Fokker-Planck studies of the superthermal electron population
in the edge of reversed field pinch plasmas

E. Martines\textsuperscript{1}, M. Shoucri\textsuperscript{2}, R. Bartiromo\textsuperscript{1}
\textsuperscript{1}Consorzio RFX, corso Stati Uniti 4, 35127 Padova, Italy
\textsuperscript{2}Centre Canadien de Fusion Magnétique, Varennes, Canada

A dynamo process takes place in plasmas confined in reversed field pinch (RFP) configuration [1]. The process drives the edge poloidal current required for sustaining the toroidal magnetic field reversal against resistive diffusion. A superthermal electron tail has been detected in the edge region of RFP plasmas. The properties of this tail have been extensively studied with electrostatic electron energy analysers (EEAs) and other edge probes. In particular, in the edge of the RFX device the ratio of the energy fluxes measured in the electron and on the ion drift sides has been found to be around 3 [2]. In the same machine EEA measurements have revealed on the electron drift side a superthermal tail with a temperature ranging for normal plasma density (3-5×10^{19} \text{m}^{-3} \text{for } 600 \text{kA discharges}) between 3 and 5 times the temperature measured on the ion drift side using Langmuir probes [3]. The issue of the origin of the tail is tightly related to the nature of the dynamo process. The two main theories on the RFP dynamo are the MHD dynamo and the kinetic dynamo theory (KDT). According to the former, the edge poloidal current is driven by a dynamo electric field resulting from the quadratic effect of magnetic field and plasma velocity fluctuations [1]. The latter suggests that it is due to a field-aligned electron momentum transport caused by the magnetic field stochasticity [4]. In the MHD framework the superthermal tail is thus locally generated, whereas in the KDT it is transported from the core.

In this paper we have numerically computed the electron distribution function in the MHD dynamo framework. The distribution function results from the balance between the acceleration due to the field-aligned electric field, including the dynamo field, and the slowing-down due to Coulomb collisions. The calculation is performed with a 2D Fokker-Planck code [5] which yields the electron distribution function $f(v_{//},v_{\perp})$, where $v_{//}$ and $v_{\perp}$ are the velocity components parallel and perpendicular to the magnetic field. The code treats the electron-electron collisions using the linear approximation of the Fokker-Planck collision term. The inputs to the code are the ratio $E_{//}/E_{c}$ of the parallel electric field (applied+dynamo) to the critical field for runaway generation, the effective charge $Z$ and the ratio of electron and ion temperatures $T_{e}/T_{i}$. This last parameter has been assumed to be equal to 1 in the present work. The critical electric field is defined as

$$E_{c} = \frac{1}{(4\pi\varepsilon_{0})^{2}} \frac{4\pi e^{3}n \ln \Lambda}{T_{e}}$$

where $n$ is the plasma density and $\ln \Lambda$ is the Coulomb logarithm.
The simulations have been performed on a mesh of $201 \times 100$ points in the $(v, \theta)$ space, where $v$ is the velocity modulus and $\theta$ is the azimuthal angle between $v$ and $B$. The mesh extension was from 0 up to $15 v_t$, where $v_t = (T_e/m)^{1/2}$ is the electron thermal velocity. The time step was taken equal to $\tau/2$, having defined the collision time $\tau$ as $mv/eE_c$. The code was run for 2000 time steps, which was enough to obtain a steady state. Simulations were made for $Z=1,2,3$, varying $E_\parallel/E_c$.

The curves in fig.1 display the parallel distribution function (averaged over $v^\perp$) at several values of $E_\parallel/E_c$, for the case $Z=2$. The lowest energy part, most relevant for the comparison with experimental results, is shown. As $E_\parallel/E_c$ is increased a strong tail develops for $v_\parallel > 0$ (this is referred to as electron drift side in the experimental literature). This could be the superthermal tail experimentally observed. On the ion drift side ($v_\parallel < 0$), the tail is depleted for velocities lower than a certain $v_0$, whereas at higher velocities a superthermal tail develops as well. The depletion is due to the electric field pushing electrons from the $v_\parallel < 0$ to the $v_\parallel > 0$ region of the phase space, and the tail at high energies is formed because of electrons with $v_\parallel > 0$ undergoing pitch-angle scattering. The value of $v_0$ increases with $E_\parallel/E_c$, being around 3-5 times $v_t$ for $0.03 \leq E_\parallel/E_c \leq 0.07$. As a consequence the effective temperature of the tail on the ion drift side decreases as $E_\parallel/E_c$ rises, if it is measured for energies below $mv_0^2/2$.

The current density computed from the distribution function, normalised to $en v_t$, is plotted in fig.2 as a function of $E_\parallel/E_c$. On the same graph the current density expected according to Spitzer resistivity for $Z = 1$ is also represented. For the case $Z = 1$ the code correctly reproduces the Spitzer resistivity for low $E_\parallel/E_c$, whereas at higher values a deviation is observed. Such a deviation can be attributed to the fact that the Spitzer approach is valid only to first order in $E_\parallel/E_c$, and also to the code itself.
becoming less accurate as $E_{\parallel}/E_c$ is increased, since it neglects the collisions of the distribution function tail with itself.

The energy flux collected by a floating probe is given by

$$q = 2\pi \int_0^\infty v_\perp dv_\perp \int_{-eV_f} v_{\parallel f} \left( \frac{1}{2}mv^2 + eV_f \right) v_{\parallel f} f(v_{\parallel f}, v_\perp)$$

In this expression $V_f$ is the floating potential, which is evaluated imposing equal electron and ion fluxes on the probe, and therefore depends as well on the electron distribution function shape. In the present context, it has been evaluated in the simplifying assumption of negligible secondary electron emission. The ratio of the energy fluxes on ion and electron energy sides, called $q_1$ and $q_2$ respectively, is plotted in fig.3 as a function of $E_{\parallel}/E_c$. This ratio grows very fast when the electric field, and easily attains the values between 3 and 10 which in all RFP experiments have been interpreted as a signature of the superthermal electron population. In particular, the values measured in RFX [2] suggest that in this machine $E_{\parallel}/E_c \approx 0.03$. This result allows to infer some information on the actual value of the dynamo field in RFX. In fig.4 contour lines at equal $E_c$ in the $n-T_e$ plane are shown, together with experimental points obtained with Langmuir probes in the RFX edge [2]. The points have been collected at different radial positions in the last centimetre of confined plasma. The graph shows that in this region the critical electric field varies by a large extent, between 30 and 300 V/m. In the same region, $q_2/q_1$ is almost constant, and so is, according to fig.2, $E_{\parallel}/E_c$. It is therefore possible to conclude that $E_{\parallel}$ is also rapidly changing with $r$ in the last centimetre of plasma, rising from 1 to 10 V/m (comparable to the toroidal on-axis electric field). In this calculation a contribution of the inductively applied electric field has been included. This contribution is around 0.4 V/m (for $\Theta = 1.5^\circ$ and $F = -0.2$). The current
density associated to this electric field is in the range 150-600 kA/m², compatible with estimates coming from EEA data [3].

Another important information is that concerning the slope of the tail on the electron drift side. Since this is usually compared to the temperature measured on the ion drift side (considered free of superthermal electrons), we have first estimated this quantity as the slope $T_1$ of $f(v_\parallel)$ on the ion drift side computed for velocities smaller than $v_0$. The results, normalised to the electron temperature $T_e$, are shown in fig.5.

Fig.5: Normalised temperature measured on the ion drift side plotted as a function of $E_\parallel/E_c$.

For the RFX case the Langmuir probe measurement is smaller than the true temperature by a factor between 10% and 50%, depending on the effective $Z$.

The slope $T_2$ of the $f(v_\parallel)$ normalised to $T_1$ is shown in fig.6. This quantity best compares to the experimental data regarding superthermal electrons in RFP's. The slope has been computed for velocities ranging between 3 and 15 times $v_i$, a range comparable to that used in EEA measurements. The ratio can reach high values, especially at low $Z_{eff}$. For the RFX case, the values are comparable to the experimental ones, assuming $Z_{eff}$ to be lower than 2.

In conclusion, the data about superthermal electrons in the RFP edge can be explained, at least for the RFX device, assuming a local balance between dynamo electric field and collisions.

References