Method of density profile reconstruction using pulse broadening measurements, and density fluctuation effects

S. Hacquin, S. Heuraux, G. Leclert*, M. Colin and I. Boucher

Laboratoire de Physique des Milieux Ionisés, Université Henri Poincaré de Nancy I
Unité CNRS 7040 BP 239 54 506 Vandoeuvre-lès-Nancy Cedex (France)
(*) Laboratoire PIIM (Turbulence Plasma), Unité 6633 CNRS-Université de Provence,
Centre de St-Jérôme, Case 321, 13 397 Marseille Cedex 20 (France)

1. Introduction
Reflectometry is a promising diagnostic for density profile determination on the future generation of fusion devices. Pulse radar reflectometry has the greatest advantage that all density profiles can be probed at a high repetition rate, thus enabling the study of fast phenomena in the plasma [1]. However, time of flight measurements are presently possible at only few discrete frequencies (typically about 10 frequencies), which can lead to a bad spatial resolution of the reconstructed profile. In order to improve the profile determination, it is proposed here to use a dispersive effect of higher order, namely the pulse broadening. As dispersive effects decrease when the pulse spatial length becomes larger, there is an optimal length for the probing pulse which leads to a minimal reflected pulse length and so, to a minimized error on time of flight measurements. Moreover, additional information deduced from the optimal pulse length can be used for the density profile reconstruction. It is shown that the profile reconstruction can be improved by using both time of flight and optimal pulse length measurements. The effects of measurement errors are presented; it turns out that the reconstruction method based on the optimal length is more accurate in the presence of these errors. It is also discussed that FM-CW techniques allow, in principle, to evaluate the optimal pulse length by means of pulse compression method, thus enabling the use of dispersive effects to improve the profile determination.

2. Use of pulse broadening for density profile reconstruction
In pulse radar reflectometry, because of the short probing wave duration, a broadband frequency-spectrum is launched towards the plasma. Like for any other dispersive medium, the refractive index of the plasma depends on the wave frequency. Thus, all frequencies contained in the probing pulse propagate with different group velocities and the pulse can be significantly broadened in the plasma. For a gaussian shape of the incident wave, it can be shown that the reflected pulse length at half amplitude is equal to [2]:

\[ L_{\text{ref}} = L_{\text{inc}} \left[ 1 + \frac{8c^2 \ln 2 \varphi''(\omega_0)}{L_{\text{inc}}^2} \right]^{\frac{1}{2}} \]  

(1)

where \( L_{\text{inc}} \) is the incident pulse (spatial) length, \( c \) the speed of light in vacuum, \( \varphi'' \) the second derivative of the phase and \( \omega_0 \) the frequency of the probing wave. The pulse broadening caused by dispersive effects is related to the second term of the square root and so, depends on the second derivative of the phase. The validity of relation (1) implies that the dispersive effects must not be too strong, so that the reflected pulse keeps a gaussian shape. In order to
verify this condition, the width $\Delta f$ of the frequency-spectrum has to be much smaller than the probing frequency $f_0$, which is often true in pulse reflectometry experiments.

From the expression (1), it can be noticed that the reflected pulse presents a minimal length. The incident pulse length that leads to the minimal length of the reflected pulse is the so-called “optimal length”. An analytical expression of the optimal length for the incident pulse can be deduced from (1):

$$L_{\text{opt}}(f_0) = \sqrt{8c^2 \ln 2 \frac{\varphi'(\omega_0)}{\omega_0}}$$

(2)

The second derivative of the phase can be deduced from the optimal length measurement, thus providing an additional information on the density profile. Moreover, as the optimal pulse length gives a minimal length of the reflected pulse, the error on the time of flight is minimized.

From time of flight or optimal pulse length measurements alone, linear piecewise methods have been developed [3]. If pulse radar reflectometry allows simultaneously time of flight and optimal length measurements for different probing frequencies, a double information on the profile can be deduced. Then, the profile can be reconstructed by a parabolic piecewise method. Moreover, profile initialization can be made without any help from other diagnostics, thus allowing a complete profile determination only from reflectometry measurements.

The parabolic piecewise method has been tested on a large range of theoretical profiles of general form:

$$n_e(x) = n_0 \left(1 - \left(1 - \frac{x}{R}\right)^\alpha\right)^\beta$$

(3)

An example of density profile obtained by this method is compared to a profile reconstructed by a linear piecewise method on figure 1. As expected, a parabolic piecewise determination gives improved results, especially in regions where the density profile presents locally a strong curvature.

![Figure 1: Example of profiles reconstructed from linear and parabolic piecewise methods](image)

$n_0=6.10^{19} \text{ m}^{-3}$, $R=0.5 \text{ m}$, $\alpha=2$, $\beta=3$
3. Effects of reflectometry measurement errors on the profile determination

In reflectometry experiments, measurements are usually perturbed for several reasons. First of all, instrumental errors occur due to the electronic equipment and cannot be discriminated. Still the main cause of inaccuracy is induced by the plasma itself. In addition to plasma radiation which can affect the measurements, it is well-known that density fluctuations play an important role on reflectometry measurement perturbations. To check the sensitivity of the profile reconstruction to these various errors, arbitrary measurement errors have been introduced. Thus, errors up to 10% on the time of flight have been chosen to agree with typical predictions in reflectometry experiments. As no measurement of the optimal pulse length has been made yet in an experiment, errors up to 20% have been considered. The effects of such errors on both profile reconstruction and profile initialization are separately analyzed to evaluate their respective contribution.

First of all, errors on all points have been introduced, excepted for the three lowest frequencies so that the profile initialization is not affected. It has been noticed that errors on times of flight have the major contribution. However, the profile reconstruction remains acceptable for time of flight errors up to 10%. The role of optimal length errors on the reconstructed profile is insignificant even for errors up to 20%. In fact, the times of flight contribute strongly to the profile determination. The optimal lengths affect mainly the shape of the reconstructed profile. Nevertheless, the initialization can be quite improved by using optimal lengths. So, it is important to study the sensitivity of initialization profile in the presence of error measurements.

In order to study the error measurement effects on initialization profile, only errors on the time of flight and the optimal length for the lowest probing frequencies have been assumed. As expected, the profile initialization plays a crucial role on the accuracy of profile measurements. To show the interest of using the optimal length for the profile initialization, a comparison has been made between an usual Abel method using a parabolic extrapolation and the parabolic piecewise method with the optimal length data. Figure 2 shows the results for various samples where errors (± 10% on times of flight, ± 20% on optimal lengths) on the three lowest frequencies (Abel method) or the two lowest frequencies (parabolic piecewise method) have been introduced.

![Figure 2: Measurement error effects on profile initialization for an usual Abel inversion and a parabolic piecewise method](image)

It results that the method using both time of flight and pulse broadening generally leads to a better profile reconstruction. As expected, the reconstructed profiles becomes better and better for the higher densities, where the initialization contribution is negligible.
4. Determination of the optimal pulse length from FM-CW reflectometry data

FM-CW reflectometry can, in principle, be used to extract the pulse broadening by means of pulse compression techniques [4]. Indeed, with a heterodyne detection, it is possible to measure amplitude and phase of the reflected signal of a FM-CW reflectometer. Then, if an incident pulse is decomposed into several discrete frequencies, a FFT technique from the corresponding phase and amplitude permits to obtain the reflected pulse shape.

Simulations of pulse compression method have been made from a code solving the 1D Helmholtz equation. The phase measurements, which give sufficient information in the 1D case, have been realized for a set of discrete frequencies. Defining the $k$-spectrum of a gaussian incident pulse, a FFT with a standard zero padding technique is then used to obtain the reflected pulse shape. The frequency steps usually employed in FM-CW reflectometry (typically about 1-50 MHz) permit to reconstruct the reflected pulse with a good accuracy. As a consequence, the optimal pulse length can be extracted by sweeping the k-spectrum length. Figure 3 gives an example of optimal pulse length evaluation from 200 phase measurements over the probing frequency range 28-32 GHz.

![Figure 3: Pulse broadening calculated from a pulse compression technique for a density profile defined by $n_0=6.10^{19}$ m$^{-3}$, $r=0.7$ m, $\alpha=\beta=1$ and a probing frequency equal to 30 GHz.](image)

The parabolic piecewise method using both optimal lengths and times of flight can thus be used from FM-CW reflectometry measurements to improve the density profile determination, especially for the profile initialization problem.

References