Electron Heat Transport In Stochastic Magnetic Layer.

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1. Introduction.

The operation of ergodic divertor on tokamak Tore Supra permits power and particle control of the plasma boundary [1]. The Tore Supra ergodic divertor consists of six multipolar coils located at the low field side and distributed regularly in toroidal direction. The magnetic field perturbation generated by ergodic divertor contains a radial component, leading to the islands topology of the magnetic field. In the ergodic zone the islands overlap (Chirikov parameter: \( \sigma_{\text{Chir}} > 1 \)), creating the stochastic behaviour of the field lines at the plasma edge [2].

The perturbation spectrum can be presented as follows:

\[
\frac{\delta B_r}{B} = \sum_{m,n=\pm \infty} b(r) b_{mn} e^{i(m\theta + n\phi)}
\]

Here \( r \) is a magnetic surface label, \( \theta, \phi \) are the intrinsic poloidal and toroidal angles determined in the unperturbed equilibrium by condition: \( \frac{d\phi}{d\theta} = q(r) \), \( q \) is a safety factor, \( b(r) \) is exponentially decreasing function, being at the edge \( b(a) \sim 10^{-3} \). For typical Tore Supra ergodic divertor configuration: \( q(a) = 3 \), main poloidal and toroidal numbers are \( m = 18 \) and \( n = 6 \). For more details concerning ergodic divertor spectrum see [2].

It was shown theoretically that the perpendicular energy transport through the ergodic layer is amplified by the diffusion of the magnetic field lines, since the parallel transport has a radial component and acts in addition to the usual transverse transport [3,4,5]. According to the quasi-linear theory the temperature profile was expected to be flat, due to the large effective transverse diffusion at the edge [3,4]:

\[
\chi_{\text{erg}}^{-1} = \chi_{\|} \frac{D_{\text{FL}}}{L_T} \left( 1 + \frac{L_K}{L_T} \right) \gg \chi_{\perp}
\]

Here \( L_K \approx \frac{\pi q R_{\text{tor}}}{\pi \sigma_{\text{Chir}}} \) is a Kolmogorov length, \( L_T \approx L_K \log \left( \frac{\chi_{\|} D_{\text{FL}}}{\chi_{\perp} L_K} \right) \) is parallel correlation length and \( D_{\text{FL}} = \pi q R_{\text{tor}} b(r) b_{mn}^{2} \) is the quasi-linear field line diffusion coefficient on the resonant surface \( q = m/n \). If the heat flux from the central plasma is fixed, this decreasing of the edge temperature gradient would lead to the temperature degradation in the centre. In fact, it was shown experimentally on that the edge temperature is lower in the ergodic divertor configuration in comparison with the limiter one, but without central degradation of the electron temperature [2,5] (Fig.1). On the other hand the local temperature profile measurements with Langmuir probes on the tokamaks Tore Supra, TEXT and CSTN-II [2,5,6] demonstrated the existence of stationary periodic radial, poloidal and toroidal structures with the magnitude about 5-20eV, which are larger at the low densities (Fig.2). The small-scale temperature structures in the ergodic zone and barrier creation near separatrix could not be described in the terms of the effective ergodic thermal diffusion coefficient from the quasi-linear theory [3,4] because of the averaging over the ergodic zone. In order to describe this experimental facts the local heat transport is considered numerically, taking into account the magnetic field lines...
2.3-D non-linear heat transport modelling in ergodic zone.

3-D code ERGOT is based on the heat transport equation in the following form:

\[
\frac{\partial}{\partial t} \left( \frac{3}{2} n_e T \right) = \nabla \parallel (\chi \nabla \parallel T) + \nabla \perp (\chi \nabla \perp T) \tag{3}
\]

Here \( n_e \) -is a plasma density, T- electronic temperature, the parallel heat conductivity is \( \chi \parallel = 2 \times 10^{22} T_{eV}^{5/2} \text{[m}^{-1}\text{s}^{-1}] \) and the perpendicular one is taken here as \( \chi \perp \approx 1 \text{[m}^2\text{s}^{-1}] \cdot n_e \), operators \( \nabla \parallel \), \( \nabla \perp \) parallel and transverse to the magnetic field \( \vec{B} \) are given by:

\[
\nabla \parallel = \frac{\vec{B}}{B} \left[ \frac{\partial}{\partial r} + \frac{1}{R_{\text{tor}}} \left( -\frac{1}{q} \frac{\partial}{\partial \theta} + \frac{\partial}{\partial \phi} \right) \right]
\]

and \( \nabla \perp \approx \vec{v}_r \frac{\partial}{\partial r} \). Expanding the temperature field as the magnetic perturbation (1):

\[
T = \sum_{m,n=\pm \infty} T_{mn}(r)e^{i(m\theta+n\phi)}
\]

one can present (2) in the form of the system of equations for harmonics amplitudes, depended on \( r \), which is treated by finite differences in the region \( r: [r_{\text{min}},r_{\text{max}}] \), here safety factor is \( q(r_{\text{min}}) \approx 2, q(r_{\text{max}}) \approx 3 \).

Using the symmetry of the spectrum (1) [see 2,5] one can write:

\[
ib_{mn} = B_{-m-n} = -B_{mn} \quad \text{and} \quad T_{-m-n} = T_{mn}. \tag{4}
\]

The coefficients in (4) are:

\[
\gamma_1 = -\chi \parallel b(r) \left( \frac{1}{\chi} \frac{\partial}{\partial r} k_{m'n'} - \frac{m'}{R_{\text{tor}}} \frac{\partial}{\partial \theta} \left( \frac{1}{q} \right) \right) (B_{m-n} - B_{m+m'n'})
\]

\[
\gamma_2 = -\chi \parallel b(r) (k_{m+m'n+n'} B_{m-m-n'} + k_{m-m'n} B_{m+m'n+n'}) \tag{5}
\]
Here: $k_{mn} = \frac{1}{R_{\text{tor}}} \left( -\frac{m}{q} + n \right)$; $b_o^2 = 2 \sum_{m \geq 0, n \geq 0} |B_{mn}|^2$.

The stochastic properties of the magnetic field in the ergodic zone are introduced consistently in the heat transport equation (4) by non-linear coupling of the temperature perturbation modes, which appears due to the presence of the radial component of the magnetic field perturbation (1). The boundary conditions are: heat flux from the plasma core is fixed: $-\chi_{\perp} \frac{\partial T_{00}}{\partial r} \bigg|_{r_{\text{min}}} = \text{const}$ and at the edge typically: $T_{00} \big|_{r_{\text{max}}} \approx 45-10\text{eV}$, on both boundaries the temperature perturbations are considered to be small: $T_{mn} \big|_{r_{\text{min}},r_{\text{max}}} \approx 0$, since $T_{mn} << T_{00}$ and $T_{mn}$ is localised near the surface $q=m/n$. The linear density profile is taken here for simulations ($n_e = 2.5 - 1.1 \times 10^{19} \text{m}^{-3}$). The numerical method developed for solving non-linear system is based on the implicit time-relaxation from initial temperature profile to the stationary one, using the inversion of multi-block matrix on each time-step.

3. Numerical results

The present numerical simulations reproduce the averaged temperature profile flattening in the ergodic zone, without degradation in the plasma centre, the fact observed in Tore Supra experiments (Fig 3). Taking the averaged radial heat flux as: $<Q_{\perp}>_{\theta,\phi} = -\chi_{\text{erg}}^{\text{num}} \frac{\partial T_{00}}{\partial r} = \text{const}$ one can estimate the effective transport coefficients from numerical solution. The amplification of the transverse heat transport conductivity in the ergodic zone was estimated as $\chi_{\text{erg}} / \chi_{\perp} \sim 50$ for Chirikov parameter $\sigma(a) \sim 3$ and edge temperature $\sim 45\text{eV}$ (Fig.4). The comparison with quasi-linear effective coefficient (2) is also presented on Fig.4.

Each temperature perturbation harmonic is localised of the corresponding resonant surface: $q=m/n$, which explains radial modulations on the temperature field (Fig.5-8). Notice, however, that in the simulations the radial modulations ($\sim 3-5 \text{eV}$) are smaller then the experimentally observed ($5-20\text{eV}$). The poloidal and toroidal modulations exhibit the same periodicity as the magnetic perturbation on the rational surface: $q=m/n$. 

The 3-D non-linear ERGOT code was developed for heat transport modelling taking into account the stochastic properties of the magnetic field, which are introduced consistently in the heat transport equation by non-linear coupling of the temperature perturbation modes governed by the radial component of the magnetic field perturbation of ergodic divertor. The present numerical results show the edge temperature profile flattening in ergodic zone without degradation of the central temperature, and the existents of poloidal, toroidal and radial small scale temperature field modulations, the features reported in the Tore Supra ergodic divertor experiments.

References.