NONLINEAR CALCULATION OF THE COLLISIONALITY DEPENDENCE OF NEOCLASSICAL TEARING MODES

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The paper 1 has suggested to describe the island width evolution of neoclassical tearing modes (NTMs) by the equation of the following structure

\[ \frac{dw}{dt} \sim \frac{\Delta'}{4} + \Delta_{bs} + \Delta_p. \]  

(1)

Here \( w \) is the island halfwidth, \( \Delta' \) is the standard matching parameter of the linear tearing mode theory (we have introduced the factor \( 1/4 \) for convenience), \( \Delta_{bs} \) and \( \Delta_p \) are the contributions of bootstrap current and polarization current, respectively. It was assumed in [1] that \( \Delta_p \) is proportional to a function \( g(\epsilon, \nu_i) \) characterizing the collisionality dependence of NTMs,

\[ \Delta_p \sim g(\epsilon, \nu_i), \]  

(2)

where \( \nu_i \) is the ion collision frequency, \( \epsilon \) is the inverse aspect ratio for the resonant magnetic surface. Constructing the factor \( g(\epsilon, \nu_i) \), reference 1 was guided by the results of [2, 3]. Reference 2 derived the factor \( g(\epsilon, \nu_i) \) for the case of weak ion collisions, \( \nu_i / (\epsilon \omega) \ll 1 \), solving the ion drift kinetic equation (\( \omega \) is the island rotation frequency in the reference frame where the equilibrium radial electric field vanishes). According to [2], within the accuracy of a numerical factor of the order of unity, \( g(\epsilon, \nu_i) \approx \epsilon^{3/2} \). In contrast to [2], reference 3 dealt with the strongly collisional version of the neoclassical MHD (magnetohydrodynamics) and showed that in this case \( g(\epsilon, \nu_i) \approx 1 \). It was assumed in [1] that this estimate, i.e., \( g(\epsilon, \nu_i) \approx 1 \), is valid up to \( \nu_i / (\epsilon \omega) \approx 1 \), so that one can introduce a critical ion collision frequency \( \nu_i^{crit} \approx \epsilon \omega \) separating the regions of small and large values of the factor \( g(\epsilon, \nu_i) \), i.e., \( g(\epsilon, \nu_i) \approx \epsilon^{3/2} \) and \( g(\epsilon, \nu_i) \approx 1 \), respectively. At the same time, to interpret the experimental data on NTMs, described in [1], it was noted in this reference that \( \nu_i^{crit} \) should be shifted to a value smaller than \( \epsilon \omega \), taking \( \nu_i^{crit} \approx 0.3 \epsilon \omega \).
Moreover, as explained in [4], for better interpretation of the experimental data, one should take \( \nu^\text{crit} \approx 0.03 \epsilon \omega \).

The problem of the collisionality dependence of the factor \( g(\epsilon, \nu_i) \) was then analyzed in [5]. The idea of such an analysis has been based on that the value \( \Delta_p \) calculated in [2, 3] can be estimated by means of the linear theory of neoclassical MHD modes using some “correspondence rules”. Then, allowing for the results of this theory given in [6] and papers cited therein, one can find that, really, the critical ion collision frequency \( \nu^\text{crit}_i \) should be shifted from “its natural value” \( \epsilon \omega \), however, one should shift this collision frequency only to larger values, i.e., \( \nu^\text{crit}_i > \epsilon \omega \), but not to smaller values suggested in [1, 4]. Therefore, according to [5], the experimental data described in [1, 4] could not be interpreted in the scope of the theories developed in [2, 3], while the collisionality dependence \( g(\epsilon, \nu_i) \) needed for satisfactory interpretation of these data should be considered as an \textit{ad hoc} dependence not following from the above theories.

In its turn, it is questionable that whether the estimations of linear theory are suitable for the problem of magnetic islands which are essentially nonlinear phenomena. Therefore, it seems to be reasonable to develop the magnetic island theory for arbitrary collisionality and to find the function \( g(\epsilon, \nu_i) \) directly from such a theory. This is the goal of the present work. As a result, we show that the nonlinear theory corroborates the above suggestion of [5].

Our main task is to calculate the polarization current density \( J_p \). Then the value \( \Delta_p \) entering (1) is obtained by means of the standard formula

\[
\Delta_p = -\frac{\pi}{2c\psi} \sum_{\sigma_x} \sigma_x \int_\psi^{\infty} d\psi \int \frac{d\xi}{\psi_x} J_p \cos \xi. \tag{3}
\]

All the definitions can be found in [6]. We look for \( J_p \) using the neoclassical current continuity equation

\[
\nabla || J_p = -\frac{e \rho_0}{B_0} \frac{\partial}{\partial x} \left( \frac{d_0 V_{||}}{dt} \right), \tag{4}
\]

where \( V_{||} \) is the parallel plasma velocity. To find \( V_{||} \) we use the MHD neoclassical model following from chapter 19 of [7]:

\[
\frac{d_0 V_{||}}{dt} = -\frac{\epsilon}{q} \tilde{\chi}_\theta \left( V_{E\theta} + \frac{\epsilon}{q} V_{||} \right), \tag{5}
\]

where

\[
\tilde{\chi}_\theta = \chi_\theta + \frac{q^2}{\epsilon \nu^{3/2}} \frac{d_0}{dt} \tag{6}
\]
and \( \chi_\theta = q^2 \nu_i / \varepsilon^{3/2} \). For our problem (5) reduces to

\[
\omega h' \psi_x \frac{\partial V}{\partial \xi} = \varepsilon^{1/2} \nu_i \nabla V = -\varepsilon^{1/2} \frac{\omega^2 B_0 \tilde{\psi}}{L_y} (h')^2 \sin \xi - \frac{q \nu_i \omega}{\varepsilon^{1/2} k_y} h' (\psi_x - \langle \psi_x \rangle),
\]

(7)

where, as in [8, 6], \( h' \) is the cross-field velocity profile function.

Instead of cyclic variable \( \xi \), we introduce the canonical cyclic variable \( \tau \) explained in [9]. Then (7) is transformed to

\[
y + \frac{f \omega}{\gamma_T S} \frac{\partial y}{\partial \tau} = -\frac{2^{3/2}}{\kappa} \left( \frac{dn}{\kappa} - \langle dn \rangle \right) - \frac{2^{3/2}}{\kappa} \frac{f \omega}{\gamma_T S} \frac{dn}{d\tau},
\]

(8)

where \( y \) and \( f \) are normalized \( V_\parallel \) and \( h' \), respectively, \( \gamma_T = \varepsilon^{1/2} \nu_i \), \( \kappa \) is the standard dimensional parameter marking the island magnetic flux surfaces (see in detail [9]), \( S = S(\kappa) \) is the function explained in [9], \( \frac{dn}{dn} = \frac{dn}{\kappa} \) is the Jakobian elliptical function, \( u = K \tau / \pi \), \( K = K(\kappa) \) is the complete elliptical integral of the first kind. We use the expansion [10]

\[
dn - \langle dn \rangle = \frac{2\pi}{K} \sum_{l=1}^{\infty} \Lambda_l \cos (l\tau),
\]

(9)

where the explicit form of \( \Lambda_l \) can be found in [10]. It then follows from (8) that

\[
y = \sum_{l=1}^{\infty} y_l \cos (l\tau),
\]

(10)

where

\[
y_l = -\frac{4}{S} \Lambda_l \frac{1 + l^2 (f/S)^2 \varepsilon^{3/2} (\omega/\gamma_T)^2}{1 + l^2 (f/S)^2 (\omega/\gamma_T)^2}.
\]

(11)

To use the above results one should know the profile function \( h' \). Assuming this function to be regularized by the hyperviscosity, we take \( h' \) in the form given by Eq. (77) of [8]. Then we calculate numerically the value \( \Delta_p \) as a function of the parameters \( \nu_i / \omega \) and \( \epsilon \), \( \Delta_p = \Delta_p (\nu_i / \omega, \epsilon) \). The function \( g(\epsilon, \nu_i) \) is defined by

\[
g(\epsilon, \nu_i) = \frac{\Delta_p (\nu_i / \omega, \epsilon)}{\Delta_p (\infty, \epsilon)}.
\]

(12)

This function is plotted in Fig. 1 together with similar function predicted by the linear theory [5]. One can see from this figure that the predictions of the linear and nonlinear theories are qualitatively the same.
Fig. 1. Collisionality dependence of the polarization current contribution $\Delta_p$ in linear (dashed, black, right curve) and nonlinear (solid, red, left curve) theory.