Neoclassical Transport in Helical Magnetic Axis System
in the Low-collisionality \((1/\nu)\) Regime

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Abstract

The neoclassical transport in the L=1 helical axis stellarator is investigated. The effective
toroidal curvature term \(\epsilon_T\) defined as the sum of usual toroidal curvature and one of the
nearest satellite harmonics of helical field, determines the confinement properties of localized
trapped particle. The reduced \(\epsilon_T\) configuration which is attained in negatively pitch-
modulated L=1 torsatron, is found to improve the collisionless and \(1/\nu\) regime collisional
particle confinement.

1. Introduction

There exist two characteristic transport features appropriate to 3-D helical magnetic axis
system. The first is the formation of the largest magnetic islands at the lowest-order rational
surfaces because they couple nonlinearly most readily to the nonresonant vacuum magnetic
Fourier components, helical magnetic axis field and toroidal field, which cause indirect
resonant pressure driven currents at every rational surface and form the islands. The width of
island decreases with increasing \(N\) (field period number). The second is the role of the
effective toroidal curvature term \(\epsilon_T\) for localized trapped particles, which determines the
collisionless confinement conditions of helically trapped particles. This paper describes the
neoclassical transport in the \(1/\nu\) collisionality regime for the reduced \(\epsilon_T\) configuration of
negatively pitch-modulated L=1 helical magnetic axis torsatron with \(N=17\), coil aspect
ratio \(R/a \sim 8.4\), in which helically trapped particles are completely collisionless confined.

2. Effective Toroidal Curvature

The magnetic field strength \(B\) in magnetic coordinates \((\psi, \theta, \phi)\) on a given flux surface
\(\psi = \text{const.}\) is represented by its Fourier components \(B_{n,m}(\psi)\) as follows,

\[
B(\psi, \theta, \phi) = B_{0,0} + 2 \sum_{(n,m) \neq (0,0)} B_{n,m}(\psi) \cos(n\phi - m\theta).
\]
With the rotational transform \(\pi/2 \ll N\) in present case, we can set \(\Theta \sim N\Phi\) [1,2], so that the main L=1 field is rewritten by
\[
B \sim B_{0,0} [1 + \varepsilon_r \cos \Theta + \varepsilon_L \cos(N\Phi - \Theta) + \cdots].
\]
Here, \(\varepsilon_r\) is defined as \(2(B_{0,1} + B_{N,0})/B_{0,0}\), which gives an effective toroidal curvature term for localized trapped particles rather than usual toroidal curvature term \(\varepsilon_t(\equiv 2B_{0,-1}/B_{0,0} < 0)\). The term \(\varepsilon_L\) represents \(2B_{N,1}/B_{0,0}\). Defining \(\varepsilon_0\) as \(2B_{N,0}/B_{0,0}\), we obtain the relation \(\varepsilon_r = \varepsilon_t + \varepsilon_0\). These field components for \(\alpha^* = -0.2\) (coil winding law \(\Theta = N\Phi + \alpha^* \sin N\Phi\)) are shown in Fig.1, where \(\psi\) is normalized to the outermost flux surface \(\psi = 1\). It is noteworthy that \(\varepsilon_0\) is positive in the case of \(\alpha^* = -0.2\), hence \(\varepsilon_r\) becomes small. We have reported the collisionless localized trapped particle confinement is improved by this term[2], and the pitch-modulated L=1 system becomes near omnigenous system[3,4].

3. Neoclassical Transport

When we consider the collisional plasma, the \(1/\nu\) collisionality regime, where the effective collision frequency \(\nu_{\text{eff}}\) of the helically trapped particles is less than their bounce frequency \(\omega_b\) and more than the poloidal drift frequency \(\omega_d\) (\(\omega_b > \nu_{\text{eff}} > \omega_d\)), is characteristic for standard stellarators due to the symmetry break effect of satellite harmonics \(B_{N,0}\) and \(B_{N,2}\). In this regime, both particle and heat fluxes are proportional to the neoclassical transport surface integral \(S\) [5], which depends only on the geometric parameters \(\varepsilon_r\), \(\varepsilon_L\) and \(\varepsilon_{L,i}\) (\(i \neq 0\), satellite harmonics). Figure 2 shows that the transport of the \(\alpha^* = -0.2\) case is the smallest of the three \(\alpha^*\) cases in the core plasma region (\(\psi < 0.5\)). We can see also the enlargement of radial transport in the outer region (\(\psi > 0.5\)). This causes mainly by the \(\varepsilon_2(\equiv 2B_{N,2}/B_{0,0})\) term as described in the next section, but the total loss is insignificant by consideration of the particle density in the outer region.

3. Effects of Satellite Term on the Radial Transport

These facts suggest the important role of \(\varepsilon_r\) on \(S\) even in the presence of collisionality. One of the nearest satellite harmonics of main helical field \(B_{N,1}\), \(B_{N,0}\) has been reported to contribute to the reduction of \(\varepsilon_r\), and lead to the good collisionless confinement of helically trapped particles. On the other hand, the another nearest satellite harmonic \(B_{N,2}\) enhances the
drift width of helically trapped particles by a factor of \( \sim 3 \) in maximum (for deeply trapped particles) compared with that without \( B_{N,2} \) harmonic, although it does not give any effect on the confinement conditions of helically trapped particles. This enhancement of drift width suggests the degradation of diffusion coefficient. Figure 3 shows the level contours of \( S(\varepsilon_r,\varepsilon_0) \) at \( \psi \sim 0.5 \) in \( \alpha^* = -0.2 \) case. Though the parameters \( \varepsilon_r \) and \( \varepsilon_0 \) are dependent each other in the real system, these are changed independently with keeping the other Fourier components. It is noticed that the level of the real \( \alpha^* = -0.2 \) system is near the lowest level of these hypothetical systems, and the region of the low level is along the line \( \varepsilon_0 \approx -\varepsilon_r \), which is corresponded to \( \varepsilon_r \approx 0 \). These results means the role of \( \varepsilon_r \) on the transport integral even in the presence of collision, and the significantly reduced \( \varepsilon_T \), \( L=1 \) configuration (\( \alpha^* = -0.2 \)) is near the lowest \( S \) region (Fig.3). We evaluate \( S \) in three configuration models for \( \alpha^* = -0.2 \) case. Those are the single helicity model including only \( \varepsilon_r, \varepsilon_{L(=1)} \) terms, the two helicities model including \( \varepsilon_r, \varepsilon_1, \varepsilon_0 \) terms, and the multi helicities model including all helical magnetic Fourier terms. In case of the single helicity model, the value of \( S \) becomes slightly larger than the multi helicities model. On the contrary, the two helicities model reduces the value significantly. As the results of the research concerning \( S \), the two satellite harmonics of \( L=1 \) helical field affect the transport in the \( 1/\nu \) collisionality regime so that \( \varepsilon_0 \) makes it decrease by the reduction of \( \varepsilon_T \) combining with \( \varepsilon_r \) even in the presence of collision, and \( \varepsilon_2 \) increase(decrease) when \( \varepsilon_2 < 0 \)(when \( \varepsilon_2 > 0 \)). Its deviation from the lowest \( S \) value is mainly due to \( \varepsilon_2 \) harmonic and the increasing factor of \( S \) is \( \sim 3.0 \) on \( \psi \sim 0.5 \) due to the negative value of \( \varepsilon_2 \) in case of \( \alpha^* = -0.2 \) as shown in Fig.4.

4. Conclusion

In conclusion, the reduced \( \varepsilon_T \) configuration is found to be good radial transport properties even if collisonal regime. The two satellite harmonics of \( L=1 \) helical field affect the transport in \( 1/\nu \) collisionality regime so that \( \varepsilon_0 \) makes it decrease by the reduction of \( \varepsilon_T \), combining with \( \varepsilon_r \), and \( \varepsilon_2 \) increase when \( \varepsilon_2 \) is negative. Our new approach would give rise to the possibility of stellarator design study in a wider parameter domain than quasisymmetry approaches.
Fig. 1 The main field components and the effective Toroidal curvature $\varepsilon_T$.

Fig. 3 The contours of transport integral $S$.

Fig. 2 The transport integral $S$ versus $\psi$. The three pitch-modulation cases are shown.

Fig. 4 The transport integral $S$ versus $\psi$. The $\varepsilon_2$ term increase the radial transport.

References