MODELLING OF CORE-LOCALIZED ALPHA DRIVEN ALFVÉN EIGENMODES IN TFTR-LIKE TOKAMAKS

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1. INTRODUCTION

Experiments in JET, TFTR, NSTX etc. demonstrated the possible destabilization of toroidicity induced Alfvén eigenmodes (TAE) by energetic ions. Here we examine alpha driven TAEs that were first observed in TFTR with reduced central magnetic shear and elevated central safety factor \( (q(0)>1) \) following the end of neutral beam injection [1]. Our numerical modeling is based on the alpha distribution function obtained from a 3D Fokker-Planck equation numerically solved both in the axisymmetric limit [2] and in the presence of weak TF ripples [3]. Stochastic and collisional ripple diffusion are also considered. Previous results [4,5] showed the alpha driven TAE mode growth rate to be strongly affected by the magnetic field geometry as well as by finite orbit width effects. Here we additionally analyze possible mechanisms of TAE mode damping and compare the corresponding damping rates with the mode growth rates in order to clarify the TAE destabilization by fast alphas.

2. METHOD USED

Following [6] the alpha driven TAE mode growth rate \( \gamma_\alpha \) may be expressed as

\[
\frac{\gamma_\alpha}{\omega} = \frac{\text{Im} \delta W_k \left( \xi^*, \xi_n, \omega_n \right)}{2 \omega_n^* \delta K \left( \xi^*, \xi_n \right)}
\]

(1)

where \( \xi \) is the plasma displacement, \( \omega \) the TAE eigenfrequency and \( \delta K = \int d^3x |\xi|^2 \); the subscript 0 denotes the zeroth order of a small parameter related to weak kinetic effects. The kinetic integral \( \delta W_k \) was expressed and evaluated as in [4,5] as a function of the distribution function \( F_\alpha \) of high-energy alphas, of the poloidal and toroidal mode numbers \( m \) and \( n \), and of the alpha drift motion invariants. As phase space we chose the constants-of-motion space with the particle velocity \( V \), the normalized adiabatic invariant \( \lambda = \mu B_o / E \) and the square root of the maximal toroidal flux on the trajectory, \( R_m \), as coordinates. \( F_\alpha \) was derived from a 3D Fokker-Planck equation both in the axisymmetric limit as well as involving toroidal field ripple effects [3]. The alpha source profile was taken as a combination of beam-plasma and bulk plasma sources [2]. For the model profile of the safety factor we assumed
\[ q(r) = q(0) + [q(1) - q(0)] r^2 \] with \( r \) denoting the flux surface radius normalized to the plasma radius \( a \).

Considering TAE damping mechanisms we note that, according to [7,8], continuum damping of TAE modes should be substantial for low-\( n \) as for high-\( n \) modes. Referring to core localized TAE’s with \( n \approx 4 \), as treated in this study, the continuum damping may be described (for a magnetic shear \( S = \frac{r q'}{q} \geq 0.3 \) and \( 1 < \hat{m} \varepsilon < 20 \)) by [8]

\[
\frac{\gamma_c}{\omega} = -0.8 \varepsilon S^2 \left[ \frac{\partial \ln (q^2 / V_A^2)}{m \varepsilon} \frac{\partial \ln (q^2)}{m \varepsilon} \right]^{3/2} = -\frac{0.8 \varepsilon S^2}{(m \varepsilon)^{3/2}}
\] (2)

where \( \varepsilon \) is the poloidal harmonics toroidicity coupling strength and \( V_A \) the Alfvén velocity.

The usual electron Landau damping is small here due to \( V_{e,\text{thermal}} >> V_A \), but Landau damping on resonant electrons becomes asymptotically larger as \( S \to 0 \) [9]. As TAE damping due to thermal particles we will consider Landau damping on bulk plasma ions and collisional damping on trapped electrons. Since here is \( V_i << V_A \) we use the expression for ion Landau damping on the sideband resonance \( V_A / 3 \) given in [10] and describe the TAE damping caused by the collisionality of trapped electrons also according to [10].

**3. RESULTS OF MODELING**

We used typical TFTR plasma parameters and supposed that the hypothetic TAE modes with \( n=4 \) are strongly localized near three given radial positions, i.e. \( r = 0.3, 0.4 \) and 0.5. The mode resonant structure as varied by the value of \( q(0) \) is illustrated in Fig. 1 for a mode localization at \( r =0.3 \). Fig.2 exhibits the total mode growth rate calculated for a plasma current \( I_p = 2 \text{MA} \) at different radial mode localizations. Strong destabilization by alphas is seen at \( r =0.4 \) due to the enhanced ripple induced radial transport of fast alphas generating a substantial additional anisotropy of \( F_\alpha \) in this region.

To clarify the possibility of TAE mode destabilization by high-energy alphas we compare mode growth and damping rates. Their dependences on the shear of the safety factor are displayed in Fig.3. The modes localized at \( r =0.3 \) may be destabilized by energetic alphas only in a narrow region around \( S=0.24 \). Shifting the mode resonant position to \( r=0.4 \) the total damping rate remains practically unchanged for the same magnetic shear value, however, as plotted in Fig.4, there occurs a significant enhancement of the mode growth rate, which is caused by the ripple induced anisotropy of \( F_\alpha \). Thus at this radial position fast alphas may destabilize TAE modes in a wide range of shear values \( \leq S=0.6 \). With a further shift to \( r=0.5 \) we observe by inspection of Fig.5 again only little variation of the total damping rate,
whereas the TAE growth rate is drastically decreased in this region. We may conclude that energetic alphas will not be able to destabilize TAE modes localized far away from $r = 0.4$.

Determining the influence of high-energy alpha particle finite orbit width effects and their transport processes on the results derived above, we chose a case for which the TAE growth rate is just below the total damping rate and performed a set of calculations for different total plasma currents $I_p$. We see in Fig.6 that, in the axisymmetric approximation, the calculated TAE mode growth rate decreases with a reduced plasma current, which proves the stabilizing role of the finite orbit width for alpha-driven TAE modes. On the other hand, accounting for TF ripple effects our calculations demonstrate enhanced growth rates due to the ripple induced anisotropy in $F_\alpha$. As evident from Fig.6, an initially stable mode at $I_p = 2 \text{MA}$ (starting point with $S = 0.3$ in Fig.3) may be destabilized by energetic alphas with plasma current increase.

4. CONCLUSION

The presented numerical modeling gives at least a good qualitative picture of the core localized TAE mode destabilization by energetic alphas. For TFTR experimental conditions it predicts that fast alphas may destabilize TAE modes localized at $-0.4a$ and for a safety factor shear less than 0.6 at this point; hence the central safety factor should be elevated to about 1.3 for a total plasma current $\sim 2 \text{MA}$. The maximum of wave-particle interaction is shifted to higher bounce frequency numbers as $q(0)$ is increased. Our results are in satisfactory agreement with corresponding experimental observations in TFTR [1].

REFERENCES


Acknowledgement

This work has been partially carried out within the Association EURATOM-OEAW project P4 and the impact project Investigation of Charged Fusion Product Confinement in JET funded by the Austrian Academy of Sciences under GZ 4229/1-VIII/A/5/2000.
Fig. 1: Partial TAE mode growth rates versus the bounce frequency harmonic number $p$ for several values of the central safety factor. The maximum of the wave-particle interaction rate is shifted to the higher harmonic number region as $q(0)$ increases.

Fig. 2: TAE mode growth rate as a function of the safety factor at the magnetic axis for different radial positions of mode localization. Maximum of the growth rate is observed at $r=0.4$.

Fig. 3: TAE mode growth and damping rates versus the shear of the safety factor for the mode localized at the radial position $r=0.3$. One may expect mode destabilization by alphas for $S$ about 0.25.

Fig. 4: TAE mode growth and damping rates versus the shear of the safety factor for the mode localized at the radial position $r=0.4$. Fast alphas destabilize the mode for $S<0.6$.

Fig. 5: TAE mode growth and damping rates versus the shear of the safety factor for the mode localized at $r=0.5$. Fast alphas cannot destabilize the mode.

Fig. 6: TAE mode growth and damping rates versus the total plasma current for the mode localized at $r=0.3$ with $S=0.3$ for $I_p=2$MA.