COUPLED PEELING-BALLOONING MODES: A MODEL FOR ELMs AND THE TEMPERATURE PEDESTAL?

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In this paper we discuss the MHD stability of the tokamak plasma edge, and the possible implications for ELMs and the temperature pedestal. ELMs are a particular concern for Next Step tokamaks (eg ITER) because of the large transient heat loads they can cause, while the temperature pedestal is important because it has a large influence on the overall confinement. The two key MHD instabilities we explore are the kink, or peeling, mode (current-driven) and the ballooning mode (pressure driven), both of which can be unstable at moderate to high toroidal mode number, \( n \) at the edge [1,2]. Our computational studies will consider \( n \) in the range 5<\( n <31 \), spanning the range typically observed to be involved in ELMs.

When considered individually, the equations describing peeling and high \( n \) ballooning modes can be reduced to a 1D system, but generally these modes couple through toroidicity and then the mode structure is truly 2D [1]. Thus, in order to analyse moderate to high \( n \) edge MHD modes, we have developed the ELITE code. ELITE is based on the linearised ideal MHD equations and, for a given \( n \), employs an expansion in poloidal Fourier harmonics (making use of a ‘straight field line’ angle). The variational energy is expanded in \( n \), retaining the first two orders, from which the set of Euler equations for the radial variation of the Fourier amplitudes, \( u_m \), is derived. At high \( n \) there is a need to retain many poloidal Fourier harmonics but, because of the strong effect of field line bending, each \( u_m \) tends to be localised about its own rational surface (ie where \( m=nq \), \( q \) is the safety factor); as a result, only a limited number of poloidal harmonics are important at each radial position. ELITE exploits this to allow efficient retention of a large number of poloidal harmonics. We have performed a careful benchmark with the ideal MHD version of the MISHKA code [3]. A circular cross-section equilibrium was chosen, calculated to be unstable to \( n=\infty \) ballooning modes, and the predictions for the growth rate as a function of \( n \) were compared. Figure 1 shows the good agreement achieved over a range of \( n>4 \). Figure 2 shows the mode structure obtained from ELITE for \( n=20 \); Fig 2a shows the radial variation of the individual Fourier harmonics which clearly illustrates their localised nature, while Fig 2b shows the mode
structure in the poloidal cross-section
(the shaded area indicates the plasma region analysed by ELITE), confirming
the ballooning nature of the mode.
The benchmarked code ELITE has been
used to explore the edge stability of
shaped tokamak plasmas. The important
variables which govern stability are the
density and pressure gradient. In order to parameterise these
in terms of variables which are more accessible experimentally, we first fix the form of the
density and temperature profiles (the density profile being rather flat, and the temperature
profile being somewhat more peaked, but with a broad pedestal region penetrating ~20% of
the minor radius). An up-down symmetric ‘D’-shaped cross-section plasma was chosen,
having major radius $R=3m$, elongation $\kappa=1.6$ and aspect ratio $A=3$, and two triangularities,
$\delta=0.3$ and $0.5$, were studied. The current profile consists of the sum of Pfirsch-Schlüter,
diamagnetic, bootstrap and Ohmic contributions (the latter two taken from neo-classical
theory, allowing for collisionality corrections), so that the edge current density is largely
determined by normalised pressure, $\beta_N$, and the pedestal temperature, $T_{\text{ped}}$, while the edge
pressure gradient is parameterised by $\beta_N$. Thus we can characterise the stability of the
plasma edge in terms of these two parameters. An $n=\infty$ ballooning stability analysis
indicates that this class of equilibria has access to the second stability regime.
Figure 3 shows the results of stability analyses for a range of equilibria, obtained by scaling
the density and temperature profiles to map out a space in $T_{\text{ped}}$-$\beta_N$. The boundary between
the squares (unstable) and triangles (stable) represents the stability boundary; \( n \) in the range \( 5 < n < 31 \) were considered. Note that \( T_{\text{ped}} \) is limited by the edge MHD stability, though higher \( T_{\text{ped}} \) is tolerable at higher \( \delta \). At the lower \( \beta_N \) values the eigenfunction is extremely localised in the last few percent of the plasma minor radius (Fig 4a). This instability is essentially a peeling mode, driven by the edge current density (higher \( T_{\text{ped}} \) reduces the edge collisionality, enhancing the bootstrap current). As \( \beta_N \) is increased, two things happen: (1) a higher \( T_{\text{ped}} \) is achievable because of the stabilising influence of the Mercier coefficient [1]; (2) the modes begin to couple strongly to the ballooning mode, and thereby become more radially extended (Fig 4b). Indeed, for the intermediate \( n \) modes, a qualitative WKB analysis indicates that the radial extent of the mode may be comparable to the pedestal width and therefore these modes are likely to have a rather deleterious effect on both confinement and the exhaust power loading.

Turning now to the pressure limiting modes, at higher \( T_{\text{ped}}, \beta_N \) is limited by intermediate \( n \) modes, typically \( n \approx 6-8 \) (Fig 5); note that in this region the finite edge current would be sufficient to provide second stability access for \( n = \infty \) ballooning modes. At lower \( T_{\text{ped}}, \beta_N \) is limited by higher \( n \) modes, and this is consistent with reduced access to second stability. The

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**Figure 3:** Stability diagram for peeling-ballooning modes for (a) \( \delta=0.3 \) and (b) \( \delta=0.5 \).

**Figure 4:** (a) Mode structure for a \( n=13 \) peeling-type mode with \( T_{\text{ped}}=2.2keV, \beta_N=1.5 \) and (b) for a \( n=8 \) coupled peeling-ballooning mode with \( T_{\text{ped}}=3.4keV, \beta_N=2.5 \)
drive for the intermediate $n$ modes at higher $T_{\text{ped}}$ is expected to be due to the current gradient drive, which increases with $\beta_N$ due to the bootstrap current. To verify this, we have provided a small amount of negative edge-localised current drive (~7% of the plasma current); this does indeed stabilise the intermediate $n$ modes, allowing ~25% increase in $\beta_N$ before the $n=6$ mode is again destabilised. However, too much edge current drive is also bad; for example doubling the negative edge current drive is again destabilising for intermediate $n$ modes.

So far we have restricted consideration to an ideal MHD plasma model. However, we can estimate the importance of diamagnetic effects [4,5] by comparing the MHD growth rate with the ion diamagnetic frequency, $\omega_i$. We find that $\omega_i$ is indeed in the range where diamagnetic stabilisation of even the intermediate $n$ modes is possible, particularly for higher triangularity (where MHD growth rates are smaller) and steeper pedestal profiles. This suggests a scaling for reduced ELM amplitudes due to diamagnetic stabilisation (ideal MHD instabilities would then likely be replaced by weaker, dissipative instabilities):

$$nq_A \frac{P_i}{L_p} \sqrt{\beta} > 2 \frac{\gamma_{\text{MHD}}}{\omega_A} \quad \left( \text{typically } \frac{\gamma_{\text{MHD}}}{\omega_A} \ll 1 \right)$$

where $\rho_i$ is the ion larmor radius, $L_p$ is the pressure length scale and $\omega_A$ is the Alfvén frequency.

In summary, we have shown that edge MHD instabilities can limit $T_{\text{ped}}$, and that the amplitude and radial extent of ELMs (and hence their impact on confinement and exhaust) can be influenced by the operating point in $\beta_N$-$I_{\text{edge}}$ space and diamagnetic effects. Indeed, negative current drive at the plasma edge helps to stabilise the edge MHD and therefore may also be beneficial for ELM control [6]; furthermore the effect of varying the pedestal width may also have an influence, and this, together with diamagnetic effects, will be the subject of future work.

**References**


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