WAVES

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The particle flows produced by the nonlinear ponderomotive (PM) force in a magnetized plasma are studied. An EC wave resonantly absorbed produces a localized wave-amplitude gradient about the resonant surface. A Lagrangian formalism is used to describe average particle trajectories, in order to determine the drifting motion of the guiding center. The equations of motion are numerically solved to show particle orbits for the O- and X-mode. It is shown that the X-mode is more appropriate to produce particle flows.

1 Introduction

The use of Electron Cyclotron Waves for plasma heating is now a common feature of several fusion devices and modern gyrotrons can deliver relatively high powers, ranging in the few to several MW, and they will soon be capable of delivering tens of MW. The use of free electron lasers in the microwave region would deposit up to a few GW of pulsed power. At these powers, nonlinear effects are expected to play an important role in the plasma dynamics. The effects of particle trapping in the wave field have been studied previously, and it has been shown that electron heating takes place in a non-diffusive way [1]. In these studies the wave field amplitude was assumed not to vary with position. However, it is well known that a large spatial dependence of an oscillating field gives rise a nonlinear effect known as the ponderomotive (PM) force. This force is the one responsible for transferring momentum from a wave to the plasma in a non-resonant way, and can produce plasma flows, as it has been shown in several works. This flow generation is relevant for creating high confinement modes in tokamak plasmas, by establishing a sheared radial electric field. In most models the momentum transfer is directly in the poloidal or toroidal direction to produce poloidal or toroidal flows. We have proposed a rotation mechanism [2] based on the action of the radial component of the PM force of electron cyclotron (EC) waves, which can give rise to plasma spin-up, due to the poloidally asymmetric nature of the wave injection. The idea is to launch an EC wave radially inwards, propagating perpendicularly to the magnetic field, which is absorbed at the resonant surface, thus producing a radial gradient of the wave amplitude. For large amplitude microwaves, the associated PM force can induce poloidal particle drifts, which in presence of friction produce the radial plasma flow necessary to trigger the spin-up mechanism [2].

Here we study the particle flows directly from the wave fields, considering both the O-mode and the X-mode, in order to clarify their origin, and to determine which mode is more efficient for the spin-up. Our intention is to show evidence of the PM force, by
analyzing directly the particle orbits. Klima [3] has studied the particle drifts in a high-frequency wave and found the PM force and its resulting drift. We follow a Lagrangian description to obtain the average particle motion, and then visualize the orbits with numerical computations.

2 Particle dynamics

The wave is described through a vector potential \( \mathbf{A} \). The background magnetic field \( B_0 \) is along the \( z \)-axis and the wave propagates along the \( x \)-direction. It is absorbed at the resonant surface \((x = 0)\) and its amplitude is thus a function of \( x \). For the \( O \)-mode we use \( \mathbf{A} = B_0 \hat{x}y + f B_1(x) \cos(kx - \omega t)dx \hat{z} \), where the wave amplitude, \( B_1(x) \), is a specific function of \( x \) given by the way it is absorbed. For our purposes we can assume an exponential dependence: \( B_1(x) = B_1 \exp(-ax) \) with the wave propagating from the left. Since the field gradient in the \( x \)-direction, this will be the direction of the PM force too \((bF_p \sim \nabla E^2)\), and we expect it to produce a drift in the \( y \)-direction. The phase space relativistic Lagrangian, \( L(Q, P, \hat{Q}, t) = P \cdot \dot{Q} - H(P, Q, t) \), with \( P = p + (q/c)A \) the generalized momentum and \( p = m \gamma \mathbf{v} \), the linear momentum, is written, for an electron \((q = -e)\),

\[
L_O = [p - \frac{e}{c} B_0 \hat{x}y - \frac{e}{c} \int B_1(x) \cos(kx - \omega t)dx \hat{z}] \cdot \dot{Q} - mc^2 + p^2)^{1/2},
\]

where the scalar potential is fixed to zero. Now we go to guiding-center coordinates \([1]\), \( p = p_\perp \cos \theta \hat{x} + p_\parallel \sin \theta \hat{y} + u_\parallel \hat{z}\), \( Q = R + (p_\perp/m\Omega) \sin \theta \hat{x} + (p_\parallel/m\Omega) \cos \theta \hat{y}\), where \( \Omega \) is the gyrofrequency and \( R \) the guiding center position, and take the average over the rapidly varying \( \theta \). The wave term can be expanded in series of Bessel functions, while the exponential decay is expressed in a series of modified Bessel function. When averaged, the only term that survives is the one in resonance \( \omega = n\Omega \). For the first harmonic, \( n = 1 \),

\[
\langle L_O \rangle = p_\parallel \dot{z} - c mc^2 + p^2)^{1/2} + \frac{p_\perp^2}{2m\Omega} \frac{\dot{\theta}}{c B_0 XY} - \frac{e B_1 \dot{z}}{c a^2 + k^2} I_0 \left( \frac{a p_\perp}{m\Omega} \right) J_1 \left( \frac{kp_\perp}{2m\Omega} \right) \](1)

\[
e^{aX} \times \left( a \cos(kX + \psi) - k \sin(kX + \psi) \right),
\]

where \( J_1 \) and \( I_0 \) are Bessel and modified Bessel functions which can be approximated by the small argument form: \( J_1(x) \approx x^2/2, I_0(x) \approx 1 \), and \( \psi = \theta - \omega t \) is the relative phase shift. With this Lagrangian we can obtain the six Euler-Lagrange (EL) equations for the variables \( X, Y, z, p_\perp, p_\parallel \) and \( \theta \). We are interested in the drifting motion which comes from the \( X \)-EL-equation which is,

\[
\dot{Y} = \frac{B_1 \dot{z}}{B_0} \frac{k p_\perp}{2m\Omega} e^{-aX} \cos(kX + \psi).
\]

This is still a function of \( \dot{z} \), so we need an equation for it which is obtained from the \( p_\parallel \)-EL-equation, giving: \( \dot{z} = u_\parallel/m \gamma \). Thus the equation for \( u_\parallel \) is now needed which comes from the \( \dot{z} \)-EL-equation. The resulting equation for \( \dot{z} \) is,

\[
\dot{z} = \frac{e B_1}{c \gamma 2m\Omega a^2 + k^2} \left( a \cos(kX + \psi) - k \sin(kX + \psi) \right).
\]

(2)
Inserting eq. (2) into eq. (1) gives the drift velocity in the y-direction,

\[ \dot{Y} = \frac{B_i^2}{B_0} \frac{e}{m\gamma c} \left( \frac{k p_\perp}{2m\Omega} \right)^2 e^{-2\alpha X} \cos(kX + \psi) (a \cos(kx + \psi) - k \sin(kX + \psi)), \]

which is proportional to the gradient of the wave field squared (∼ (B_i e^{-\alpha x})^2), as it would be expected for the PM-force-induced drift. Therefore, this represents the particle flux produced by the wave absorption near the resonant surface. By contrast, the equation for \( \dot{X} = 0 \). The other EL equations describe the way particles interchange energy with the wave in the perpendicular plane, and are important to study plasma heating. There are some trapping regions in the phase space \( p_\perp - \theta \) that redistribute the electron population, which initially have low energy, thus giving a net energy gain. This regions also affect the orbits, modifying them from the uniform cyclotron motion, as it will be shown in the next section.

Now we turn to the X-mode. The potential is taken as, \( \mathbf{A} = -A_1(x) \cos(kx - \omega t) \mathbf{x} + (B_0 x + A_1(x) \sin(kx - \omega t) \mathbf{y} \), which gives a circularly polarized \( E \)-field in the plane perpendicular to \( \mathbf{B_0} \), in the electron gyro-motion direction. This is used in the relativistic Lagrangian as before, and the later is transformed to guiding center variables and then averaged over the gyro-angle. Here again, the wave contributions are expanded in Bessel and modified Bessel functions, surviving the resonant terms, obtaining now the more complicated Lagrangian,

\[ \langle L_x \rangle = p_{\parallel} \dot{z} - c(m^2 + p_{\parallel}^2)^{1/2} + \frac{p_{\perp}^2}{2m\Omega} - \frac{e}{c} B_0 \dot{X} Y + \frac{e}{c} A_1 e^{-\alpha x} J_n \left( \frac{k p_\perp}{m\Omega} \right) \left\{ I_0 \left( \frac{ap_\perp}{m\Omega} \right) \right\}, \]

where \( \Psi \equiv kX + \psi. \) In this case, since the X-mode is more absorbed at the second harmonic, we evaluate the Lagrangian at \( n = 2 \) and take the approximations \( J_2(x) \approx x^2/8 \) and \( I_1(x) \approx x/2. \) The EL equation that gives the drift velocity in \( y \) is,

\[ \dot{Y} = \frac{k^2 p_{\perp}^2 \dot{B}}{8m^2 \Omega^2 B_0 + k^2 p_{\perp}^2 \dot{B} \left( 2m^2 \Omega^2 + \frac{p_{\perp}^2}{k \cos \Psi - a \sin \Psi} \left( (1 - \left( \frac{2m\Omega}{ap_\perp} \right)^2 \right) \cos \Psi + \frac{k}{a} \sin \Psi \right)] \]

where \( \dot{B} = A_1 e^{-\alpha X} \left( k \cos \Psi - a \sin \Psi \right). \) This is dependent on \( \dot{\theta} \) and \( \dot{p}_{\perp}, \) and not on the parallel motion (\( \dot{z} \)) unlike the O-mode, so the EL equations for these variables are needed. The coupling of all four equations for \( X, Y, \theta \) and \( p_\perp \) makes the resulting expression for \( \dot{Y} \) very cumbersome, involving many different terms. Because of the different effects present in eq.(4), we expect the drift to be larger for the X-mode than for the O-mode. We point out that there is a term in eq.(4) that is the product of the field amplitude and its gradient, and is directly related to the PM force: \( \dot{Y} \sim \dot{B} d\dot{B}/dx. \)

3 Numerical computations of orbits

In order to visualize the particle orbits and to have a clearer view of the origin the drifts obtained with the Lagrangian method, we now integrate the equations of motion
numerically with the wave-fields of the two EC modes considered before. The orbits are computed in the three-dimensional space but we only show the projection in the $x - y$ plane, normal to the background field. In Figure 1 we show electron trajectories, as well as the velocity vector, for the two EC modes. The magnetic field points inwards, and the wave comes from the left. The spatial dependence of the amplitude is $e^{-ax}$ for $x > 0$ and constant for $x < 0$ with $a$ taken equal to the electron gyroradius. This simulates a wave being absorbed at a resonant surface, within a distance of order $a$, but unaffected at larger distances before reaching the surface. The wave field is 20% of $B_0$. For the O-mode we considered a wave in resonance at the fundamental harmonic, $w \equiv \omega/\Omega = 1$. As we can see there is a drift motion in the direction of the negative $y$, but since the particle is gaining energy from the wave its gyroradius is increasing, and therefore the particle itself does not have a displacement from the top, although the guiding center does. When the resonance condition is not satisfied, the gyroradius is kept approximately constant and the particles are displaced downwards, too. This drift is the result of the $x$-dependence of the amplitude, and therefore it may be interpreted as the result of the $F_p \times B$ drift of the PM force $F_p$. For the X-mode, we also show a resonant orbit but for the second harmonic, $w = 2$. The behavior is similar to that for the O-mode except that the drifting motion is faster. This indicates that the X-mode is more efficient is producing particle fluxes, as it was also determined in the previous section. We also note that the irregular orbits, which are more prominent for resonant particles, are the result of trapping effects in the wave fields, that can be studied with the Lagrangian description.

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